FSM Analysis for Box Girder Bridges

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Abstract – The finite strip method has already proven to be the most efficient numerical technique for the static analysis of bridges. In fact the method is even more efficient for dynamic analysis of bridges. The structure can be divided into a number of finite strips. In each strip the displacement components at any point are expressed in terms of the displacement parameters of nodal lines by means of simple polynomials in the transverse direction and a continuously differentiable smooth series in the longitudinal direction. Thus, the number of dimensions of the analysis is reduced by one. The minimum number of degrees of freedom along a nodal line in the finite strip method is equal to twice the number of terms used in the series and this is normally much less than that for finite element method, which requires a minimum of three times the number of nodes along the same line. Hence the size and the bandwidth of the matrices are greatly reduced, and consequently it can be handled by personal computers and solved in much shorter time.

In this study the finite strip method is applied for analysis of box girder bridges subjected to dead load and IRC live load. The computer program for the analysis of box girder bridges is based on the lower order rectangular bending strip and plane stress strip. The program can be used to analyse the box girder for dead loads and IRC live loads. For this purpose the loading of IRC class 70R (tracked and wheeled), class AA (tracked and wheeled) and the class A has been incorporated in program. The impact factor has also been considered for live load analysis. The analysis program calculates bending stresses, shear stresses, bending moments and torsion moments at the desired sections along the span. However, the program is limited to simply support conditions without intermediate diaphragms.

Keywords – finite strip method, box girder, simply supported, IRC loads.

I. INTRODUCTION

Cheung has introduced the use of the finite strip approach to analyse the folded plate structures in 1969, followed by the box girder in 1971. He combined the bending strip and plane stress strip to form the shell strip. The lower order finite strip for shell was then applied to analyse the orthotropic folded plate and eccentric folded plate in 1970. Higher order finite strip element with one internal nodal line was introduced by Loo and Cusens in 1971. This approach was then used by Cheung and Cheung in 1971 to analyse cut conical shape structures.

The objective of this study was to elaborate the behaviour of rectangular and trapezoidal box girder bridges due to various positions of IRC load. Besides, this study was also aimed to reveal the characteristics of wheel load distribution on the right box girder bridges due to various loading positions in accordance to IRC loadings for Bridges. For these purposes, the Finite Strip Method was applied.

II. BASIC CONCEPTS

Following the normal procedures of the finite strip method, a right box girder bridge structure is first divided into a number of lower order rectangular strips (LO2) bounded by nodal lines. Within each strip, the in-plane displacements are obtained by linear interpolation between corresponding values at the nodal lines, whereas the out-of-plane displacements are given by an interpolation scheme using polynomials. The nomenclature of displacements at a nodal line and within a strip is illustrated in Figure 1 and 2 respectively. Under this scheme of discretisation, the displacement function at a section is fully described by the displacements at the nodal lines. Hence the number of independent variables is four times the number of nodal lines. If the displacement function can be expressed as a combination of only a few essential shape functions, the resulting bandwidth will then be much reduced, although some loss of accuracy may be anticipated.
Fluid viscous dampers have the unique ability to simultaneously reduce both stress and deflection within a structure subjected to a transient. This is because a fluid viscous damper varies its force only with velocity, which provides a response that is inherently out-of-phase with stresses due to flexing of the structure.

The \( m \)th term for the plate in the simply supported box girder can be written as:

\[
\begin{bmatrix}
[S]_{11} & [S]_{12} \\
[S]_{21} & [S]_{22}
\end{bmatrix}
\begin{bmatrix}
\delta_{1m} \\
\delta_{2m}
\end{bmatrix} = \begin{bmatrix}
F_{1m} \\
F_{2m}
\end{bmatrix}
\]

or \( [S]_{mm} \{\delta\}_{m} = \{F\}_{m} \)  

In which

\[ [\delta]_{m} = [u_{1}, v_{1}, w_{1}, 1, u_{2}, v_{2}, w_{2}, 2]^{T}_{m} \]
\[ [F]_{m} = [u_{1}, v_{1}, w_{1}, M_{1}, u_{2}, v_{2}, w_{2}, M_{2}]^{T}_{m} \]

