

# Mathematical Modelling For EEG Source Localization Using Subspace Method

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**Abstract:** A subspace based source localization method is proposed to estimate the location of sources of electrical activity within brain from EEG signals recorded by an array of sensors. Main intention is to localize sources with a minimum of localization error. The proposed method here is the Multiple Signal Classification (MUSIC) algorithm that locates multiple dipole sources from EEG data. In this, multiple dipole locations are found by scanning potential locations using a simple one dipole model. This method provides high spatial resolution than conventional methods.

**Keywords:** subspace source localization, MUSIC, subspace, array manifold, EEG, snapshot

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## 1. INTRODUCTION

Electroencephalography (EEG) source localization is an inverse problem. In both neuroscience research [1] and clinical applications [2], location of sources of electric activity within the brain is of great interest. Source localization plays an important role in location of origins of neurological disorders. Locating the neural electrical activities effectively and precisely is a critical issue for clinical neurology and neuroscience and it is a challenging problem. This uses voltage potential measurements at various locations on the scalp and then applies signal processing techniques to estimate the current sources inside the brain that best fit this data. The neural activity can be modelled as currents. Localization problem consists of two sub problems. First one is the forward problem which determines the electric field at the scalp from known sources and volume conductor. Second one is the inverse problem which estimates the location of sources in the brain from recorded EEG signals. For estimating the location of sources of electrical activity in the brain, first up all a source and head model needs to be assumed [4]. This problem has no unique solution. That means different source configurations can produce same potential distribution on the scalp. So the problem

is ill-posed. The estimation of sources becomes possible if some priori assumptions are made. It should be physiologically reasonable. By knowing from where and how the signals are generated inside the brain these assumptions can be introduced. It is up to the user to decide if the constraints used in the inverse solution are physiologically reasonable. The inverse solution will be different if different head or source model is used. Head model should be chosen carefully. Head model describes the varying conductivity layers in the head.

## 2. METHODOLOGY

A mathematical representation of the data received by the electrode array is the data model. Source localization consists of forward and inverse problem. First forward model that is the electrode potential needs to be obtained. Then the source model and head model has to be assumed. The next step is to calculate the inverse solution for the location of the sources by using the potential obtained and volume conductor assumed. Here, multiple signal classification (MUSIC) algorithm [3] is proposed for source localization. This is one of the first subspace source localization algorithms.

This gives high spatial resolution between closely spaced dipoles.

### 3. MUSIC ALGORITHM

MUSIC is a subspace based method. The subspace based methods became more prominent among the dipole source localization algorithms. It has the ability to distinguish between more closely spaced sources. This is a new approach to three-dimensional (3D) dipole source localization. These subspace-based methods will perform certain projections onto the estimated noise-only subspace to find the location of sources as the peaks.

In this a single dipole is scanned through a grid. Here, a 3-D head or source volume is confined as a grid. The forward model for a dipole at each location of this grid is calculated and is projected against a noise-only subspace. After performing all the projections, find out the location on the grid where the forward model gives the better projection onto the noise-only subspace. That location on the grid will give the source or dipole location. To get the orientation of the dipole, it is not necessary to find out all possible dipole orientation at each location. The solution of a generalized eigen value problem will give the best orientation of the dipole.

### 4. BASIC STEPS IN MUSIC ALGORITHM

The basic idea of MUSIC is that the eigenvalues and eigenvectors of a signal covariance matrix are used to estimate the location of multiple signals received by the sensor array. Flowchart for steps in MUSIC algorithm is given in Figure 1.

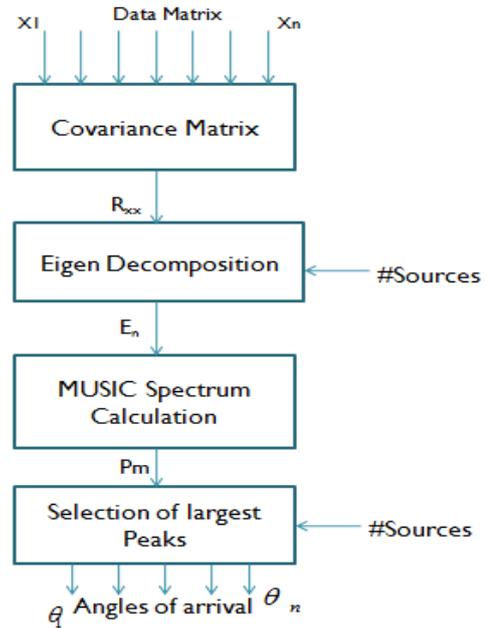


Figure 1: The Basic Steps In MUSIC Algorithm.

#### A. Calculation of Covariance Matrix From Data Matrix

A covariance matrix can be calculated by multiplying a time sample of the data matrix and the Hermitian adjoint of that time sample. This product value can be taken over a number of time samples to reduce the disturbing effect of the noise. A covariance matrix is calculated by:

$$R_{xx} = 1/m \sum_{i=1}^m x_i x_i^H$$

Where  $R_{xx}$  is the  $n$  by  $n$  covariance matrix,  $X_i$  is the  $i^{\text{th}}$  snapshot containing  $n$  elements, and  $m$  is the number of snapshots.

