Design of an Adaptive Nonlinear Controller for an Autonomous Underwater Vehicle

Fahimeh Rezazadegan, Khoshnam Shojaee & Abbas Chatraei
Department of Electrical Engineering, Islamic Azad University, Najafabad Branch, Najafabad, Iran
E-mail: fahime.rezazadegan@gmail.com, khoshnam_shojaee@gmail.com, abbas.chatraei@gmail.com

Abstract – The paper addresses the problem of trajectory tracking control of underactuated underwater vehicles in the presence of parametric uncertainty. Based on backstepping design approach, an adaptive control law for 6 DOF model is derived for the trajectory tracking problem. The desired trajectory does not need to be of a particular type and in fact can be any sufficiently smooth bounded curve parameterized by time. The reference yaw velocity has not to satisfy various kinds of persistently exciting conditions which is often needed in previous works. It is assumed that system parameters are unknown and have to be estimated accurately. The stability analysis is carried out using the Lyapunov theorem. The effectiveness of the proposed control method is demonstrated through numerical simulations.

Keywords – Adaptive control, Back-stepping, Lyapunov theory, Trajectory tracking, Underactuated underwater vehicle

I. INTRODUCTION

In recent years, there has been a growing interest in the development of unmanned underwater vehicles (UUVS) especially Autonomous Underwater Vehicles (AUVs).

AUVs have been applied to many domains, such as scientific of deep sea, discovering of offshore oil field, long range survey, underwater pipelines tracking, oceanographic mapping, exploitation of underwater resources, military missions and etc.

Besides their numerous practical applications, AUVs present a challenging control problem since most of them are underactuated, i.e., they have fewer actuated inputs than degrees of freedom (DOF), imposing nonintegrable acceleration constraints. In addition, AUVs’ kinematic and dynamic models are highly nonlinear and coupled [1], and hydrodynamic coefficients of vehicles are difficult to be accurately estimated.

In the past variety of design techniques based on feedback linearization, sliding mode control, adaptive control, robust control, optimal control have been attempted [2], [3], [4]. But traditional modern strategies, requires an accurate model of the system to be controlled which is hardly possible. So the researchers resorted to adaptive control to estimate the system parameters and then construct controllers [5], [6], [7], [8], [9].

For the control of nonlinear deterministic and uncertain MIMO systems, often a backstepping design technique [10] is used. This is a sequential design process applicable to systems with matched as well as unmatched uncertainties. It provides flexibility in the choice of Lyapunov functions and stabilizing virtual control signals at each step of the design process for shaping the closed-loop responses.

Recently the design of a composite adaptive control system using backstepping technique has been also considered [11]. This approach combines the direct and indirect adaptive schemes for faster convergence of estimated parameters and the tracking error. The trajectory planning and tracking control of AUVs in the horizontal plane without parameter uncertainties using backstepping procedure has been attempted [9]. In [12] a back-stepping nonlinear controller with certain parameters without any disturbances was proposed. However, there has been very few works in the literature considering six DOF trajectory tracking control of AUVs.

In this paper we considering six DOF model to design a controller for an AUV. In addition we have assumed that the system parameters are unknown, and the vehicle experiences external disturbance forces. On the other hand we extending the control design was proposed in [12] with considering parametric uncertainties. The desired trajectory does not need to be of a particular type and can be any sufficiently smooth
bounded curve parameterized by time. In fact an adaptive back-stepping controller using parameter estimation with projection algorithm is proposed in this paper. Then with the suggestion of a novel Lyapunov function, stability analysis will carry out using Lyapunov theory and Barbalat’s lemma. The results verify our control design robustness in the presence of parameter uncertainty and illustrate that tracking remain very satisfactory.

II. PROBLEM STATEMENT

A. Coordinate Frames

When analyzing the motion of marine vehicles in six degrees-of-freedom (6 DOF), it is convenient to define two coordinate frames as indicated in Fig. 1. The moving coordinate frame X_o Y_o Z_o is conveniently fixed to the vehicle and is called the body-fixed reference frame. The origin 0 of the body-fixed frame is usually chosen to coincide with the center of gravity (CG) when the CG is in the principal plane of symmetry or at any other convenient point if this is not the case.

