Mathematical Representations of 1D, 2D and 3D Wavelet Transform for Image Coding

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Abstract – The central theme of this paper is not only show how to represent 1D, 2D and 3D Discrete Wavelet Transform (DWT) mathematically but also focus on how these wavelet transform are constructed and used for image signal representation in general and how it is utilized for image compression in particular. This paper also presents a sequence of mathematical development followed by image coding examples from both the spatial and transform domain perspectives. The DWT is currently a standard tool to study time-frequency produced by a wide range of non-stationary dynamical systems and frameworks. A multi-dimensional transform domain for separable image decompositions were also investigated in an image representation context under mathematical contemplations and after that again under image coding objectives. This mathematical study report and review will be beneficial and handy not only for image coding domain but also the other applications where DWT is used, for instance, digital water marking, image fusion, image segmentation, image registration and content based image retrieval,

Keywords – 1D, 2D, 3D, DWT, STFT, Fourier Transform, scaling and wavelet functions

I. INTRODUCTION

It is well known that wavelet analysis has turned into a global focus in diverse research fields widely and profoundly in mathematics, engineering and many other disciplines by scholastic and academic researchers, scientists and engineers [1]-[3],[5][8]. It is also observed that wavelet techniques have demonstrated the ability to provide not only high coding efficiency but also spatial and quality scalability features [6].

Wavelet Transform gives better analysis of signals which is a function of time and frequency and also allows complex information such as music, speech, images and patterns to be decomposed into elementary forms at different positions and scales and subsequently reconstructed with high precision and exactness [6],[10]. It is no doubt that Fourier transform is a powerful tool for analyzing the components of a stationary signal. Be that as it may, it is fizzled and failed for analyzing the non-stationary signal whereas wavelet transform allows the components of a non-stationary signal to be analyzed such as images and pictures.

A wavelet is a waveform or small wave of limited duration or little influx of constrained that has an average value of zero. Sinusoids (Fourier waveform) hypothetically reach out from minus to plus infinity yet wavelets have a starting and an end. Scientifically a "wave" is communicated as a sinusoidal (or wavering) capacity of time or space [19]-[20]. Wavelets are sporadic, restricted term and frequently non-symmetrical. Wavelet transform is utilized to break down the signal both in frequency and time space. Wavelets are not only a powerful tool for studying time-frequency behaviour of finite-energy signals but also provide a coherent set of concepts, methods, and algorithms that are adapted to a variety of non-stationary signal processing [11],[13]-[15].

Wavelet systems have already been utilized and acknowledged as a part of a different applications regions, for example, image compression, advanced signal processing (speech and image processing), image analysis and picture investigation,, statistics, and modeling of nonlinear dynamic processes in engineering and scientific domain.

The rest of the paper is organized as follows: Section II deals with the basics of wavelet transform. Section III gives the complete picture on the evolution of wavelet form and its salient features. Section IV explains about the One Dimensional wavelet transform. Section V and VI deals with 2D and 3D respectively. Section gives the conclusion and future scope of this research study.

II. WAVELET TRANSFORM

A. WAVELET CHARACTERISTICS

An imperative property of wavelet transform is the preservation of energy (total of square of pixel values) [6]. In wavelet transform, the energy of the signal is divided into approximation and detail components but the total energy remains constant. Secondly, wavelet transform supports energy compaction. Energy compaction is one of the important and essential properties for image compression because the more energy compaction into the approximation component of an image, the higher compression efficiency may be acquired [8]-[10]. Wavelet transform decomposes the signal into various sub bands, each of which has its own spatial orientation feature that can be efficiently used for image coding.

Another property of wavelet transform is that quantization error [15]. This support the ability to generate lower level coefficients from the higher level coefficients and this can be achieved by the use of tree-like structured chain of filters called Filter Banks. Wavelets have space-frequency localization. Most of the energy of a wavelet is confined in a finite interval and the transform contains frequencies from a certain frequency band [16]-[17.