A typical sub matrix \([S]_{ij}\) of the matrix \([S]_{mm}\) is made up from appropriate sub matrix of the bending stiffness \([S]^{b}_{ij}\) and plane-stress matrix \([S]^{p}_{ij}\) Thus

\[ [S]_{mm} = \begin{bmatrix}
[S]^{b}_{ij} \\
[S]^{p}_{ij}
\end{bmatrix} \]

For the whole structure, the equation can be constructed by assembling contribution from each plate:

\[
\sum_{m=1}^{n} [S]_{mm} \{\delta\}_{m} = \{F\}_{m}
\]

Matrix equation as shown in Equation 1 is written in the local coordinate, whereby different plates may use different local coordinate systems. To assemble, it will be easier if all of them are transformed into the same coordinate system, called global coordinate system. Equation 1 is to be transformed into the global coordinate system. Transformation is carried before the assembling process. Once the global displacement matrix found, it can be easily transformed back into the local coordinate system, to determine the local displacement matrix as well as stresses.

3.1 Analysis of Plane Stress Plate

The displacement within the strip can be related with nodal line displacement.

\[
(u) = \sum_{m=1}^{n} \begin{bmatrix}
1 \\
-u_{2m}^{b} \frac{v_{m}}{b}
\end{bmatrix} \begin{bmatrix}
N_{m}^{b} \\
N_{m}^{p}
\end{bmatrix} [x']
\]

\[
(v) = \sum_{m=1}^{n} \begin{bmatrix}
1 \\
v_{2m}^{d} \frac{u_{m}}{b}
\end{bmatrix} \begin{bmatrix}
N_{m}^{b} \\
N_{m}^{p}
\end{bmatrix} [x']
\]

3.2 Analysis of Bending Plate

A suitable displacement function can be written as

\[
[W]_{m} [N] [\delta]_{m} = \sum_{m=1}^{n} N_{m} \{\delta\}_{m}
\]
In which \( C_1, C_2 \) given as
\[
[C_1] = [(1 - 3x^2 + 2x^3), x(1 - 2x + x^2)]
\]
\[
[C_2] = [(3x^2 - 2x^2), x(x - x^2)]
\]
\[
(\delta)^T_m = [W_{1m} \quad 1m \quad W_{2m} \quad 2m].
\]

If the thickness of a strip is assumed to be constant then the stiffness matrix can be formed as
\[
[S]_{mn} = \int [B]^T[D][B] d (area)
\]

For the special case of two opposite ends simply supported, \([S]_{mn}\) becomes zero for \( m \neq n \).

The consistent load matrix is obtained as follows,
\[
\{F\} = \int [N]_m^T[q] d (area)
\]

\[
\begin{pmatrix}
F_1 \\
F_2 \\
F_3 \\
F_4
\end{pmatrix} = \int \begin{pmatrix}
(N_1)^T \\
(N_2)^T \\
(N_3)^T \\
(N_4)^T
\end{pmatrix}[q] d (area)
\]

Where, \( q = \) load intensity

IV. COMPUTER PROGRAMMING

A computer program for the analysis of box girder bridges has been written in FORTRAN language. This program can be used to analyse box girder bridges as folded plate structures with straight geometry, by applying lower orders rectangular strip. The techniques presented in this program are quite general and can be applied to the analysis of other problems such as plates, shells, solids etc. for the analysis of box girder bridge.

The stiffness matrices are developed for a strip with an orthotropic property for both bending and in-plane analysis. For simply supported strips, the terms of series are uncoupled and the stiffness matrices of each strip are formed, assembled and solved separately. The individual stiffness matrix of a strip is normally computed with respect to the local coordinate system and then transferred to the global coordinate system.

The IRC loads of class 70R (tracked and wheeled), class AA (tracked and wheeled) and class A has been incorporated in the program. The position of front wheel of wheeled vehicle is fixed on the span for maximum moment at any section. For class 70R (wheeled) loading, the coordinates of front axle of train (80 KN) are required as input along with strip numbers. For class A loading the coordinates of front axle of train (27 KN) are to be specified. The coordinates are then stored in the corresponding elements of a two dimensional array for each and every axle of I.R.C. Load. The impact factors have been incorporated in the program as per IRC recommendations. It is computed based on the span and type of loading.

