Profit-Function of Two- Identical Cold Standby aircraft System subject to failure due to Controls damaged by engine failure and Controls damaged by structural failure of involving commercial aircraft.

Ashok Kumar Saini
BLJS COLLEGE, TOSHAM (BHIWANI) HARYANA INDIA
Email: drashokksaini2009@gmail.com

Abstract - In this paper we have taken failure due to Controls damaged by engine failure and Controls damaged by structural failure of involving commercial aircraft. When the main unit fails then cold standby system becomes operative. Controls damaged by structural failure of involving commercial aircraft cannot occur simultaneously in both the units and after failure the unit undergoes Type-I or Type-II or Type-III repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

Keywords: Cold Standby, failure due to Controls damaged by engine failure and Controls damaged by structural failure of involving commercial aircraft, first come first serve, MTSF, Availability, Busy period, Benefit-Function.

INTRODUCTION

Accidents and incidents involving commercial aircraft

Controls damaged by engine failure

- LOT Polish Airlines Flight 5055, an Ilyushin Il-62M, on 9 May 1987. According to the Polish investigatory commission, the cause of the crash was the disintegration of an engine shaft due to faulty bearings inside engine No. 2, which seized, causing extensive heat. This in turn caused the consequent damage to engine No. 1, rapid decompression of the fuselage, and a fire in the cargo hold, as well as the loss of elevator controls and progressive electrical failures. Zygmunt Pawlaczuk decided to return to Warsaw Okecie Airport using only trim tabs to control the flight of the aircraft. He lost his struggle to land about 5 km from the runway in the Kabacki Forest. All 172 passengers and 11 crew members perished.

- United Airlines Flight 232, a McDonnell Douglas DC-10, on 19 July 1989. A fan disk in the No. 2 engine fractured, severing most of the flight controls. Dennis Fitch, a deadheading DC-10 instructor who had studied the case of JAL Flight 123, was able to help the pilots steer the aircraft using throttle differential. Despite the break-up of the aircraft on landing, 175 of 285 passengers and 10 of the 11 crew members survived.

- Baikal Airlines Flight 130, a Tupolev Tu-154, on 3 January 1994. When starting the engines before takeoff, the pilots noticed a warning light signaling dangerous rotation of the starter in engine #2. Believing the warning to be false, they decided to take off anyway. During the initial climb, the starter failed and a fire broke out in the #2 engine. The fire damaged all three hydraulic lines, rendering the plane uncontrollable. After 12 minutes of the crew trying to control the sliding trajectory of the plane, it eventually crashed into a dairy farm near Mamony town at 500 km/h, killing all 125 people aboard and one man on the ground.

Controls damaged by structural failure

American Airlines Flight 96, a McDonnell Douglas DC-10, on 12 June 1972. The failure of the rear cargo door caused an explosive decompression, which in turn caused the rear main cabin floor to collapse and severed flight controls. The pilots had only limited ailerons and elevators; the rudder was jammed. The number two engine also ran down to idle at the time of decompression. The aircraft landed safely at Detroit-Metropolitan Airport.

Japan Airlines Flight 123, a Boeing 747, on 12 August 1985. A faulty repair years earlier had weakened the aircraft's rear pressure bulkhead, which failed in flight. The vertical stabilizer and much of the aircraft's empennage was blown off during the decompression. The decompression also ruptured all four hydraulic lines which controlled the aircraft's mechanical flight controls. The pilots were able to continue flying the aircraft with very limited control, but after 32 minutes the aircraft crashed into a mountain, killing 520 of the 524 people aboard in the deadliest single aircraft disaster in history.
Turkish Airlines Flight 981, a McDonnell Douglas DC-10, on 3 March 1974. Similar to American Airlines Flight 96, the flight experienced a explosive decompression, when flying over the town of Meaux, France, caused by a rear cargo door failure. The rear main cabin floor collapsed and severed all flight controls. While the plane went into a vertical dive, the captain called for “Speed!” meaning increasing engine thrust to pull the plane's nose up. The plane began to level out, but had lost too much altitude and slammed into the Ermenonville Forest. All 346 people on board were killed upon impact, and it became the worst single aircraft disaster without survivors, and the fourth deadliest aviation death count ever. Aleeutian Airways Flight 8, a Lockheed L-188 Electra, on 8 June 1983. Flying over Cold Bay, Alaska, the plane's number 4 engine propeller separated and cut a hole in the plane, causing an explosive decompression, jammed flight controls, snapped throttle cables, and left the flight deck crew of three with only autopilot that had no lateral control. After managing to wrench the ailerons and elevators into minimal working condition, the crew tried to land at Anchorage at high speed. They had to make a go-around, but landed on the second attempt, saving all 10 passengers on board.