B. WAVELET ANALYSIS

Wavelet analysis has become an essential mathematical tool, providing effective solution for various problems related to the study and diagnostics of complex nonlinear processes, as well as digital signal processing. Over the past few decades, wavelet analysis has been widely considered as an interdisciplinary technique. Wavelet analysis decomposes signals into component waves of varying durations, called wavelets. These wavelets are localized into detail and approximate signals [22]. The important part of wavelet analysis is multiresolution analysis. Multiresolution analysis is the decomposition of a signal (such as an image) into sub signals (sub images) of different size resolution levels [23].

C. WAVELET HISTORY

Wavelets were introduced relatively recently, in the beginning of the 1980s. They attracted considerable interest from the mathematical community and from members of many diverse disciplines in which wavelets had promising applications. Their name itself was coined approximately a decade ago (Morlet, Arens, Fourgeau, and Giard (1982), Morlet (1983), Grossmann and Morlet (1984) [15-[20]. In the last ten years interest in them has grown at an explosive rate. There are several reasons for their present success. On the one hand, the concept of wavelets can be viewed as a synthesis of ideas which originated during the last twenty or thirty years in engineering (subband coding), physics (coherent states, renormalization group), and pure mathematics (study of Calderon-Zygmund operators). As a consequence of these interdisciplinary origins, wavelets appeal to scientists and engineers of many different backgrounds. On the other hand, wavelets are a fairly simple mathematical tool with a great variety of possible applications. Already they have led to exciting applications in signal analysis (sound, images) (some early references are Kronland-Martinet, Morlet and Grossmann (1987), Mallat (1989b), (1989c); more recent references are given later) and numerical analysis (fast algorithms for integral transforms in Beylkin, Coifman, and Rokhlin (1991); many other applications are being studied [21]-[24].

III. EVOLUTION OF WAVELET TRANSFORM

It has taken after a long course from Fourier analysis to wavelet analysis. In fact, as early as 1946, Gabor has found out that Fourier analysis is lacking to localize signals in the time domain. He applied Gaussian function as a "window" to improve the Fourier transform. It is referred to as Gabor transform and is applied to analyze and enhance the transient signals [16]-[18]. Thereafter, Gabor transform led to the general window Fourier transform (or short-time Fourier transform) by the replacement of Gaussian function with other localized window functions. However, the size of the window in the window Fourier transform is fixed, it did not improve Fourier transformation thoroughly.

The wavelet transform is only one of a few existing time-frequency transforms. Every one of them have been created to conquer and overcome the drawback of the Fourier transform and to be specific the absence of time data of the frequency components. A couple of case for time-frequency analysis are the windowed Fourier transform, the Gabor transform and the Wigner-Ville-transform [18]-22]. Each transform technique has got its own merits and demerits. Be that as it may, the wavelet transform has turned into the best of those, because of its solid hypothetical, theoretical and mathematical foundation and connections with the theory of frames and filter banks in the engineering field.

The term wavelet transform really portrays two in some aspects different transforms, specifically the continuous wavelet transform (CWT) and the discrete wavelet transform (DWT). The Discrete Fourier Transform (DFT) is typically identified and related to the discrete signal but the DWT is a discrete transform of a continuous signal. The DWT is definitely an exceptional instance of the CWT, however with a great deal of hypothetical outcomes, which will likewise be analyzed and talked about briefly.

The principles of wavelet transform are similar to those of Fourier analysis, which was first developed in the early part of the 19th century. Wavelets have proven to be highly effective and viable at extracting frequency information from data. Their multi-scale nature enables the efficient description of both transient and long-term signals. Furthermore, only a small number of wavelet coefficients are needed to describe complicated signals and the wavelet transform is computationally efficient.