For marine vehicles, the body axes X_o, Y_o and Z_o coincide with the principal axes of inertia, and are usually defined as follows:

X_o : longitudinal axis (directed from aft to fore)
Y_o : transverse axis (directed to starboard)
Z_o : normal axis (directed from top to bottom).

Fig.1: Earth-fixed frame and Body-fixed frame for AUV.

B. AUV Equations of motion

We adopt the standard notation for the equations of motion of an AUV, see [12], [13], [14]. Linear velocity v = [u v w]^T consists of surge, sway and heave, angular velocity ω = [p q r]^T consists of roll rate, pitch rate and yaw rate, and attitude η = [ψ θ ψ]^T consists of roll angle, pitch angle and heading angle. Furthermore we assume that the center of gravity and the center of buoyancy are located vertically on the O_z Z_o-axis, and that there are no couplings (off-diagonal terms) in the matrices M, D, and D_y(v).

The mathematical model of an AUV in 6 DOF can be described as:

\[ \dot{\eta}_1 = J_1(\eta_2) v_1, \quad \dot{\eta}_2 = J_2(\eta_2) v_2; \]
\[ M \ddot{v} = -C(v) v - D(v) v - g(\eta) - \tau \]

(1)

where \( \eta = [\eta_1, \eta_2]^T \) with \( \eta_1 = [x, y, z]^T \) and \( \eta_2 = [\varphi, \theta, \psi]^T \) is the position and orientation vector in earth-fixed frame, \( v = [v_1, v_2]^T \) with \( v_1 = [u, v, w]^T \) and \( v_2 = [p, q, r]^T \) is the velocity and angular rate vector in body-fixed frame, the positive definite inertia matrix \( M = M_{BB} + M_A \) includes the inertia \( M_{BB} \) of the vehicle as a rigid body and the added inertia \( M_A \) due to the acceleration of the wave, the skew symmetrical matrix \( C(v) \) is the matrix of Coriolis and centripetal, the hydrodynamic damping term \( D(v) \) takes into account the dissipation of energy due to the friction exerted by the fluid surrounding AUV. The vector \( g(\eta) \) is the gravitational forces and moments vector, \( \tau \) is the input torque vector, and the transformation matrices \( J_1(\eta_2) \) and \( J_2(\eta_2) \) are as following:

\[ J_1(\eta_2) = \begin{bmatrix} c(\psi) c(\theta) & -s(\psi) c(\varphi) + s(\varphi) s(\psi) c(\theta) & s(\psi) s(\varphi) + s(\varphi) c(\psi) c(\theta) \\ s(\psi) c(\theta) & c(\psi) c(\varphi) + c(\varphi) s(\psi) c(\theta) & -c(\psi) s(\varphi) + c(\varphi) s(\psi) c(\theta) \\ -s(\theta) & c(\theta) & s(\theta) \end{bmatrix} \]
\[ J_2(\eta_2) = \begin{bmatrix} 1 & s(\varphi) t(\theta) & c(\varphi) t(\theta) \\ 0 & c(\theta) & -s(\theta) \\ 0 & s(\theta) / c(\theta) & c(\theta) / c(\theta) \end{bmatrix} \]

(2)

Where \( c(\cdot) = \cos(\cdot) \), \( s(\cdot) = \sin(\cdot) \), \( t(\cdot) = \tan(\cdot) \).

So, the general mathematical model of an underactuated AUV in surge, sway, heave and heading motion with ignoring roll motion is:

\[ \dot{x} = \cos(\psi) \cos(\theta) u - \sin(\psi) v + \sin(\theta) \cos(\psi) w \]
\[ \dot{y} = \sin(\psi) \cos(\theta) u + \cos(\psi) v + \sin(\theta) \sin(\psi) w \]
\[ \dot{z} = -\sin(\theta) u + \cos(\theta) w \]
\[ \dot{\psi} = q \]
\[ \dot{\theta} = \frac{r}{\cos(\theta)} \]
\[ \dot{\varphi} = \frac{m_2}{m_1} \left( vr - \frac{m_3}{m_1} wq \right) - \frac{m_1}{m_1} r_e - \frac{d_1}{m_1} u + \frac{1}{m_1} r_e \]

