Abstract - Orthogonal Frequency Division Multiplexing (OFDM) is a popular method for high data rate wireless transmission. It can be thought of as a combination of modulation and multiple access schemes that segments a communication channel in such a way that many users can share it. It segments according to frequency. It is a technique that divides a spectrum into a number of equally spaced tones, and carries a portion of user’s information on each tone. In the most basic form a tone may be present or disabled to indicate a one or zero bit of information, however, either Phase Shift Keying or Quadrature Amplitude Modulation is typically employed. An OFDM system takes a data stream and splits it into n parallel data streams each at a rate of 1/n of original rate. Each stream is then mapped to a tone which is at a unique frequency and combined together using the Inverse Fast Fourier Transform to obtain the time domain waveform to be transmitted. This requires Antenna arrays at the transmitter and receiver to enhance the system capacity on frequency selective channels resulting in a Multiple Input Multiple Output (MIMO) configuration. As there are several antennas at receiver and transmitter, MIMO systems can be employed for diversity. This spatial multiplexing method transmits several parallel information streams at same transmit power. A number of design trade-offs must be considered when developing an OFDM based system. This paper explores MIMO system model, MIMO receivers, SVD of MIMO channel and system capacity, Beam forming techniques, OFDM system design using IFFT with MCM technique, OFDM with Cyclic Prefix, method of MIMO-OFDM system design including physical channel measurements, space time coding techniques, frequency synchronization, and finally types of distortions in OFDM technique and possible remedies to avoid those distortions.

Key words: Key words --- Multiple Input Multiple Output system (MIMO), Orthogonal Frequency Division Multiplexing (OFDM), space-time coding, Cyclic Prefix, Types of distortions in OFDM.

I. INTRODUCTION:
The key objective of future wireless communication systems is to transmit information streams at a high data rate maintaining its high quality as well like video calling. This is the basis for all the new 3G & 4G wireless communication systems. The drawbacks in narrow band system like GSM (Global system for Mobile Communications) and wide band system like CDMA is that GSM uses Narrowband channels in which frequency reuse is not possible which leads to inefficient use of total bandwidth. Total network is decomposed into high SINR point-to-point links because of which adjacent cells cannot be assigned the same channel. In CDMA the frequency reuse problem of narrowband system is fixed i.e. all users, both within a cell and across different cells, transmit and receive on the entire bandwidth. The signal of each user is modulated onto a pseudo noise sequence so that it appears as white noise to other users. The interference due to universal frequency re-use in CDMA is managed via averaging of the effects of multiple interferences which also allows statistical multiplexing of users and thus increase in the system capacity. The below mentioned two schemes in combination gives a new technology making use of advantages of the above mentioned old technologies.

Multiple Input Multiple Output (MIMO) Systems have multiple transmit antennas and receive antennas having same Transmitter and Receiver. It transmits Several Information streams in parallel in free space so called Spatial Multiplexing. MIMO system plays a key role in 3G & 4G wireless systems. MIMO can increase data rate by transmitting several symbols in parallel. Singular Value Decomposition of MIMO channel solves the problem of interfering all symbols at every receiver.

Orthogonal Frequency Division Multiplexing (OFDM) is a multi-carrier modulation technique. It splits the transmitted data into several interleaved lower rate data streams and transmits them in parallel by using several narrowband sub-carriers. The advantage of OFDM over Frequency Division Multiplexing (FDM) is that sub-carriers are Orthogonal to each other. Because of Orthogonality, Inter-carrier Interference is avoided, spectral overlap is possible and spectral efficiency is enhanced. OFDM converts frequency selective channel into parallel flat fading channels. OFDM does not require guard bands between sub-carriers.

MIMO-OFDM is a combination of MIMO communication with OFDM. MIMO-OFDM converts MIMO frequency selective channel into set of parallel flat fading MIMO channels maintaining universal frequency reuse.
II. MIMO:

MIMO systems are a special class of wireless communication systems which has multiple antennas at transmitter as well as receiver. This system can be employed for diversity gain as it uses multiple antennas in data transmission and reception. MIMO became the key concept for basis of 3G & 4G systems due to its unique SPATIAL MULTIPLEXING feature. Spatial Multiplexing is a process in which we can increase the data rate by transmitting several information streams in parallel and at same transmit power of SISO. This is possible through multidimensional signal processing.