#### B. Eigen value decomposition of the covariance matrix

An eigen decomposition is used to determine the eigen values and eigenvectors of the covariance matrix. Let  $k$  be the number of sources and  $n$  be the number of electrodes in the sensor array. Then,  $k$  largest eigen values and their corresponding eigenvectors are assigned to the sources and the  $n-k$

other (smallest) eigen values and their corresponding eigenvectors are assigned to the noise. The eigenvectors assigned to the noise are combined in matrix  $E_n$  (the noise subspace). Each eigenvector is a column in  $E_n$ . The number of snapshots used to calculate the covariance matrix determines the accuracy of the eigen values and eigenvectors.

If an infinite number of snapshots are used, the eigen values of the noise converge to the same value, namely the variance of the noise. Otherwise, for determining the number of sources threshold value can be used. The SNR determines the level of the threshold value. The number of eigen values greater than the threshold value determines the number of sources.

### C. MUSIC Spectrum Calculation

After the eigen decomposition, the MUSIC spectrum is calculated. This is done by:

$$P_m(\theta) = \frac{1}{a^H(\theta) E_n E_n^H a(\theta)}$$

where  $P_m(\theta)$  is the measure for the MUSIC spectrum,  $E_n$  is the noise subspace and  $a(\theta)$  is a steering vector of the array manifold. Predefined steering vectors are stored in an array manifold. An angle in the MUSIC spectrum is represented by these steering vectors. The steering vectors of the array manifold are same as a steering vector of a source. For every vector in the array manifold  $P_m(\theta)$  is calculated. The maximal spatial resolution of the MUSIC algorithm is determined by the angle between two adjacent steering vectors in the array manifold. To reduce the number of computations, from the array manifold all the vectors other than the area of interest can be removed.

### D. Selection of Largest Peaks

The last step in the MUSIC algorithm is the selection of the k largest peaks in the MUSIC spectrum. The denominator of the equation for

spectrum calculation is an inner product. The steering vectors are projected against the noise subspace. When a steering vector is orthogonal to the noise subspace the denominator of the spectrum equation becomes zero. This gives infinity as the answer of spectrum equation. Therefore, the peaks in the spectrum give the source locations.

## 5. REDUCING THE NUMBER OF SNAPSHOTS

To perform the source localization in a successful manner the signals should not be correlated. No peaks should be appeared in the spectrum if all the signals are fully correlated. The correlation between the signals is determined by the number of time samples used to estimate the covariance matrix. Signals become more correlated if small number of snapshots is used. In MUSIC, size of all the peaks is determined by the level of correlation between the signals. If the level of correlation is low, high peaks will appear in the spectrum and vice versa.

## 6. RESULTS AND DISCUSSIONS

A model of MUSIC algorithm is made in MATLAB to analyze the performance on the source localization.

### A. The MUSIC spectrum

Three sources are assumed. MUSIC spectrum is obtained. The Figure.2 shows the MUSIC spectrum with three sources located at -30, 35 and 45 degrees.

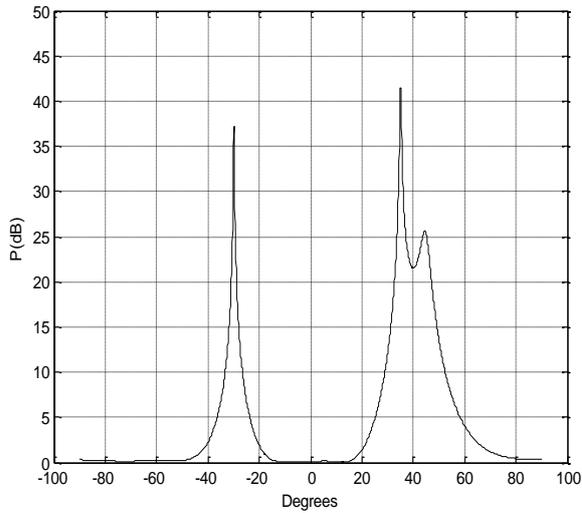


Figure 2: The MUSIC spectrum

**B. Electrode model**

For the 10-20 system of Electrode placement, sensors coordinates are calculated. The radius of the sphere is taken as 10cm. The spherical view of the model is shown in Figure 3 and plan view is shown in Figure 4.

The sensors placement is shown in Figure 4. From left to right along coronal plane: T3, C3, C4, T4. From the nasion to the inion along sagittal plane: F0, C0, P0. From right to left FP1, F7, T3, T5, O1, O2, T8, T4, F8, FP2. Frontal plane: F4, F3. Parietal plane: P4, P3. Fp, F, C, T, P, O, A indicates frontal pole, frontal, central, temporal, parietal, occipital, ear lobe respectively.