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The positive constant terms $d_i$ and $m_i$ (i=1, 2, 3, 4, 5) denote the hydrodynamic damping and AUV inertia including added mass in surge, sway, heave, pitch and heaving, respectively. The available controls are the surge force $\tau_x$, pitch moment $\tau_p$ and the yaw moment $\tau_z$. Since AUVs do not have independent actuators in the sway and heave axes, the vehicle represented by the mathematical model (1) is underactuated. Our objective is to design a controller $(\tau_x, \tau_p, \tau_z)$ to track the reference trajectory generated by the virtual vehicle model (4).

$$\dot{x}_e = \cos(\psi_e)\cos(\theta_e)u - \sin(\psi_e)\psi_e + \sin(\theta_e)\sin(\psi_e)w_d$$
$$-\sin(\theta_e)u + \cos(\theta_e)w_d$$

$$\psi_e = \frac{\tau_d}{\cos(\theta_e)}$$

$$\dot{u}_d = \frac{m_{12}}{m_{11}}v_d - \frac{m_{30}}{m_{11}}q_d - \frac{d_{11}}{m_{11}}u_d + \frac{1}{m_{11}}\tau_{vd}$$

$$\dot{v}_d = \frac{m_{11}}{m_{12}}u_d - \frac{d_{23}}{m_{12}}v_d$$

$$\dot{w}_d = \frac{m_{30}}{m_{12}}u_d - \frac{d_{33}}{m_{12}}w_d$$

$$\dot{q}_d = \frac{m_{33} - m_{11}}{m_{35}}u_d - \frac{d_{35}}{m_{35}} - \frac{\rho g \Delta \bar{M}}{m_{35}} \sin(\theta_e) + \frac{1}{m_{35}}\tau_{qd}$$

$$\dot{r}_d = \frac{m_{13} - m_{12}}{m_{36}}u_d V_d - \frac{d_{36}}{m_{36}}q_d + \frac{1}{m_{36}}\tau_{rd}$$

Now with substituting (7) in the system equations (6), error system equations (8) is achieved:

$$\dot{x}_e = u_e - (\sigma_2 - \sigma + \cos(\theta_e)(\sigma_e - \sigma))\sigma^{-1}u_e + \cos(\theta_e)\delta\sigma\dot{u}_d - \sigma_1\psi\sigma^{-1}w_d + \sin(\theta_e)\sigma\dot{w}_d - \sigma_1\psi w_d$$

$$\dot{y}_e = v_e - \cos(\theta_e)\delta\dot{u}_d \sigma^{-1}y_e - (\sigma_2 - \sigma)\sigma^{-1}v_d + \sin(\sigma_1)\delta\dot{w}_d \sigma^{-1}y_e - (\sigma_2 - \sigma)\sigma^{-1}w_d$$

$$\dot{z}_e = w_e - (\sigma_2 - \sigma)\sigma^{-1}z_e$$

The positive constant terms $d_i$ and $m_i$ (i=1, 2, 3, 4, 5) denote the hydrodynamic damping and AUV inertia including added mass in surge, sway, heave, pitch and heaving, respectively. The available controls are the surge force $\tau_x$, pitch moment $\tau_p$ and the yaw moment $\tau_z$. Since AUVs do not have independent actuators in the sway and heave axes, the vehicle represented by the mathematical model (1) is underactuated. Our objective is to design a controller $(\tau_x, \tau_p, \tau_z)$ to track the reference trajectory generated by the virtual vehicle model (4).
\[
\begin{align*}
\dot{v}_z &= -\frac{m_{11}}{m_{22}} (u_r r_x + u_r r_y + u_i r_z) - \frac{d_{22}}{m_{22}} v_x, \\
\dot{w}_e &= \frac{m_{11}}{m_{33}} (u_i q_r + u_i q_d + u_i q_e) - \frac{d_{33}}{m_{33}} w_e, \\
\dot{q}_e &= \frac{m_{33} - m_{11}}{m_{33}} u_e w_d - \frac{d_{35}}{m_{33}} q_i - \rho \eta d_\text{air} \sin(\theta) + \frac{1}{m_{33}} \tau_{\text{act}} - \dot{q}_i \\
\dot{r}_e &= \frac{m_{11} - m_{22}}{m_{66}} r_v - \frac{d_{66}}{m_{66}} r_i + \frac{1}{m_{66}} \tau_{\text{act}} - r_d.
\end{align*}
\]