III. MIMO SYSTEM MODEL:

The modelling of a practical MIMO channel includes the transmit vector, receive vector, channel matrix and most importantly the inherent property of every wireless channel, Noise. The MIMO system can be characterized by following model:

$$\begin{bmatrix}
 y_1 \\
 y_2 \\
 \vdots \\
 y_r
\end{bmatrix} = \begin{bmatrix}
 h_{11} & h_{12} & \ldots & h_{1t} \\
 h_{21} & h_{22} & \ldots & h_{2t} \\
 \vdots & \vdots & \ddots & \vdots \\
 h_{r1} & h_{r2} & \ldots & h_{rt}
\end{bmatrix} \begin{bmatrix}
 x_1 \\
 x_2 \\
 \vdots \\
 x_t
\end{bmatrix} + \begin{bmatrix}
 n_1 \\
 n_2 \\
 \vdots \\
 n_r
\end{bmatrix}$$

Or equivalently

MIMO vector equation: $$\hat{y} = Hx + \hat{n}$$

Here we will assume that noise component at each pair of receive antennas is uncorrelated that is the noise is white across space and also assume it to be uncorrelated over time which results in a spatio-temporally white noise. The covariance at receiver thus can be given as:

$$R_n = E(\hat{n}\cdot\hat{n}^H) = \sigma_n^2[I]$$

IV. MIMO EQUALIZERS (RECEIVERS):

At the receiver, before demodulating the received signal we have to perform the equalization process. The goal of equalization is to mitigate the effects of Inter Symbol Interference (ISI) which is due to spatial diversity of the MIMO channel. Equalization can be done both linearly and non-linearly. In the Linear category we have 2 basic equalizing algorithms 1: Zero Forcing (ZF-equalizer) 2: Minimum Mean Square Error (MMSE equalizer).

A. ZF-Equalizer:

This equalizer chooses the minimum deterministic squared error vector amongst all possible transmit vectors \(\hat{x}\). I.e. norm of error \(||y-H\hat{x}|||^2\) should be minimized and from the vector differentiation techniques approximate error minimizing transmit vector can be given as:

$$\hat{x} = ((H^H H)^{-1} H^H) \hat{y}$$

Which is nothing but performing multiplication operation of a pseudo inverse of channel matrix with received vector. This can successfully mitigate the ISI but it results in enormous noise enhancement. Since the new noise vector is

$$\hat{n} = ((H^H H)^{-1} H^H)^{-1} \hat{y}$$

So, if the channel transfer function is highly attenuated at any frequency within bandwidth of interest, as is common in frequency selective channels, noise power will be significantly increased. We need an equalizer that better optimizes between ISI mitigation and noise enhancement. One such equalizer is MMSE-Equalizer.

B. MMSE-Equalizer:

This equalizer is designed such that it minimizes the average mean square error between transmitted symbol \(x\) and its estimate \(\hat{x}\) at output of equalizer. i.e. Now the problem is

Minimizing \(E(||x - \hat{x}||^2)\) and the resulting estimating vector is \(\hat{x}_{\text{MMSE}} = P_d (P_d H^H H + \sigma_n^2[I])^{-1} H^H \hat{y}\)

Where \(P_d\) is the symbol power. In this case as \(h>0\) the estimated transmit symbol value will be bounded only. Thus MMSE estimator is ROBUST to noise. More over at high SNR applications it works as a ZF-receiver only as its noise component will be dominated by the symbol power. And at low SNR it implies a matched filter with transfer function \(H^H\).

V. SINGULAR VALUE DECOMPOSITION (SVD) OF MIMO:

Unlike SISO, Multiple antenna structures like MISO or SIMO employ in diversity gain. In addition to this, in MIMO systems there is other kind of gain called multiplexing gain. It is because a MIMO channel can be decomposed into certain number of independent channels. By sending independent information streams through these independent channels simultaneously we can obtain the multiplexing gain. So, SVD is a key
concept of MIMO. A matrix $\mathbf{H}$ of any order can have a SVD as follows: $\mathbf{H} = \mathbf{U} \Sigma \mathbf{V}^H$

To have SVD we assume that the channel gain matrix $\mathbf{H}$ is known at the transmitter and also at the receiver. SVD follows conversions at channel input and receiver input through transmit precoding and receiver shaping.