A. FOURIER SERIES

The most well known of these is Fourier analysis, which breaks down and separates a signal into constituent sinusoids of various frequencies. Another approach to consider Fourier analysis is as a mathematical technique for transforming our view of the signal from time-based to frequency-based. The Fourier Series is the main branch of mathematics leading to wavelets began with Joseph Fourier (1807) [25] with his hypotheses of frequency analysis, now often referred to as Fourier synthesis. He affirmed that any 2π-periodic function f(x) is the sum

$$f(x) = a_0 + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx)$$

of its Fourier series. The coefficients a_0, a_k and b_k are calculated by

$$a_o = \frac{1}{2\pi}\int_0^{2\pi} f(x)dx$$

$$a_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos(kx)dx$$

$$b_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin(kx)dx$$

Fourier's assertion assumed a vital part in the advancement of the thoughts mathematicians had about the functions. He opened up the way to another new functional universe [25]-[26].

B. FOURIER ANALYSIS

Fourier's representation of functions as a superposition of sines and cosines has become ubiquitous for both the analytic and numerical solution of differential equations and for the analysis and treatment of communication signals. Fourier and wavelet analysis have some very strong links. Fourier analysis consists of breaking up a signal into sine waves of various frequencies. For many signals, Fourier analysis is extremely useful because the signal's frequency content is of great importance. So why do we need other techniques, like wavelet analysis?

Fourier analysis has a serious drawback. In transforming to the frequency domain, time information is lost. When looking at a Fourier transform of a signal, it is impossible to tell when a particular event took place. If the signal properties do not change much over time — that is, if it is what is called a stationary signal — this drawback isn't very important. However, most interesting signals contain numerous non-stationary or transitory characteristics: drift, trends, abrupt changes, and beginnings and ends of events. These attributes are regularly the most imperative part of the signal, and Fourier examination is not suited to recognizing them [27].

Similarly, wavelet analysis is the breaking up of a signal into shifted and scaled versions of the original (or mother) wavelet. This gives wavelets a distinct advantage over a purely frequency domain analysis. Since Fourier investigation presumes that any sample is an independent drawing, Fourier analysis requires "covariance stationarity", whereas wavelet analysis may analyze both stationary and long term non-stationary signals. This methodology gives a helpful approach to represent complex signals. Expressed differently, spectral decomposition methods perform a global analysis, whereas wavelet methods act locally in both frequency and time. Fourier analysis can relax local non-stationarity by windowing the time series as was indicated above.

C. FOURIER TRANSFORM (FT)

The Fourier transform's utility lies in its ability to analyze a signal in the time domain for its frequency content. The transform works by first translating a function in the time domain into a function in the frequency domain. The signal can then be analyzed for its frequency content because the Fourier coefficients of the transformed function represent the contribution of each sine and cosine function at each frequency. An inverse Fourier transform just exactly what you would expect, transform data from the frequency domain into the time domain [27].

Let us introduce the mathematical definition of the Fourier transform for a signal f(t):

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t} dt$$

The inverse Fourier transform is used to obtain the time domain representation of the signal:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{j\omega t}\, d\omega$$

The Fourier transform is one of the tools used to analyze and to break down the frequency components of the signal. However, if we take the Fourier transform over the whole time axis, we cannot tell at what instant a particular frequency rises. Short-time Fourier transform (STFT) uses a sliding window to find spectrogram, which gives the information of both time and frequency. But still another problem exists: The length of window limits and restricts the resolution in frequency. Wavelet transform seems to be a solution to the problem above. Wavelet transforms are based on small wave (wavelets) with limited duration. The translated-version wavelets locate where we concern. Whereas the scaled-version wavelets allow us to analyze the signal in various scale (multiresolution).

D. SHORT TIME FOURIER TRANSFORM (STFT)

In an effort to correct this deficiency, Dennis Gabor (1946) adapted the Fourier transform to analyze and investigate only a small section of the signal at a time — a technique called windowing the signal. Gabor's adaptation is called as the Short-Time Fourier Transform (STFT) which maps a signal into a two-dimensional function of time and frequency [28].

The STFT represents a sort of compromise between the time and frequency based views of a signal. It provides some information about both when and at what frequencies a signal event occurs. However, one can only obtain this information with limited precision, and that precision or accuracy is determined by the size of the window.