Where:
\[
\sigma = \sqrt{1 + x^2 + y^2 + z^2}, \quad \sigma_1 = \sqrt{1 + x^2 + (1 - \delta z^2) y^2 + z^2}
\]

And,
\[
\begin{align*}
\sigma_1 &= -((\cos(z) - 1)) \sigma_2 - \sin(z) z_x + \cos(\theta) \cos(\phi) \times ((\cos(z) - 1)) \sigma_1 + \sin(z) z_d \\
\sigma_2 &= \cos(\phi) \sin(z) \sigma_1 - ((\cos(z) - 1)) \delta_1 y_t \sigma_1 - \sin(z) (\sin(z) - 1) \delta_1 y_t \sigma_1 - \sin(z) (\delta_1 y_t) \sigma_1 - \sin(z) (\sin(z) - 1) \sigma_1
\end{align*}
\]

\[
\begin{align*}
\sigma_3 &= -((\cos(z) - 1)) \sigma_2 + \sin(z) \delta_1 z_x + \sin(\theta) \cos(\phi)) ((\cos(z) - 1)) \sigma_1 - \sin(z) \delta_1 z_x + \sin(z) \delta_1 z_x + \sin(z) (\delta_1 y_t) \sigma_1 - \sin(z) (\sin(z) - 1) \sigma_1 + \sin(z) (\delta_1 y_t) \sigma_1 - \sin(z) (\sin(z) - 1) \sigma_1
\end{align*}
\]

\[
\begin{align*}
\sigma_4 &= \delta_1 \sigma_1 - \sigma_1 (z_x - y_x - \delta_2 y_t x + y_x - y_t - x + z_x + x) + \cos(\phi) (2 - \delta_2 z_x + \tan(\theta)) (\sin(z) - 1) \sigma_1
\end{align*}
\]

III. ADAPTIVE NONLINEAR CONTROL DESIGN

First, the virtual velocity controls of \(u_e, q_r, r_z\) are designed to asymptotically stabilize \(x_e, y_e, z_e, \dot{z}_e, \ddot{z}_e, w_e\) and \(v_r\) at the origin. Then based on the back-stepping technique, the controls \(r_e, \tau_p\) will be designed to make the errors between the virtual velocity controls and their actual values exponentially vanish. It is clear that \(u_e\) enters the \(v_r\) and \(w_e\) dynamics, so to simplify the stability analysis, we will design a bounded virtual control of \(u_e\). The virtual controls of \(q_r\) and \(r_z\) are chosen to stabilize the \(z_l\) and \(z_r\) dynamics.

A. Assumption 1

The reference signals \(u_d, q_d, r_d, u_d, \dot{q}_d\) are bounded. There exists a strictly positive constant \(u_{d,\text{min}}\) such that \(|u_d(t)| \geq u_{d,\text{min}}, \forall t \geq 0\). This condition is much less restrictive than a persistently exciting condition on the yaw reference velocity. The reference sway and heave velocities satisfy
\[
|v_w(t)| < |u_w(t)|, |w(t)| < |u_w(t)|, \forall t \geq 0.
\]

B. Assumption 2

The reference pitch angle satisfies
\[
|\theta_p(t)| \leq 0.5 \pi, \forall t \geq 0 \text{ because of singularity avoidance.}
\]

Define the virtual control errors as:
\[
\tilde{u}_e = u_e - u_e' \quad \tilde{q}_r = q_i - q_r' \quad \tilde{r}_e = r_e - r_e'.
\]

where \(u_e', q_r'\) and \(r_e'\) are the virtual velocity controls of \(u_e, q_r, r_z\) respectively.