- Air Midwest Flight 5481, a Beechcraft 1900D, on 8 January 2003. On takeoff from Charlotte/Douglas International Airport, it pitched up into a vertical ascent and stalled, only 37 seconds later smashing into a US Airways hangar, despite the captain applying full elevator down. There were 21 fatalities. The NTSB found out that the plane had been overweight and that during maintenance, the tension turnbuckles that governed elevator movement had been set incorrectly by an inexperienced mechanic. This caused the elevators to lose control upon takeoff.

- Air Transat Flight 961, an Airbus A310, on 6 March 2005, catastrophic structural failure: the rudder detached from the aircraft with a loud bang. The pilots regained enough control to land the aircraft safely.

Stochastic behavior of systems operating under changing environments has widely been studied. Dhillon, B.S. and Natesan, J. (1983) studied an outdoor power systems in fluctuating environment. Kan Cheng (1985) has studied reliability analysis of a system in a randomly changing environment. Jinhua Cao (1989) has studied a man machine system operating under changing environment subject to a Markov process with two states. The change in operating conditions viz. fluctuations of voltage, corrosive atmosphere, very low gravity etc. may make a system completely inoperative. Severe environmental conditions can make the actual mission duration longer than the ideal mission duration. In this paper we have taken failure due to Controls damaged by engine failure and Controls damaged by structural failure of involving commercial aircraft. When the main operative unit fails then cold standby system becomes operative. Failure due to controls damaged by structural failure of involving commercial aircraft cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of Type- II by ordinary repairman or Type III by multispecialty repairman in case of failure due to Controls damaged by engine failure involving commercial aircraft immediately. The repair is done on the basis of first fail first repaired.

**Assumptions**

1. \( \lambda_1, \lambda_2 \) are constant failure rates due to Controls damaged by engine failure and Controls damaged by structural failure of involving commercial aircraft respectively. The CDF of repair time distribution of Type I, Type II and multispecialty repairmen Type-III are \( G_1(t) \), \( G_2(t) \) and \( G_3(t) \).

2. The failure due to controls damaged by structural failure of involving commercial aircraft is non-instantaneous and it cannot come simultaneously in both the units.

3. The repair starts immediately after critical failure of ISS without astronauts and works on the principle of first fail first repaired basis.

4. The repair facility does no damage to the units and after repair units are as good as new.

5. The switches are perfect and instantaneous.

6. All random variables are mutually independent.

7. When both the units fail, we give priority to operative unit for repair.

8. Repairs are perfect and failure of a unit is detected immediately and perfectly.

9. The system is down when both the units are non-operative.

**Notations**

\( \lambda_1, \lambda_2 \) - failure rates for failure due to Controls damaged by engine failure and Controls damaged by structural failure of involving commercial aircraft respectively.

\( G_1(t), G_2(t), G_3(t) \) – repair time distribution Type –I, Type-II, Type III due to Controls damaged by engine failure and Controls damaged by structural failure of involving commercial aircraft , repair by the multispecialty repairman respectively.

\( p, q \) - probability of failure due to Controls damaged by engine failure and Controls damaged by structural failure of involving commercial aircraft respectively such that \( p+ q=1 \)

\( M_i(t) \) System having started from state i is up at time t without visiting any other regenerative state

\( A_i(t) \) state is up state at instant t

\( R_i(t) \) System having started from state i is busy for repair at time t without visiting any other regenerative state.