While the STFT compromise between time and frequency information can be useful. The main drawback is that a specific size for the time window is set or selected ,that window size is the same for all frequencies. Many signals processing frameworks require a more flexible approach. It is necessary that one may vary or fluctuate the window size to determine more accurately either time or frequency. Recall the Fourier transform is defined as;

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

and the STFT is defined by;

$$STFT(\tau, f) = \int_{-\infty}^{\infty} f(t)\ g^*(t-\tau)e^{-j\omega t}\,dt$$

The difference between the Fourier transform equation and the STFT is the function $g^*(t-\tau)$. The '*' indicates a complex conjugation. The function $g^*(t-\tau)$ can be thought of as a window that is shifted along the signal f**(t).** For each shift, the Fourier transform of the product function $f(t)\ g^*(t-\tau)$ is computed. The variables $(t-\tau)$ are the transform variables from the single time domain variable t. Therefore, it could be said that the STFT transforms a time-variable function (t) into a time-frequency function. The signal $f(t)$ may then be reconstructed using:

$$f(t)\, g^*(t-\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{-j\omega t}\,dt$$

The STFT does provide some information on both time locality and frequency spectra, but it does have a limitation. The limitation is that once a particular window size is chosen the same window is used to analyze every frequency of the signal. A small window results in good time resolution but poor frequency resolution. A large window gives good frequency resolution but poor time resolution[29]-[30].

IV. THEORETICAL ASPECTS OF WAVELET TRANSFORM

Wavelet techniques enable us to divide a complicated function into several simpler ones and study them separately. This property, along with fast wavelet algorithms, which are comparable in efficiency to fast Fourier transform algorithms, makes these techniques very attractive in analysis and synthesis problems. Different types of wavelets have been used as tools to solve problems in signal analysis, image analysis, medical diagnostics, boundary value problems, geophysical signal processing, statistical analysis, pattern recognition, and many others. While wavelets have gained popularity in these areas, new applications are continually being investigated.

The wavelet transform is defined as a mathematical technique in which a particular signal is analyzed (or synthesized) in the time domain by using different versions of a dilated (or contracted) and translated (or shifted) basis function called the wavelet prototype or the mother wavelet. A wavelet function ψ(t) is a small wave, which is oscillatory in some way to discriminate between different frequencies. The wavelet contains both the analyzing shape and the window [28]-[30].

A. CONTINUOUS WAVELET TRANSFORM (CWT)

CWT is an implementation of the wavelet transform that uses arbitrary scales and nearly arbitrary wavelets. The data obtained by this transform are highly correlated and the wavelets used are not orthogonal. For the discrete time series we use this transform , with a limitation that the smallest wavelet translations should be equal to the data sampling. This is called Discrete Time Continuous Wavelet Transform (DT-CWT) and it is the most appropriate way of computing CWT in real applications [ ]. Wavelets are mathematical functions formed by scaling and translation of basis functions $\psi(t)$ in the time domain. $\psi(t)$ is also called the mother wavelet and satisfies the following conditions:

1. The total area under the curve $\psi(t)$is equal to zero

$$\int_{-\infty}^{\infty} \psi(t)dt = 0$$

2. The function has a finite energy, i.e., it is square-integral

$$\int_{-\infty}^{\infty} |\psi(t)|^2 dt < \infty$$

The wavelet is defined by the formula

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right)$$

where a and b are two arbitrary real numbers used as scaling and translation parameters, respectively. The factor $a^{-1/2}$ also represents a normalization factor, which allows the energy of the wavelet function to remain independent of parameter a. For the values $0 < a < 1$, the basis function shrinks in time, while for $a > 1$, it spreads in time.