Since the standard application of back-stepping lead to complex controller, we have chosen a simple virtual control law \(u_e'\) without canceling some known terms.
\[ u_i^d = -\delta_0 \sigma_i^{-1} x_i + (\sigma_i z_i - \sigma_i \cos(\theta) \cos(\theta_i) (\sigma_i - \sigma_i)) \sigma_i^{-1} u_i - \\
\cos(\theta) \varepsilon_i \varphi_i \sigma_i^{-1} y_i - (\delta_i \sigma_i - \cos(\theta) \sin(\theta_i) (\sigma_i - \sigma_i)) \sigma_i^{-1} w_i \]

\[ q_i^d = q_i + q_i^r, \quad r_i^d = r_i + r_i^r, \]

\[ r_{i2}^d = (\delta_i \sigma_i^{-1} \sigma_i^{-2} x_i \dot{q}_i^r - h_i) \cos(\theta) \]

\[ g_{i2}^d = (\delta_i \tan(\theta) \sigma_i^{-1} y_i \dot{r}_i^r - \delta_i \sigma_i^{-1} \sigma_i^{-2} x_i \dot{u}_i^r - h_i) \]

\[ q_{i2}^d = (\delta_i \sigma_i^{-2} \sigma_i^{-2} y_i \dot{r}_i^r - \delta_i \sigma_i^{-1} \sigma_i^{-2} x_i \dot{u}_i^r - s_i) \]

Where \( \delta_i, c_i \), and \( c_2 \) being positive constants.

Remark 1. \( q_i^d \) and \( r_i^d \) exponentially converge to zero when \( z_i \) and \( z_i \) do so and make stability analysis simpler.

Remark 2. \( q_i^d \) and \( r_i^d \) are Lipschitz in \((x_i, y_i, z_i, v_i, w_i)\) that plays a crucial role in the stability analysis of the closed loop system. It could be shown that the virtual control \( u_i^d \) is bounded as:

\[
\left| u_i^d \right| \leq \delta_0 (1 + \delta_i^r + \delta_i^\theta) |u_i| + |\delta_i v_i| + \left| \dot{z}_i \right| \leq \delta_0 u_0
\]

\[
\dot{e}_i = \dot{m}_i \left( -\rho_i \dot{w} - \frac{m_{i2}}{m_{i1}} \varphi_i + \frac{m_{i3}}{m_{i1}} \varphi_i + \frac{\dot{d}_{i1}}{m_{i1}} (w_i \dot{u}_i - \dot{w}_i) + \dot{z}_i - \dot{z}_i \right) + \delta_i \sigma_i^{-1} \sigma_i^{-2} x_i \dot{u}_i^r + \delta_i \sigma_i^{-1} \sigma_i^{-2} y_i \dot{r}_i^r
\]

\[
\dot{r}_i = \dot{m}_{i6} \left( -\rho_i \ddot{r}_i - \frac{m_{i1} - m_{i2}}{m_{i6}} \varphi_i + \frac{\dot{d}_{i6}}{m_{i6}} (r_i \dot{u}_i + r_i \dot{w}_i) + \dot{r}_i \dot{z}_i - \dot{z}_i \right)
\]

\[
\dot{q}_i = \dot{m}_{i5} \left( -\rho_i \dot{q}_i - \frac{m_{i3} - m_{i5}}{m_{i5}} \varphi_i + \frac{\dot{d}_{i5}}{m_{i5}} (q_i \dot{u}_i + q_i \dot{w}_i) + \rho \Delta \varphi \dot{\varphi} \sigma_i^{-1} \sigma_i^{-2} x_i \dot{u}_i^r + \rho \Delta \varphi \dot{\varphi} \sigma_i^{-1} \sigma_i^{-2} y_i \dot{r}_i^r \right)
\]

IV. STABILITY ANALYSIS

In this section, we prove that the control signals defined above are well defined, bounded, and that the closed loop system is asymptotically stable at the origin. To simplify stability analysis of this closed loop system, we consider two subsystems \((x_i, y_i, z_i, v_i, w_i)\) and \((z_i, \dot{z}_i, \dot{u}_i, \dot{q}_i, \dot{r}_i)\) -subsystem then move to \((x_i, y_i, z_i, v_i, w_i)\) - subsystem.

