State predictive LQI congestion control for TCP/AQM networks considering effective equilibrium points

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Abstract—The purpose of this paper is to design state predictive LQI congestion controllers by considering effective equilibrium points of nonlinear models of TCP/AQM networks and to illustrate the effectiveness of the designed congestion controllers via the ns-2 (Network Simulator ver.2) simulator. First nonlinear models of TCP/AQM networks are linearized at equilibrium points and a method to configure the equilibrium points of the models of TCP/AQM networks is proposed by focusing on the effective probabilities of packet marks. Second congestion controllers are designed by using state predictive control and LQI control. Finally the designed controllers are implemented in the router of TCP/AQM networks and simulation results are shown to illustrate the effectiveness of the designed congestion controllers by using SIMULINK and the ns-2 simulator.

Index Terms—Computer networks, congestion control, state predictive control, control engineering.

I. INTRODUCTION

Recently data in computer network increases rapidly and a lack of reliability and efficiency by congestion in computer network appears as problems to be solved [1]. Congestion is a traffic jam in computer network. If congestion occurs, a number of data, which is called as packets, exceeding the capacity of computer networks concentrates at nodes such as routers. Packets are kept waiting for long time and dropped in computer networks. Therefore congestion control at nodes plays a significant role to use network resources efficiently and fairly [2].

TCP (Transmission Control Protocol) is used on the Internet as a protocol and it ensures reliable data transfer. In TCP, the receiver sends back an acknowledgement (ACK) if the receiver has received data correctly. Thus the Internet can be treated as a large-scale feedback system. Moreover though TCP has congestion control mechanisms, it has a defect as a lack of efficiency in communication because the congestion control mechanisms work after congestion occurs in computer networks. Thus effective congestion control problems are discussed as problems to design feedback controllers based on systems and control theories [3-8].

On the other hand, AQM (Active Queue Management) seems effective because AQM is mechanism that the routers having local information in computer network control network [9]. Congestion control for TCP/AQM networks which are composed of TCP and AQM mechanism is expected to use network resources efficiently and fairly. In papers [10-16], these AQM mechanisms are designed based on systems and control theories to achieve better performances in computer networks. The efficacy of AQM mechanisms are verified in experiments [17][18]. However those congestion controllers were not designed by considering effective equilibrium points of the nonlinear models for TCP/AQM networks.

In this paper, state predictive LQI congestion controllers are designed by considering effective equilibrium points of nonlinear models of TCP/AQM networks. First nonlinear models of TCP/AQM networks derived in the paper [10] are linearized at equilibrium points. Moreover a condition to configure the equilibrium points of the models of TCP/AQM networks is derived by focusing on the effective probabilities of packet marks. Second congestion controllers are designed by using state predictive control [19][20] and LQI control with Kalman filters [21]. Finally the designed controllers are implemented in the router of TCP/AQM networks and some simulation results are shown via SIMULINK and the ns-2 (Network Simulator ver.2) simulator to demonstrate the effectiveness of the designed controller.

II. DYNAMICS OF TCP/AQM NETWORKS

A. Nonlinear models

In this article, congestion controllers are designed based on the following nonlinear models which describe dynamics of a TCP/AQM network in congestion [10].
\[
\dot{w}_s(t) = \frac{1}{R(t)} - \frac{w_s(t) w_s(t-R(t))}{2} p(t-R(t))
\]

Here \(w_s(t)\) is an average TCP window size, \(q(t)\) is an average queue size and \(p(t)\) is a probability of packet mark. \(C(t)\) is the queue capacity and \(N(t)\) is the number of TCP sessions. The time-delay \(R(t)\) is given as the following linear function of the queue size \(q(t)\),

\[
R(t) = \frac{q(t)}{C(t)} - T_p
\]

where \(T_p\) is the propagation delay of the link and constant. If the queue size and the queue capacity are constant, that is \(q(t)=q_0\), \(C(t)=C\), the time-delay \(R(t)\) also becomes constant.

\[
R_0 = \frac{q_0}{C} - T_p
\]