The wavelet transform of the signal f(t) is mathematically described by the expression:

$$W(a,b) = \int_{-\infty}^{\infty} \psi_{a,b}(t) f(t) dt$$

W(a,b) is called the continuous wavelet transform (CWT), where a and b are continuous variables and f(t) is a continuous function. The inverse wavelet transform is obtained as:

$$f(t) = \frac{1}{C}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \psi_{a,b}(t) W(a,b)\, da\, db$$

where,

$$C = \int_{-\infty}^{\infty} \frac{|\Psi(\omega)|^2}{|\omega|} d\omega$$

where $\Psi(\omega)$is the Fourier transform of $\psi(t)$. The inverse continuous wavelet transform exists if the parameter C is positive and finite.

B. Discrete Wavelet Transform (DWT)

The DWT is an implementation of the wavelet transform that uses a discrete set of the wavelet scales and translations following some defined rules. The transform decomposes the signal into mutually orthogonal set of wavelets, which is the difference from CWT, or its implementation for the discrete time series known as discrete-time continuous wavelet transform (DT-CWT).

V. ONE DIMENSIONAL DISCRETE WAVELET TRANSFORM (1D-DWT)

Discrete Wavelet Transform is a technique to transform image pixels into wavelets, which are then used for wavelet-based compression and coding. The DWT is defined as [Gonzalez]:

$$W_{\phi}(j_o,k)=\frac{1}{\sqrt{M}}\sum_{x} f(x)\,\phi_{jo,k}(x)$$

$$W_{\psi}(j,k)=\frac{1}{\sqrt{M}}\sum_{x} f(x)\,\psi_{j,k}(x)$$

for j≥ j_0 and the Inverse DWT (IDWT) is defined as:

$$f(x)=\frac{1}{\sqrt{M}}\sum_{x} W_{\phi}\,(j_o,k)\phi_{j_{o},k}(x) + \frac{1}{\sqrt{M}}\sum_{j-j_o}^{\infty}\sum_{k} W_{\psi}\,(j,k)\phi_{j,k}(x)$$

where $f(x)$, $\phi_{jo,k}(x)$ and $\psi_{j,k}(x)$ are functions of the discrete variable.The $\phi_{jo,k}(x)$ is a member of the set of expansion functions derived from a scaling function $\phi(x)$, by translation and scaling using:

$$\phi_{j,k}(x)=2^{j/2}\phi(2^{j}x-k)$$

The $\phi_{j,k}(x)$ is a member of the set of wavelets derived from a wavelet function $\psi(x)$, by translation and scaling using:

$$\psi_{j,k}(x)=2^{j/2}\psi(2^{j}x-k)$$

The DWT can be formulated as a filtering operation with two related FIR filters, low-pass filter (h_{ϕ}) and high-pass filter (h_{ψ})

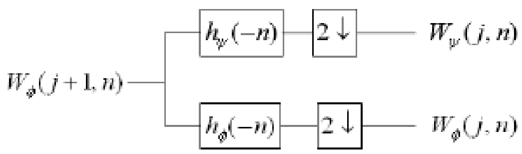

Fig 1. Analysis Filter bank of 1D

Both $W_{\phi}(j,k)$ and $W_{\psi}(j,k)$, the scale j approximation and the detail coefficients, can be computed by convolving $W_{\phi}(j+1,k)$, the scale j + 1 approximation coefficients, with the time-reversed scaling and wavelet vectors, $h_{\phi}(-n)$and $h_{\psi}(n)$ and sub-sampling the results by 2, as expressed in Equations (6) and (7) and illustrated in [29]-[30]

$$W_{\psi}(j,k)=h_{\psi}(-n)*W_{\phi}(\mathrm{j}+1,n)\mid_{n=2k,k\geq 0}$$

$$W_{\phi}(j,k)=h_{\phi}(-n)*W_{\phi}(j+1,n)\mid_{n=2k,k\geq 0}$$

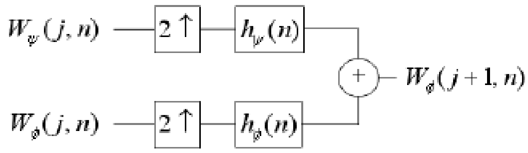

Fig 2. Synthesis Filter bank of 1D

The filter bank in Fig. (1 and 2) can be iterated to implement multi-resolution analysis. The IDWT can be implemented by up-sampling and synthesis filtering. The one-dimensional DWT and IDWT can be extended to two-dimensional [36]-[37].