A. Stability Analysis of \((z_i, \dot{z}_i, \dot{u}_i, \dot{q}_i, \dot{r}_i)\) - subsystem

From the closed loop system equations, it is direct to show that this subsystem is exponentially stable at the origin by using the following Lyapunov function:

\[ V_1 = \frac{m_{i1}}{2} \dot{z}_1^2 + \frac{m_{i6}}{2} \dot{z}_2^2 + \frac{m_{i1}}{2} \dot{u}_1^2 + \frac{m_{i6}}{2} \dot{q}_1^2 + \frac{m_{i6}}{2} \dot{r}_1^2 \]

\[ + \frac{1}{2} \sum_{i \neq q, r} \delta_i \Gamma_i \dot{\varphi}_i^r. \]

Where \( \dot{\varphi}_i = \theta_i - \tilde{\theta}_i \). From (15) estimation rule is derived:

\[ \sum_{i \neq q, r} \dot{\varphi}_i = -\Gamma_i (\varphi_i \dot{u}_i + \varphi_i \dot{q}_i + \varphi_i \dot{r}_i). \]

\[ \theta_i = \left[ \begin{array}{c} m_{i22} \quad m_{i33} \\ m_{i22} m_{i33} \quad \frac{d_{i33}}{m_{i1}} \quad m_{i22} m_{i33} \quad m_{i22} m_{i33} \quad \frac{d_{i33}}{m_{i1}} \end{array} \right]. \]

\[ \theta_i = \left[ \begin{array}{c} m_{i33} - m_{i1} \\ \frac{d_{i33}}{m_{i1}} \quad m_{i33} m_{i22} \quad \frac{d_{i33}}{m_{i1}} \quad m_{i22} m_{i33} \quad \frac{d_{i33}}{m_{i1}} \quad m_{i22} m_{i33} \quad \frac{d_{i33}}{m_{i1}} \end{array} \right]. \]
Where:
\[
\sigma_1 = \min \left( c_1, c_2, (\rho_1 + d_1 n_1^{m_1^{-1}}), (\rho_2 + d_2 s^{m_2^{-1}}), (\rho_3 + d_3 o^{m_3^{-1}}) \right)
\]

Considering (16), we can observe \( z_i, z_k, \tilde{u}_i, \tilde{q}_i, \tilde{r}_i \in L_1 \), and \( \tilde{\theta} \in L_2 \Rightarrow \tilde{\theta} \in L_1 \Rightarrow V_i \in L_1 \), then with integrating from the last equation of (16):
\[
\int_0^\infty \frac{\partial V_i}{\partial t} dt \geq \int_0^\infty \left( -c_1 z_i^2 - c_2 z_k^2 - (\rho_1 + d_1 n_1^{m_1^{-1}}) \tilde{u}_i^2 - (\rho_2 + d_2 s^{m_2^{-1}}) \tilde{q}_i^2 - (\rho_3 + d_3 o^{m_3^{-1}}) \tilde{r}_i^2 \right) dt
\]

Since Left side of (18) is bounded, it could be written \( z_i, z_k, \tilde{u}_i, \tilde{q}_i, \tilde{r}_i \in (L_2 \cap L_1) \)

Also, considering five last equations of closed loop system, it is not hard to illustrate that:
\[
z_i, z_k, \tilde{u}_i, \tilde{q}_i, \tilde{r}_i \in L_2
\]

Now using (19), (20) and Barbalat Lemma, asymptotically convergence to zero with an appropriate choice of the design constants \( \delta_0, \delta_1, \delta_2 \) is proved.

**B. Stability Analysis of \((x_e, y_e, z_e, v_e, \omega_e)\) subsystem**

From the closed loop system equations, it is direct to show that this subsystem is exponentially stable at the origin by using the following Lyapunov function:
\[
V_2 = 1 + x_e^2 + y_e^2 + z_e^2 - 1 + \frac{1}{2} \delta_3 (v_e^2 + \omega_e^2)
\]

V. SIMULATION RESULTS

In this section, to illustrate the performance of the tracking control algorithm, computer simulations using matlab software have been carried out assuming that the vehicle is directly actuated in force in the \( x_B \) direction and in torque about the \( y_B \) and \( z_B \) axes. Results are shown in Fig. 2, 3, 4, 5, 6 verified that desired trajectory does not need to be of a particular type and in fact proposed adaptive controller can track any sufficiently smooth curve parameterized by time accurately. Reference trajectory generated by (4) is considered as:
\[
\begin{align*}
\tau_{ud} &= -(m_{12} w_{ud} - m_{13} w_{ud} q_{ud}) + 5 d_{11} \\
\tau_{wd} &= -(m_{13} + m_{11}) w_{ud} w_{ud} - d_{22} s_{ud} - 69.42 \sin(\theta_{ud}) + m_{22} (-\theta_{ud} + 0.2 - \dot{\theta}_{ud}) \\
\tau_{rd} &= -(m_{11} - m_{22}) u_{ud} v_{ud}
\end{align*}
\]