$$\hat{y} = \mathbf{H}\hat{x} + \hat{n}$$
$$\tilde{y} = \mathbf{U} \Sigma \mathbf{V}^H \hat{x} + \tilde{n}$$

Applying $\mathbf{U}^H$ both sides:

$$\mathbf{U}^H \hat{y} = \mathbf{U}^H \mathbf{U} \Sigma \mathbf{V}^H \hat{x} + \mathbf{U}^H \hat{n}$$

Now the new channel model is:

$$\tilde{y} = \sum \hat{x} + \tilde{n}$$

Where transmit pre-coding is $\hat{x} = \mathbf{V}^H \hat{x}$ and receiver shaping is $\tilde{y} = \mathbf{U}^H \tilde{y}$ and $\sum$ is the diagonal matrix of singular values of $\mathbf{H}$ with $\sigma_i$ on the $i^{th}$ diagonal. Now the channel looks as:

![Figure: Parallel Decomposition of MIMO-channel](image)

So, unlike earlier MIMO channel, now there is no interference of transmitted symbols at receivers. More over the noise is appeared to be changed in terms of power but actually there is no change in noise power even after this beam forming because of the identity property: $\mathbf{U}^H \mathbf{U} = [I]$.

Now from the new channel model equation, SNR of the $i^{th}$ stream is $\frac{P_i \sigma_i^2}{\sigma_n^2}$. Maximum rate at which information can be transmitted is called Shannon capacity and is given by $\log_2 \left(1 + \frac{P_i \sigma_i^2}{\sigma_n^2}\right)$. Thus the NET MIMO capacity is given by

$$\text{NET MIMO capacity is}$$

$$\sum_{i=1}^{t} \log_2 \left(1 + \frac{P_i \sigma_i^2}{\sigma_n^2}\right)$$

The capacity is achieved and the next problem is how to allocate power to these $t$ independent information streams. This comes under optimal MIMO power allocation problem which is our next topic of discussion.

VI. OPTIMAL POWER ALLOCATION FOR CAPACITY MAXIMIZATION:

(Channel State Information (CSI) is known at receiver)

We have the capacity obtained by SVD of MIMO channel. In order to maximize it, we have a constraint on its maximization that is the power must be allocated at most to $t$ independent SV Decomposed channels only. i.e. $\sum_{i=1}^{t} P_i = P_t$.

The solution for this is famous as WATER-FILLING algorithm. This algorithm follows an equation to allocate power to the $t$ streams. $P_i = \left(\frac{1}{\lambda_i} - \frac{\sigma_n^2}{\sigma_i^2}\right)^+$ where $1/\lambda_i$ is a cut-off power level to allocate power in a stream and $\left(\frac{\sigma_n^2}{\sigma_i^2}\right)^+$ is inverse of instantaneous SNR of $i^{th}$ channel. This takes advantage of good channel conditions. When channel conditions are good maximum power is allocated to that channel and vice versa. When the SNR of channel falls below the cut-off level the channel is not used which makes this algorithm non-linear.

![Figure: Water-filling model for optimal power allocation](image)

VII. SPACE-TIME CODES:

(CSI is unknown at transmitter)

Unlike SIMO, MIMO needs CSI at transmitter to have transmit beam forming. But Orthogonal Space Time Block Code (OSTBC) with MIMO-OFDM has drawn much attention because it achieves maximum transmit diversity without CSI. One such basic OSTBC code is ALAMOUTI CODE.

A. ALAMOUTI-OSTBC Code:

It is a space time code proposed for $1 \times 2$ wireless systems. It achieves a maximum diversity order of 2. In alamouti encoder we send two transmit symbols $x_1, x_2$ with the following space-time code matrix: $\tilde{X} = \begin{pmatrix} x_1 & -x_2 \\ x_2 & x_1 \end{pmatrix}$. This alamouti encoded signal is transmitted from the two transmit antennas in two symbol times. So effectively we are sending two symbols at one time instant. That is why it is called a FULL-RATE code with $R=1$. In this kind of receiver the SNR found at the receiver is $\frac{\|h\|^2 P}{\sigma_n^2}$ which gives the transmit diversity of order 2. But for that it pays a factor of ½ power loss i.e. 3-dB loss in SNR.
VIII. NON-LINEAR MIMO RECEIVER: V-BLAST:

**V-BLAST** is a short form for Vertical Bell labs Layered Space-Time architecture. This employs in Successive Interference Cancellation (SIC). We remove impact of each estimated symbol on receive vector successively. This uses left inverse of channel matrix to decode each transmitted symbol as in ZF receiver but removal each decoded symbol makes it non-linear. Though it became non-linear at the end the r×t channel becomes r×1 channel as we remove remaining t-1 channels' effect after decoding them. As the channel is now of order r×1 it can give us diversity order of r which is much higher compared to linear Zero Forcing receiver.