V. TWO DIMENSIONAL DISCRETE WAVELET TRANSFORM (2D-DWT)

The scaled and translated basis elements of the 2D wavelet transform are given by [3].

$$LL=\ \phi(x,y)\ =\phi(x)\,\phi(y)$$

$$LH=\ \psi^{H}(x,y)=\ \psi(x)\phi(y)$$

$$HL=\ \psi^{V}(x,y)\ =\phi(x)\,\psi(y)$$

$$HH=\ \psi^{D}(x,y)=\psi(x)\,\psi(y)$$

where the super scripts H, V, and D refer to the decomposition direction of the wavelet. Two-dimensional wavelets are utilized as a part of image manipulation. The multiresolution representation of scaling and wavelet functions for the 2-D given below:

$$\phi_{j,m,n}(x,y)=\ 2^{j/2}\phi(2^{j}x-m,2^{j}y-n)$$

$$\psi^{i}_{j,m,n}(x,y)=\ 2^{j/2}\psi^{i}(2^{j}x-m,2^{j}y-n)$$

where I = { H,V,D}

The discrete wavelet transform function f(x, y) of size M x N is given below:

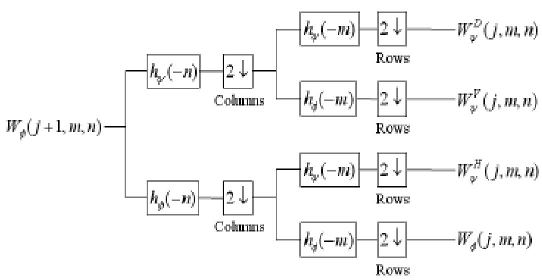

Fig 3. Analysis Filter bank of 2D

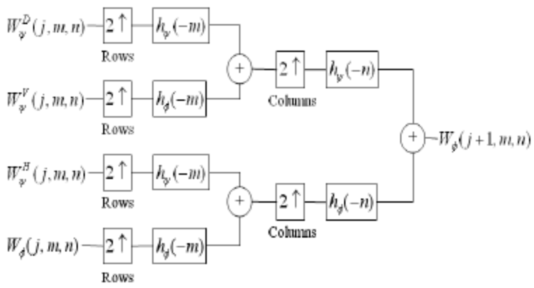

Fig 4. Synthesis Filter bank of 2D

1. Scaling function of the wavelet representation is shown as

$$W_\phi(j_o,m,n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, \phi_{j_{o,m,n}}(x,y)$$

The corresponding wavelet function of the horizontal, vertical and diagonal representation of the images are as follows:

2. Horizontal subband representation

$$W_\psi^H(j,m,n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, \psi_{j,m,n}^H(x,y)$$

3. Vertical subband representation

$$W_\psi^V(j,m,n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, \psi_{j,m,n}^V(x,y)$$

4. Diagonal subband representation

$$W_\psi^D(j,m,n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, \psi_{j,m,n}^D(x,y)$$

From the above mentioned scaling and wavelet functions, namely, W_ϕ and W_ψ , one can easily acquired through the inverse discrete wavelet transform for the image signal as

$$f(x,y) = \frac{1}{\sqrt{MN}} \sum_m \sum_n W_\phi\,(j_0,m,n)\, \phi_{j_{o,m,n}}(x,y) + \frac{1}{\sqrt{MN}} \sum_{i=H,V,D} \sum_{j=j_0}^{\infty} \sum_m \sum_n W_\psi^i\,(j,m,n)\, \psi_{j,m,n}^i(x,y)$$