for first 150 seconds, and:
\[
\begin{align*}
\tau_{ud} &= -(m_{11} - m_{22}) u_{ud} v_{ud} + 5 \sin(0.003 t) - 0.5 \sin(0.005 t)
\end{align*}
\]

for the rest of simulation time. This choice consists of straight line and helix with constant curvature and torsion in the reference trajectory. The parameters used for producing reference trajectory shown in Appendix are taken from [12]. The initial conditions are picked as:
\[
\eta_0(t_0) = [0, 0, -20, 0, 0, 10, 0, 0, 0, 0], \quad \eta(t_0) = [-50, -50, 0, 0, 0, 0, 0, 0, 0, 0]
\]

and the initial parameter estimation values are picked as 40% of real parameter values of the system.
VI. CONCLUSION

In this paper, a 6-DOF trajectory tracking control scheme for an underactuated underwater vehicle in presence of parameter uncertainties was presented. Using the Lyapunov theory and back-stepping technique, an adaptive controller was designed. Projection algorithm was used to update the estimate of the unknown parameters to avoid parameter drift instability. It was shown that in the closed-loop system including the adaptive law, the actual trajectory asymptotically converge to the reference trajectory. Simulation results verified convergence of position and orientation tracking errors asymptotically even reference trajectory consist of straight line. However, in this paper, the system matrices was assumed diagonal and roll motion was neglected. In addition we concentrated on the case of a vehicle workspace free of obstacles. In the future, more work will be done to improve the design.
VII. APPENDIX

In this section, parameters of the AUV have been used for simulations are given.

RIGID BODY AND HYDRODYNAMIC PARAMETERS OF THE AUV

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>$M$</td>
<td>1089.8</td>
<td>Kg</td>
</tr>
<tr>
<td>Length</td>
<td>$L$</td>
<td>5.56</td>
<td>M</td>
</tr>
<tr>
<td>Mass+Added mass in surge</td>
<td>$m_1 = m - X_0$</td>
<td>1116</td>
<td>Kg</td>
</tr>
<tr>
<td>Mass+Added mass in sway</td>
<td>$m_2 = m - Y_0$</td>
<td>2133</td>
<td>Kg</td>
</tr>
<tr>
<td>Mass+Added mass in heave</td>
<td>$m_3 = m - Z_0$</td>
<td>2133</td>
<td>Kg</td>
</tr>
<tr>
<td>Inertia+Added inertia in roll</td>
<td>$m_4 = I - K_0$</td>
<td>36.7</td>
<td>kgm$^2$</td>
</tr>
<tr>
<td>Inertia+Added inertia in pitch</td>
<td>$m_5 = I - M_0$</td>
<td>4061</td>
<td>kgm$^2$</td>
</tr>
<tr>
<td>Inertia+Added inertia in yaw</td>
<td>$m_6 = I - N_0$</td>
<td>4061</td>
<td>kgm$^2$</td>
</tr>
<tr>
<td>Surge linear drag</td>
<td>$d_1 = -X_0$</td>
<td>25.5</td>
<td>kgs$^{-1}$</td>
</tr>
<tr>
<td>Sway linear drag</td>
<td>$d_2 = -Y_0$</td>
<td>138</td>
<td>kgs$^{-1}$</td>
</tr>
<tr>
<td>Heave linear drag</td>
<td>$d_3 = -Z_0$</td>
<td>138</td>
<td>kgs$^{-1}$</td>
</tr>
<tr>
<td>Roll linear drag</td>
<td>$d_4 = -K_0$</td>
<td>10</td>
<td>kgm$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>Pitch linear drag</td>
<td>$d_5 = -M_0$</td>
<td>490</td>
<td>kgm$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>Yaw linear drag</td>
<td>$d_6 = -N_0$</td>
<td>490</td>
<td>kgm$^2$ s$^{-1}$</td>
</tr>
</tbody>
</table>

VIII. REFERENCES