\( B_i(t) \) the server is busy for repair at time t.
H(t) Expected number of visits by the server for repairing given that the system initially starts from regenerative state i

Symbols for states of the System

Superscripts  O, CS, CDEF, CDSF,

Operative, Cold Standby, failure due to Controls damaged by engine failure, failure due to and Controls damaged by structural failure of involving commercial aircraft respectively

Subscripts  nedef, cdef, cdfs, ur, wr, wR

No failure due to Controls damaged by engine failure, failure due to controls damaged by structural failure of involving commercial aircraft, under repair, waiting for repair, under repair continued from previous state respectively

Up states – 0, 1, 2, 3, 8, 9 ; Down states – 4, 5, 6, 7; regeneration point – 0,1,2, 3, 8, 9

States of the System

0(O_nedef, CS_nedef)

One unit is operative and the other unit is cold standby and there is no failure due to Controls damaged by engine failure in both the units.

1(CDEF cdef, ur1 , O_nedef)

The operating unit fails due to controls damaged by engine failure and is under repair immediately of Type-I and standby unit starts operating with no failure due to failure due to controls damaged by engine failure.

2(CDSF_cdef, ur11 , O_nedef)

The operative unit fails due to controls damaged by structural failure of involving commercial aircraft and undergoes repair of type II and the standby unit becomes operative with no failure due to controls damaged by engine failure.

3(CDSF_cdef, ur111 , O_nedef)

The first unit fails due to controls damaged by structural failure of involving commercial aircraft and under Type-III multispecialty repairman and the other unit is operative with no failure due to controls damaged by engine failure.

4(CDEF cdef,ur111 , CDEF_cdef,wr1)

The unit failed due to CDEF resulting from failure due to controls damaged by engine failure and is under repair immediately of Type-I and the other unit failed due to CDEF resulting from failure due to controls damaged by engine failure and is waiting for repair of Type-I.

5(CDEF cdef,ur111 , CDSF_cdef, wr1)

The unit failed due to CDEF resulting from failure due to controls damaged by engine failure and is under repair of Type-I continued from state 1and the other unit fails due to controls damaged by structural failure of involving commercial aircraft is waiting for repair of Type-II.

6(CDSF_cdef, ur111 , CDEF_cdef, wr1)

The operative unit fails due to controls damaged by structural failure of involving commercial aircraft and under repair continues from state 2 of Type –I and the other unit is failed due to CDEF resulting from failure due to controls damaged by engine failure and waiting for repair of Type-I.

7(CDSF_cdef, ur1111 , CDEF_cdef, wr1)

The one unit fails due to controls damaged by structural failure of involving commercial aircraft continued to be under repair of Type II and the other unit failed due to CDEF resulting from failure due to controls damaged by engine failure is waiting for repair of Type-II.

8(CDEF cdef,ur11111 , CDSF_cdef, wr1)

The one unit failure due to controls damaged by engine failure is under multispecialty repair of Type-III and the other unit is failed due to controls damaged by structural failure of involving commercial aircraft is waiting for repair of Type-II.

9(CDEF cdef,ur111111 , CDSF_cdef, wr1)

The one unit failure due to controls damaged by engine failure is under multispecialty repair of Type-III and the other unit is failed due to controls damaged by structural failure of involving commercial aircraft is waiting for repair of Type-I

Transition Probabilities

Simple probabilistic considerations yield the following expressions:

\[ p_{01} = \lambda_1 / \lambda_1 + \lambda_2, \quad p_{02} = \lambda_2 / \lambda_1 + \lambda_2, \quad p_{10} = pG_1^*(\lambda_1) + qG_2^*(\lambda_2), \]

\[ p_{14} = p - pG_1^*(\lambda_1) = p_{14}^{(4)}, \quad p_{15} = q - qG_1^*(\lambda_2) = p_{15}^{(5)}, \]

\[ p_{23} = pG_2^*(\lambda_1) + qG_2^*(\lambda_2), \quad p_{26} = p - pG_2^*(\lambda_1) = p_{26}^{(6)}, \]

\[ p_{27} = q - qG_2^*(\lambda_2) = p_{27}^{(7)}, \]

\[ p_{10} = p_{26} = p_{91} = 1 \]