Moreover if the number of TCP sessions is constant, that is \(N(t)=N\), the nonlinear system can be described as the following nonlinear model with a constant time-delay.

\[
\dot{w}_s(t) = \frac{1}{R_0} - \frac{w_s(t) w_s(t-R_0)}{2} p(t-R_0)
\]

\[
\dot{q}(t) = \frac{N}{R_0} w_s(t) - C
\]

B. Linearized models

The nonlinear model in the equation (3) is easier to design controllers than the nonlinear model in the equation (1). For this nonlinear model, an equilibrium point is introduced as \((w_{so}, q_0, p_0)\). Note that \(q_0\) is same as the value in the equation (2). Here the error variables are defined as follows,

\[
\delta w_s(t) = w_s(t) - w_{so},
\]

\[
\delta q(t) = q(t) - q_0,
\]

\[
\delta p(t) = p(t) - p_0.
\]

Now we define the state variable, the control input and the output as follows,

\[
x_p(t) = \begin{bmatrix} \delta w_s(t) \\ \delta q(t) \end{bmatrix},
\]

\[
u_p(t) = \delta p(t),
\]

\[
y_p(t) = \delta q(t).
\]

Then the linearized model is given as the following state-space form.

\[
\dot{x}_p(t) = A_p x_p(t) + B_p u_p(t - R_0),
\]

\[
y_p(t) = C_p x_p(t).
\]

where

\[
A_p = \begin{bmatrix} -\frac{2N}{R_0^2} & 0 \\ \frac{N}{R_0} & 0 \end{bmatrix},
\]

\[
B_p = \begin{bmatrix} \frac{R_0 C^2}{2N^2} \\ 0 \end{bmatrix},
\]

\[
C_p = [0 \ 1].
\]

In linearization of the equation (3), it is important to configure the equilibrium point because the point depends on the probability of packet marks especially. The probability \(p_0\) at an equilibrium point satisfies the following condition.

\[
p_0 = \frac{2N^2}{(q_0 + T_p C)^2}.
\]

This condition can be derived from the equations (3) at the equilibrium point \((w_{so}, q_0, p_0)\). In the TCP/AQM network, each value of \((w_{so}, q_0, p_0)\) is positive and the probability \(p_0\) is less than or equal to 1. Therefore the following theorem is satisfied.

**Theorem 1:** If the queue size at the equilibrium point \((w_{so}, q_0, p_0)\) satisfies the following condition,

\[
q_0 \geq \sqrt{2N - T_p C},
\]

then \(p_0 \leq 1\) is satisfied.

Proof: This is directly derived based on the equation (5) because the following condition

\[
1 \geq \frac{2N^2}{(q_0 + T_p C)^2}.
\]

is equivalent to the equation (6).

**Remark 1:** Note that Theorem 1 implies existence of an effective equilibrium point \((w_{so}, q_0, p_0)\) and especially the queue size \(q_0\). If the minimum value of the queue size \(q_0\) is less than the right term in the equation (6), the state space model (4) cannot have a point as the linearized model for the nonlinear model (3).

### III. DESIGN OF LQI CONGESTION CONTROL BASED ON STATE PREDICTIVE CONTROL

A. Augmented plants based on state prediction

In the previous section, the linearized model of the nonlinear model (3) can be described as a linear system with input time-delay (4) by considering Theorem 1. Moreover an integrator is introduced to improve the performance at the steady state.

\[
\dot{y}_p(t) = -y_p(t) = -C_p x_p(t).
\]
Then the augmented plant is described as the following equation.

\[
\dot{x}(t) = Ax(t) + Bu(t - R_0), \quad y(t) = Cx(t),
\]

(7)

where

\[
x(t) = \begin{bmatrix} x_p(t) \\ x_i(t) \end{bmatrix}
\]

\[
A = \begin{bmatrix} A_p & 0 \\ -C_p & 0 \end{bmatrix}, B = \begin{bmatrix} B_p \\ 0 \end{bmatrix}, C = \begin{bmatrix} C_p & 0 \end{bmatrix}
\]