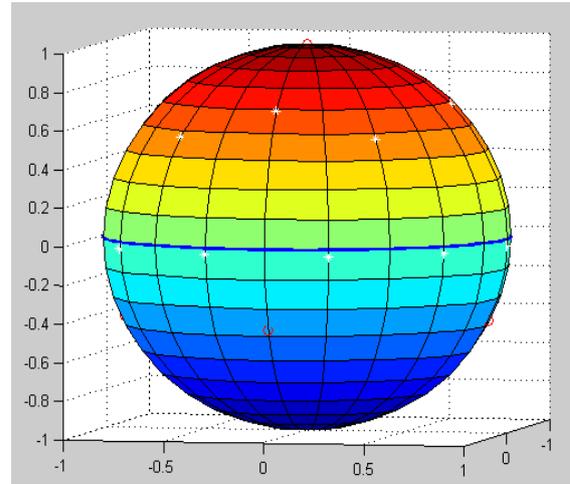


Figure 3: Spherical view

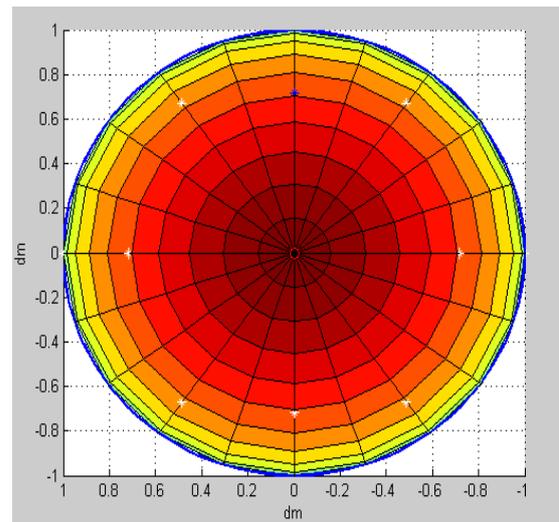


Figure 4: Plan view

**C. Effect Of Number Of Snapshots**

To find out the effect of different numbers of snapshots, EEG data base of a particular subject is selected. The data base is consisted of data of five different tasks namely baseline, multiplication, letter composing, rotating, counting. One snapshot of this data is selected and covariance matrix is calculated. The MUSIC algorithm is applied to the obtained matrix. The number of sources is assumed

as three. When this is simulated in MATLAB different sources could be detected at different spatial frequencies for each task. This result is shown in Figure 5.

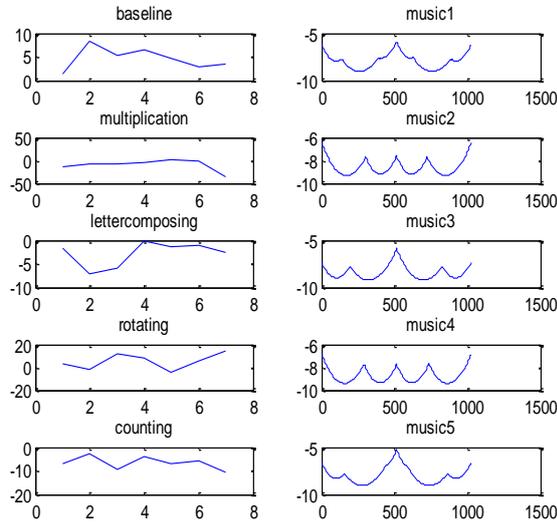


Figure 5: MUSIC simulation result with only one snapshot.

In this, the signals seem to be more correlated since only one snapshot is used. When the number of snapshots is increased to 100, sources became more prominent. That is, the correlation between the signals became less. So it results in high peaks at three frequencies, which gives the location of sources of three different signals. This result is shown in Figure 6.

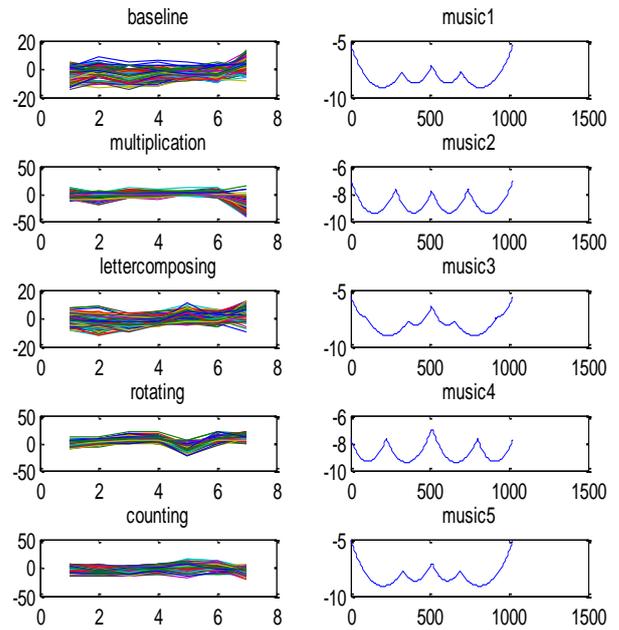


Figure 6: MUSIC simulation result with 100 snapshots.

## 7. CONCLUSION

A subspace based method; multiple signal classification algorithm is presented for locating the number of sources assumed in the data. The three sources could be located with high resolution. The electrodes coordinates are calculated and an electrode model is designed. The MUSIC algorithm simulation is performed for different snapshots. It is observed that as the number of snapshots is increased the peaks became more prominent and resolvability is increased.

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