VI. THREE-DIMENSIONAL (3D) DISCREETE WAVELET TRANSFORM

A 3-D wavelet decomposes a 3-D image set into a number of slices based on the X, Y and Z direction. Each slice contains the various frequency bands. A separable 3-D wavelet transform can be computed by extending the 1-D pyramidal algorithm[32]-[35]. The decomposed or disintegrated image slice provides an excellent representation for further quantization and coding. The scaled and translated basis elements of the 3-D wavelet transform are given by [ ]. The multiresolution representation of scaling and wavelet functions for the 3-D given below:

$$\phi_{j,m,n,l}(x,y,z) = 2^{j/2}\phi(2^j x - m, 2^j y - n, 2^j z - l)$$

$$\psi_{j,m,n,l}^i(x,y,z) = 2^{j/2}\psi(2^j x - m, 2^j y - n, 2^j z - l)$$

where i = { H,V,D}

where the super scripts H, V, and D refer to the decomposition direction of the wavelet.

$$LLL = \phi(x,y,z) = \phi(x)\,\phi(y)\,\phi(z)$$
$$LLH = \psi^1(x,y,z) = \phi(x)\,\phi(y)\,\psi(z)$$
$$LHL = \psi^2(x,y,z) = \phi(x)\,\psi(y)\,\phi(z)$$
$$LHH = \psi^3(x,y,z) = \phi(x)\,\psi(y)\,\psi(z)$$
$$HLL = \psi^4(x,y,z) = \psi(x)\,\phi(y)\,\phi(z)$$
$$HLH = \psi^5(x,y,z) = \psi(x)\,\phi(y)\,\psi(z)$$
$$HHL = \psi^6(x,y,z) = \psi(x)\,\psi(y)\,\phi(z)$$
$$HHH = \psi^7(x,y,z) = \psi(x)\,\psi(y)\,\psi(z)$$

The discrete wavelet transform function f(x, y,z) of size M x N x L is given below:

1. Scaling function of the wavelet representation is shown as

$$W_\phi(j_o,m,n,l) = \frac{1}{\sqrt{MNL}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \sum_{z=0}^{L-1} f(x,y,z)\, \phi_{j_{o,m,n,l}}(x,y,z)$$

The corresponding wavelet function of the horizontal, vertical and diagonal representation of the images are as follows:

$$W_\psi^i(j,m,n,l) = \frac{1}{\sqrt{MNL}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \sum_{z=0}^{L-1} f(x,y,z)\, \psi_{j,m,n,l}^i(x,y,z)$$

where i = H,V,D

From the above mentioned scaling and wavelet functions, namely, W_ϕ and W_ψ , one can easily acquired through the inverse discrete wavelet transform for the image signal as

$$f(x,y,z) = \frac{1}{\sqrt{MNL}} \sum_{m} \sum_{n} \sum_{l} W_{\phi}\, (j_0, m, n, l)\, \phi_{j_{o,m,n,l}}(x,y,z) + \frac{1}{\sqrt{MNL}} \sum_{i=H,V,D} \sum_{j=j_0}^{\infty} \sum_{m} \sum_{n} \sum_{l} W_{\psi}^{i}\, (j,m,n,l)\, \psi_{j,m,n,l}^{i}(x,y,z)$$

VII. CONCLUSION AND FUTURE SCOPE

In this paper, we have presented the mathematical foundation of the wavelets in general and multi-dimensional wavelets (1D, 2D and 3D) for image coding in particular. We have indicated how a few filters originate from a mathematical expression for a continuous wavelet (crude wavelets) while other wavelets start out as filters with just a few points and then are built into a suitable estimation of a continuous wavelet. We have compared the fast Fourier transform (FFT) to the continuous wavelet transform (CWT). We looked at short-time Fourier transforms and then introduced the concept of wavelet transforms by comparing them. The main drawbacks of windows Fourier transform or short Time Fourier Transform (STFT) have been summarized. We have also investigated on to the discrete wavelet transform (DWT) and showed how it is similar in many ways to the CWT.

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