Now an auxiliary variable

\[
p(t) = x(t) + \int_{-R_0}^{0} e^{A(t+\beta)}Bu(t+\beta)d\beta
\]

is introduced and the derivation of the equation is applied to the linear input time-delay system (7). This auxiliary variable is called as state prediction because the variable \(p(t)\) means the future value of the state variable \(x(t)\) [19][20]. Thus the first equation of the linear input time-delay system (7) can be transformed as the following linear time-invariant system.

\[
\dot{p}(t) = Ap(t) + e^{-AP_0} Bu(t)
\]

(8)

B. Design of LQ controllers and observers

If \((A, e^{-AP_0}B)\) is controllable in the equation (8), the stabilizing state feedback controller \(u(t) = Kp(t)\) can be designed based on the modern control theory. Because the controllability of the systems in the equation (7) and the equation (8) is equivalent, \((A, e^{-AP_0}B)\) becomes controllable if \((A, B)\) is controllable. In this article, we use the LQ control theory[21].

Now the following performance index is considered for the linear time-invariant system (8).

\[
J = \int_{0}^{\infty} \{ p^T(t)e^{T_{R_0}}Qe^{AR_0}p(t) + u(t)^T R_u(t) \}dt,
\]

\(Q \geq 0, R > 0\).

The stabilizing feedback controller gain for the system (8) is obtained as follows,

\[
K = -R^{-1}B^TPe^{AP_0},
\]

where the symmetric matrix \(P>0\) is a solution of the following Riccati equation.

\[
PA + A^TP - PBR^{-1}B^TP + Q = 0
\]

The stabilizing controller for the linear input time delay system (7) is obtained as follows,

\[
u(t) = Kp(t) = K\{x(t) + \int_{-R_0}^{0} e^{A(t+\beta)}Bu(t+\beta)d\beta\},
\]

because the stability of the system (7) is equivalent to that of the system (8). Note that the performance index for the system (7) becomes the following quadratic from,

\[
J = \int_{0}^{\infty} \{ x^T(t)Qx(t) + u(t)^T R_u(t) \}dt.
\]

Since the observer is not considered in the above discussion, the observer for the linear input time-delay system (4) can be designed as follows,

\[
\dot{\hat{x}}_p(t) = A_p \hat{x}_p(t) + B_p u(t - R_0) - L(y_p(t) - \hat{y}(t)),
\]

\[
\hat{y}(t) = C_p \hat{x}_p(t).
\]

If \((C_p, A_p)\) is observable, this observer gain L can be designed based on a Kalman filter and the optimal observer gain L is designed as

\[
L = -P_p C^T_p R_p^{-1},
\]

based on a unique positive definite solution \(P_p>0\) which satisfies the following Riccati equation.

\[
P_p A_r + A_p P_p - P_p C^T_p R_p^{-1} C_p P_p + Q_p = 0,
\]

\(Q_p \geq 0, R_p > 0\).

C. Structure of the designed congestion controllers

Summarizing the previous discussion, the following controller is designed. Fig. 1 shows the whole closed loop system and the designed LQI congestion controller is depicted as the dashed lined area.

Observer:

\[
\dot{\hat{x}}_p(t) = (A_p + LC_p)\hat{x}_p(t) + B_p u(t - R_0) - Ly(t)
\]

New state variable:

\[
\dot{x}(t) = \begin{bmatrix} \hat{x}_p \\ x_i \end{bmatrix}
\]

State prediction:

\[
\dot{p}(t) = \hat{x}(t) + \int_{-R_0}^{0} e^{A(t+\beta)}Bu(t+\beta)d\beta
\]
**State feedback gain:**

\[ u(t) = K\dot{p}(t) \]

### IV. SIMULATION RESULTS

**A. Network topology, parameters and designed controllers**

Fig. 2 is the network topology considered in this section. The parameters of the network topology are considered as 
\[ N=8 \text{ sessions}, \quad C=71.4 \text{ packets/s}, \quad T_p=0.014 \text{ second} \] and 
\[ q_{\text{max}}=80 \text{ packets}. \] The parameter \( q_{\text{max}} \) means the maximum buffer size in the bottleneck router. If \( q(t)=q_{\text{max}} \) is satisfied, then congestion occurs in the computer network.