We can easily verify that

\[ p_{01} = p_{02} = 1, \quad p_{10} + p_{14} = p_{15} = 1, \quad p_{23} + p_{26} = p_{27} = 1 \]

and mean sojourn time is

\[ \mu_0 = E(T) = \int_0^\infty P[T > t] \, dt \]

Mean Time To System Failure

\[ \Theta_0(t) = Q_{01}(t)[s] \Theta_1(t) + Q_{02}(t)[s] \Theta_2(t) \]

\[ \Theta_1(t) = Q_{10}(t)[s] \Theta_0(t) + Q_{12}(t) + Q_{23}(t) \]

\[ \Theta_2(t) = Q_{23}(t)[s] \Theta_0(t) + Q_{26}(t) + Q_{27}(t) \]

\[ \Theta_3(t) = Q_{30}(t)[s] \Theta_0(t) \]

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We can regard the failed state as absorbing

Taking Laplace-Stieltjes transform of eq. (3-6) and solving for

\[ \phi_0(s) = \frac{N_1(s)}{D_1(s)} \quad (6) \]

where

\[ N_1(s) = Q_{01}[Q_{14}(s) + Q_{15}(s)] + Q_{02}[Q_{26}(s) + Q_{27}(s)] \]

\[ D_1(s) = 1 - Q_{01}^*Q_{10} - Q_{02}^*Q_{23} - Q_{00}^* \]

Making use of relations (1) & (2) it can be shown that \( \phi_0(0) = 1 \), which implies that \( \phi_0(t) \) is a proper distribution.

\[ \text{MTSF} = E[T] = \left. \frac{d}{ds} \theta^*_0(s) \right|_{s=0} = \left( D_1(0) - N_1(0) \right) / D_1(0) \]

where

\[ \mu_0 = \mu_{01} + \mu_{02}, \quad \mu_1 = \mu_{10} + \mu_{11}(4) + \mu_{12}(5), \quad \mu_2 = \mu_{23} + \mu_{29}(7) + \mu_{29}(6) \]

**Availability analysis**

Let \( M_i(t) \) be the probability of the system having started from state \( i \) is up at time \( t \) without making any other regenerative state. By probabilistic arguments, we have

\[ M_0(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t}, \quad M_i(t) = \mu G_1(t) \quad (1) \]

\[ M_0(t) = q G_2(t), \quad M_i(t) = G_3(t) \quad (2) \]

The point wise availability \( A_i(t) \) have the following recursive relations

\[ A_0(t) = M_0(t) + q_01(t) [c] A_1(t) + q_02(t) [c] A_2(t) \]

\[ A_1(t) = M_1(t) + q_10(t) [c] A_0(t) + q_12(5) [c] A_2(t) + q_{11}(4) [c] A_1(t) \]

\[ A_2(t) = M_2(t) + q_20(t) [c] A_0(t) + q_{28}(7) [c] A_0(t) + q_{29}(6) [c] A_0(t) \]

\[ A_3(t) = M_3(t) + q_30(t) [c] A_0(t) \]

\[ A_4(t) = q_01(t) [c] A_1(t) \]

\[ A_5(t) = q_02(t) [c] A_2(t) \]

\[ A_6(t) = q_03(t) [c] A_3(t) \]

\[ A_7(t) = q_04(t) [c] A_4(t) \quad (7-11) \]

Taking Laplace Transform of eq. (7-11) and solving for

\[ \hat{A}_0(s) = \frac{N_2(s)}{D_2(s)} \quad (12) \]

where

\[ N_2(s) = \hat{M}_0 \left[ \{1 - \hat{q}_{11}(4)\} \{1 - \hat{q}_{28}(7)\} \hat{q}_{29}(6) + \hat{q}_{011} \hat{M}_1 \right] \]

\[ \hat{D}_2(s) = \{1 - \hat{q}_{11}(4)\} \{1 - \hat{q}_{28}(7)\} \hat{q}_{29}(6) + \hat{q}_{011} \hat{M}_1 \]

\[ D_2(s) = \{1 - \hat{q}_{11}(4)\} \{1 - \hat{q}_{28}(7)\} \hat{q}_{29}(6) + \hat{q}_{011} \hat{M}_1 \]

\[ A_0(t) = \lim_{s \to 0} \left[ \frac{s N_2(s)}{D_2(s)} \right] = \frac{N_2(0)}{D_2(0)} \]