IX. MIMO-BEAM FORMING:

In conventional antennae structures, beam forming refers to a technique where we can have an antenna array and we are steering a beam in certain direction. Here in MIMO beam forming refers to transmission in one spatial dimension that is in a certain abstract direction. This scheme of MIMO beam forming is termed as Maximal Ratio Transmission (MRT) and it results in simplistic transmission and reception compared to all linear and non-linear estimators. From the SVD of channel:

\[\mathbf{y} = \mathbf{U}\mathbf{S}\mathbf{V}^H + \mathbf{n}\]

Make \(\mathbf{x} = v_1\mathbf{y}\) where \(v_1\) is the dominant transmit direction in n-dimension space.

\[\mathbf{y} = \sigma_1v_1\mathbf{u}_1^T + \mathbf{n}\]

So, this gain \(\sigma_1\) is the largest possible gain in transmitting first transmit symbol which effectively becomes a SISO channel which is a simplistic transmission. With different dominant transmit modes for different transmit vectors the transmission and reception is as simple as in SISO but also with complete \(rt\) diversity order.

V. OFDM (Orthogonal Frequency Division Multiplexing)

It forms the basis for 4G wireless Communication Systems. It is a Broadband Wireless Technology which supports data rates in excess of 100 Mbps.

**Basic Idea of OFDM** is ------ Let the two sided Bandwidth available for Communication is \(B\). Divide that Bandwidth into narrow bands of bandwidth \(\frac{B}{N}\) where \(N\) is the number of Sub-carriers that is number of data streams required to be transmitted over channel.

In Single Carrier system, Single Carrier Occupies the entire bandwidth \(B\), whereas in this system on \(N\)-sub carriers \(N\)-data streams can be transmitted. This type of transmission is known as Multi-carrier Transmission or Multi-carrier Modulation (MCM).

X. MULTI-CARRIER MODULATION (MCM):

Let \(X_i\) is data transmitted on \(i^{th}\) subcarrier through the process of PSK modulation then \(S_i\) is transmitted symbol from \(i^{th}\) transmit antenna, that is \(S_i(t) = X_ie^{j2\pi ftB}\) and composite signal comprising \(N\) different streams is,

\[S(t) = \sum_i X_i e^{j2\pi ftB}\]

**SCHEMATIC DIAGRAM of MCM:**

B. TRANSMITTER and RECEIVER SCHEMATIC are
D. ADVANTAGES of MCM:

- If the Modulation Bandwidth is more than Coherence Bandwidth of the channel it causes inter-symbol interference (ISI) leads to fading that is energy from one symbol goes to another. In Single Carrier System the effect of ISI is more whereas MCM gives frequency flat-fading channels results to reduce ISI.
- Channel is free from frequency selective.
- Both MCM and SCM are exactly same in symbol throughputs.
- High Data rate than SCM system

E. PROBLEMS with MCM:

As the number of Subcarriers (N) increases then the number of Modulators and Demodulators are also increases leads to complexity in hardware.

The Key Advancement of MCM system with IDFT was made by Weinstein and Ebert in 1971. This has removed the problem with Multi-Carrier Modulation system (MCM). IDFT of input stream is nothing but MCM signal, it can be achieved without using Bank of Modulators. That is,

\[ S(t) = \sum_{i} X_i e^{j2\pi iBt} \]

By sampling the MCM signal with sampling time \( \frac{1}{B} \) then the samples of MCM signal is

\[ x(u) = \sum_{i} X_i e^{j2\pi i u} \]

Which is Inverse Discrete Fourier Transform of symbols \( X(0), X(1) = \ldots = X(N-1) \). Therefore, MCM signal generation using IFFT is called as OFDM.