The program gives the final output after summing the contribution due to various terms of Fourier series. The output includes longitudinal and transverse bending stresses, shear stresses, transverse bending moments, and longitudinal moments at different sections of the bridge along span.

V. DISCRETIZATION

The box girder bridges are to be discretized using a LO2 strip for bending stiffness’s. The box girder bridge of rectangular and trapezoidal section adopted for study is assumed to have same properties along different direction. The idealised structure is shown in Figure 4 for 2 cell vertical webs system and Figure 5 for 2 cell inclined webs system. The box girders are divided into 21 strips and there are 20 nodal lines. The top slabs are divided into finer mesh than the bottom slab. The deck slab is divided into 4 strips while the soffit slab is divided into 2 strips. Here only one strip has been used to represent adequately the web of the bridge. The nodes have been numbered to produce a matrix with the narrowest half-band width as shown in Figures 4 and 5 for both the bridge system.

![Fig 4: A typical bridge and a rectangular shell Strip](image1)

![Fig 5: A typical bridge and a trapezoidal shell Strip](image2)
VI. APPLICATION ON BOX GIRDER BRIDGE

The examples solved by this program include two bridges of span 30m having cross-section of 2-cells with vertical and inclined webs. The loading considered in the present analysis is class 70R (tracked) and class A with different disposition along transverse direction.

6.1 Two Cell Bridge with Vertical Webs for Class 70R Loading

![Diagram of Two Cell Bridge with Vertical Webs]

a) Two Cell Bridge with Vertical Webs

![Diagram of Transverse Position of Load for 70R symmetrical]

b) Transverse Position of Load for 70R symmetrical

c) Transverse Position of Load for 70R eccentrically placed

Fig 5: Two Cell Bridge with Vertical Webs

6.2 Two Cell Bridge with Inclined Webs for Class A Loading

![Diagram of Two Cell Bridge with Inclined Webs]

a) Two Cell Bridge with Inclined Webs

![Diagram of Transverse Position of Load for Class A symmetrical]

b) Transverse Position of Load for Class A symmetrical

![Diagram of Transverse Position of Load for Class A eccentrically placed]

c) Transverse Position of Load for Class A eccentrically placed

Fig 6: Two Cell Bridge with Inclined Webs

VII. DEFLECTION OF BOX GIRDER BRIDGE

From the analysis, it is seen that the from the deflection point of view, the rectangular section of box girder bridge gives greater deflection to some extend as compared to trapezoidal section for both the loadings of Class A and Class 70R. A figure shows the difference of deflection between the rectangular and trapezoidal section for both the loadings of Class A and Class 70R.
VIII. LONGITUDINAL STRESSES ($\sigma_Y$)

Figure shows longitudinal stresses for two cell vertical webs (left) and inclined webs (right) for dead load, live load of Class 70R and Class A placed symmetrically and live load of Class 70R and Class A placed with maximum eccentricity respectively.

IX. TRANSVERSE STRESSES ($M_X$)

Figure shows transverse moments for two cell vertical webs (left) and inclined webs (right) for dead load, live load of Class 70R and Class A placed symmetrically and live load of Class 70R and Class A placed with maximum eccentricity respectively.
X. CONCLUSIONS

1. The deck slab must be divided into finer mesh compared to bottom slab due to the effect of concentrated load. In the web only, one strip is regarded as essential as it saves computer time without affecting accuracy.

2. It is observed from the deflection point of view, the trapezoidal shape of box girder gives lesser deflection as compared to rectangular section.

3. Shapes of the box girder significantly change the structural behavior of the bridge. When only dead load is considered, both shapes of box subjected to maximum stresses at bottom of outer webs. However, trapezoidal section gives larger stresses at bottom of outer webs as compared to rectangular section. For upper node of middle web, it is found that trapezoidal section subjected to higher level of stresses.

4. The eccentric loads are more evenly distributed over the box cross section and the variation in longitudinal stresses with different disposition of IRC loads in transverse direction is small.

XI. REFERENCES