Using L’ Hospital’s rule we get

\[ A_0 = \lim_{s \to 0} \left[ \frac{s N_2(s)}{D_2(s)} \right] = \frac{N_2(0)}{D_2(0)} \quad (13) \]

The expected up time of the system in \((0,t]\) is

\[ \hat{\lambda}_u(t) = \int_0^s \hat{A}_0(z) dz \]

So that \[ \hat{\lambda}_u(s) = \frac{\hat{A}_0(s)}{s} \]

The expected down time of the system in \((0,t]\) is

\[ \hat{\lambda}_d(t) = t - \hat{\lambda}_u(t) \]

So that \[ \hat{\lambda}_d(s) = \frac{1}{s} - \hat{\lambda}_u(s) \quad (15) \]

The expected busy period of the server when there is failure due to Controls damaged by engine failure and failure due to Controls damaged by structural failure of involving commercial aircraft in \((0,t]\)

\[ R_0(t) = q_{01}(t) [c] R_1(t) + q_{02}(t) [c] R_2(t) \]

\[ R_1(t) = S_1(t) + q_{10}(t) [c] R_0(t) + q_{12}(5) [c] R_2(t) + q_{11}(4) [c] R_1(t) \]

\[ R_2(t) = S_2(t) + q_{20}(t) [c] R_0(t) + q_{28}(7) [c] R_0(t) + q_{29}(6) [c] R_0(t) \]

\[ R_3(t) = S_3(t) + q_{30}(t) [c] R_0(t) \]

\[ R_4(t) = S_4(t) + q_{01}(t) [c] R_1(t) \]

\[ \hat{R}_5(s) = \frac{s N_2(s)}{D_2(s)} \quad (16-21) \]

where

\[ S_i(t) = \mu G_i(t) e^{-\lambda_i t}, \quad S_i(t) = \mu G_i(t) e^{-\lambda_i t} \quad (22) \]

Taking Laplace Transform of eq. (16-21) and solving for

\[ \hat{R}_5(s) = \frac{s N_2(s)}{D_2(s)} \quad (23) \]
where

\[ N_2(s) = \hat{\alpha}_{01} \hat{S}_1(1 - \hat{\alpha}_{28}(^7)) + \hat{\alpha}_{12}(^5) \hat{S}_2 + \hat{\alpha}_{23} \hat{S}_3 + \hat{\alpha}_{28}(^7) \hat{S}_8 + \hat{\alpha}_{29}(^6) \hat{S}_9 ] + \hat{\alpha}_{10} ( \hat{S}_7 + \hat{\alpha}_{29}(^6) \hat{S}_8 + \hat{\alpha}_{28}(^7) \hat{S}_9 ) + \hat{\alpha}_{11}(^4) \hat{S}_1 \]

and \( D_2(s) \) is already defined.

(Omitting the arguments \( s \) for brevity)

In the long run, \( R_0 = \frac{N_5(s)}{D_5(s)} \) (24)

The expected period of the system under failure due to controls damaged by engine failure and failure due to controls damaged by structural failure of involving commercial aircraft is

\[ \lambda_{\text{rep}}(t) = \int_0^\infty R_3(z)dz \]

So that \( \lambda_{\text{rep}}^{-1}(s) = \frac{\hat{R}_3(s)}{s} \)

The expected number of visits by the repairman Type-I or Type-II for repairing the identical units in (0,1)

\[ H_0(0) = Q_{00}(0)[1+ H_1(t)] + Q_{01}(0)[1+ H_2(t)] \]

\[ H_1(t) = Q_{01}(0)[s]H_0(t) + Q_{12}(^5)[s] \]

\[ H_2(t) = Q_{02}(0)[s]H_3(t) + Q_{23}(^7)[s] \]

\[ H_3(t) = Q_{03}(0)[s]H_1(t) \]

\[ H_4(t) = Q_{04}(0)[s]H_0(t) \]