The Result is that there is no interference between the tones. By adding a guard time, called a cyclic prefix, the channel can be made to behave as if the transmitted waveforms were from time minus infinite, and thus ensure orthogonality, which prevents one sub carrier from interfering with another called Inter-carrier interference (ICI). The cyclic prefix is actually a copy of the last portion of the data symbol appended to the front of the symbol during the guard interval. The figure below shows three tones over a single symbol period, where each tone has an integer number of cycles during the symbol.

XI. THEORY of OFDM OPERATION:

The sinusoidal waveforms making up the tones in OFDM have the very special property of the only Eigen functions of a linear channel. This property prevents adjacent tones in OFDM systems from interfering with one another. This property, and the incorporation of a small amount of guard time to each symbol, enables the orthogonality between tones to be preserved in the presence of multipath. The figure below highlights the orthogonal nature of tones used in OFDM system.

The SCHEMATIC DIAGRAM of OFDM:

A. TRANSMITTER and RECEIVER SCHEMATIC are
The figure above is the realigned subcarriers. Multipath causes tones and delayed replicas of tones to arrive at the receiver with some delay spread. This leads to misalignment between sinusoids which need to be aligned as shown here in order to be orthogonal. The cyclic prefix allows the tones to be realigned at the receiver, thus regaining orthogonality. The cyclic prefix should be sized appropriately to serve as a guard time to eliminate ISI. This is accomplished since the amount of time dispersion from the channel (Delay spread ‘ L’) is smaller than duration of cyclic prefix. So, there must be addition of the cyclic prefix at the end of OFDM transmitter and removal of cyclic prefix at the beginning of OFDM receiver.

XII. DESIGN TRADEOFF’s:

- If the cyclic prefix is very long then there will be loss in efficiency.
  
  Loss inefficiency = \( \frac{\text{cyclic prefix length}}{\text{Total OFDM symbol length}} \times \frac{L-1}{N+L-1} \)

- As the number of Subcarriers increases, the Decoding delay of the system increases hence N must be not very large.

- For flat fading channels, Sub carrier bandwidth should be less than coherence bandwidth of the channel.

XIII. MIMO-OFDM COMMUNICATION SYSTEM:

MIMO-OFDM is a combination of MIMO communication with OFDM. MIMO-OFDM coverts wideband MIMO frequency selective channel into parallel narrow band flat-fading MIMO channels. MIMO frequency selective channel can be modelled as MIMO FIR filter. In MIMO frequency selective channel, ISI occurs between current and previous transmitted symbols. Hence in a MIMO-OFDM system one needs to perform IFFT at each transmit antenna. For MIMO detection simple zero forcing receiver or MIMO MMSE receiver can be used. First detector takes 1st carrier of all FFTs.

A. TRANSMITTER & RECEIVER SCHEMATIC

XIV. PROBLEMS IN OFDM:

A. FREQUENCY OFFSET EFFECT in OFDM:

The Presence of Carrier frequency offset at the receiver can introduce severe distortion in an OFDM system, as it results in a loss of orthogonality amongst subcarriers thereby introducing ICI. In practice, if the frequency offset is 10 \(^{-4}\)% gives the 17 to 20 % SNR decrement. Hence, Frequency synchronization in OFDM systems is very important.

B. PEAK TO AVERAGE POWER RATIO ISSUE in OFDM:

PAPR is critical factor in OFDM systems. PAPR for a SCM is 1 or 0 dB and for a OFDM system is significantly higher. Amplifiers in OFDM systems produce very high peak to peak swing around operating point (average point) leads to non-liner region (Saturation region) then Orthogonality will be lost leading to ICI and also peak distortion. SC-FDMA (Single carrier frequency division for multiple access) systems solves the PAPR problem. SC-FDMA system contains ‘ M ’ point FFT (M<N) before ‘ N ’ point IFFT. It significantly reduces PAPR of OFDM.

The figure below shows the peak distortion.
XV. CONCLUSION:
This paper discussed about MIMO modelling and MIMO receivers. Then discussed about SVD of MIMO channel used to avoid ISI. Then discussed about non-linear MIMO receivers. OFDM and transmitter and receiver schematic were discussed and finally MIMO-OFDM system and types of distortions in OFDM were covered and reducing PAPR is part of future work.

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