Taking Laplace Transform of eq. (25-30) and solving for \( H_5^0(s) \)

\[ H_5^0(s) = N_5(s) / D_5(s) \] (31)

\[ N_5(s) = \{ Q_{01}(s) + Q_{02}(s) \} \{ 1 - Q_{28}(^7)Q_{32}(s) - Q_{12}(^5)Q_{29}(s)Q_{91} \} \]

And

\[ D_5(s) = \{ 1 - Q_{21}(^4) \} \{ 1 - Q_{28}(^7)Q_{32}(s) - Q_{12}(^5)Q_{29}(s)Q_{91} \} \]

\[ Q_{01}(s) = Q_{10}(s) \{ 1 - Q_{28}(^7)Q_{32}(s) + Q_{12}(^5)Q_{23}(s)Q_{91} \} - Q_{02}(s)Q_{20}(s) \}

(OMitting the arguments \( s \) for brevity)

In the long run,

\[ H_0 = N_5(0) / D_5(0) \] (32)

The expected number of visits by the multispecialty repairman Type-III for repairing the identical units in (0,1)

\[ W_1(t) = Q_{01}(0)[s]W_0 + Q_{02}(0)[s]W_2(t) \]

\[ W_2(t) = Q_{02}(0)[s]W_1(t) + Q_{28}(^7)[s] \]

\[ W_0(t) = Q_{00}(0)[s]W_1(t) + Q_{12}(^5)[s] \]

\[ W_3(t) = Q_{01}(0)[s]W_2(t) + Q_{28}(^7)[s] \]

\[ W_4(t) + Q_{29}(^6)[t] \]

(33-38)

Taking Laplace Transform of eq. (33-38) and solving for \( H_5^0(s) \)

\[ H_5^0(s) = N_5(s) / D_5(s) \] (39)

\[ N_5(s) = Q_{01}(s) \{ Q_{23}(^5)Q_{29}(s)Q_{91} \} + Q_{02}(s) \{ Q_{23}(^5)Q_{32}(s) + Q_{29}(^6)s \} \]

\[ Q_{23}(^5)Q_{29}(s)Q_{91} \}

(OMitting the arguments \( s \) for brevity)

In the long run,

\[ W_0 = N_5(0) / D_5(0) \] (40)

where \( N_5(0) = p_{01}p_{12}(^5) + p_{02}[1 - p_{11}(^4)] \)

**Benefit- Function Analysis**

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to Controls damaged by engine failure and failure due to controls damaged by structural failure of involving commercial aircraft, expected number of visits by the repairman for unit failure.

The expected total Benefit-Function incurred in (0,1) is

\[ C(t) = \text{Expected total revenue in (0,1)} \]

- expected busy period of the system under failure due to controls damaged by engine failure and failure due to controls damaged by structural failure of involving commercial aircraft for repairing the units in (0,1)

- expected number of visits by the repairman Type- I or Type- II for repairing the identical units in (0,1)

- expected number of visits by the multispecialty repairman Type- III for repairing of identical the units in (0,1)

The expected total cost per unit time in steady state is

\[ C = \lim_{t \to \infty} \left( C(t) / t \right) = \lim_{s \to 0} \left( s^2 C(s) \right) = K_1A_0 - K_2R_0 - K_3H_0 - K_4W_0 \]

where

\[ K_1 \] - revenue per unit up-time,

\[ K_2 \] - cost per unit time for which the system is busy under repairing,

\[ K_3 \] - cost per visit by the repairman type- I or type- II for units repair,
K₄ - cost per visit by the multispecialty repairman Type- III for units repair

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate failure due to Controls damaged by engine failure and failure due to Failure due to Controls damaged by structural failure of involving commercial aircraft increases, the MTSF, steady state availability decreases and the Profit-function decreased as the failure increases.

REFERENCES