The queue sizes at the equilibrium points are 
\[ q_0 = 10, 20, 40 \] and 60 packets. In case of \( q_0 = 10 \), Theorem 1 is not satisfied and the packet mark probability \( p_0 \) becomes more than 1. The

The block diagram of the designed state predictive LQI congestion controller and the considered TCP/AQM network is shown in Fig. 1. The numerical results by using SIMULINK controller does not perform as a congestion avoidance controller. In the other cases, Theorem 1 is satisfied and each congestion avoidance controller works well. Next the designed controller gains and weighting matrixes are described as follows.

**Case 1** (\( q_0 = 10 \)):

![Graphs showing simulation results](image-url)
\[ L = \begin{bmatrix} 1.33 \\ 12.79 \end{bmatrix}, K = \begin{bmatrix} 0.0236 & 0.0043 & -0.0003 \end{bmatrix}, \]
\[ Q = \text{diag}(1,2,3,1), R = 1.10 \times 10^7. \]

Case 2 \( q_0 = 20 \):
\[ L = \begin{bmatrix} 4.44 \\ 18.48 \end{bmatrix}, K = \begin{bmatrix} 0.0352 & 0.0035 & -0.0001 \end{bmatrix}, \]
\[ Q = \text{diag}(1,2,3,1), R = 1.10 \times 10^7. \]

Case 3 \( q_0 = 40 \):
\[ L = \begin{bmatrix} 9.96 \\ 26.03 \end{bmatrix}, K = \begin{bmatrix} 0.0501 & 0.0034 & -0.0001 \end{bmatrix}, \]
\[ Q = \text{diag}(1,2,3,1), R = 1.10 \times 10^7. \]

Case 4 \( q_0 = 60 \):
\[ L = \begin{bmatrix} 15.10 \\ 34.39 \end{bmatrix}, K = \begin{bmatrix} 0.0124 & 0.0006 & -0.0001 \end{bmatrix}, \]
\[ Q = \text{diag}(100,0.1,1), R = 1.90 \times 10^8. \]

To design the observers, the following weighting matrices are used for all cases.
\[ Q_p = \text{diag}(1,1,1), R_p = 1. \]

B. Numerical simulations

By using the nonlinear model in the equation (1), simulation results using SIMULINK are shown in Fig. 3. Fig. 3 shows the queue size \( q(t) \) and the packet mark probability \( p(t) \) in Case 1, Case 2, Case 3 and Case 4. From (a), (c), (e) and (g) in Fig. 3, it can be seen that the queue sizes are stabilized at the equilibrium points \( q_0 = 10, 20, 40, 60 \) packets by using the designed congestion controllers. The overshoots becomes larger as \( q_0 \) increases. This is because the same weighting matrices for LQ control are used except Case 4. The performances of each controller can be improved by choosing the adequate weighting matrices.

Moreover it is also known that the probability of the packet mark \( p(t) \) is over 1 from (b) in Fig. 3. This result shows that Theorem 1 is effective. This result indicates that this congestion controller achieves worse performances than the other congestion controllers do in real data transfer.

C. Ns-2 simulations

Next we use the ns-2 simulator because we can estimate that the dynamical behavior of data transfer in real computer networks is different from the nonlinear model in the equation (1). Fig. 4 shows the network topology and the parameters in the ns-2 simulator. The parameters are same in the numerical simulation. In Fig. 4, the application is FTP in each sender and TCP is used for the protocol of data transfer.

Fig. 5 shows the queue size \( q(t) \) and the packet mark probability \( p(t) \) in Case 1, Case 2, Case 3 and Case 4. From (a), (c), (e) and (h) in Fig. 5, it can be seen that the queue sizes are stabilized at the near equilibrium points \( q_0 = 10, 20, 40 \) and 60 packets though the behaviors becomes oscillatory. In these cases, the congestion controller of Case 4 is best because data transfer is finished at about 100 second. Therefore the designed LQI controllers work well as congestion avoidance controllers. The different performances are caused by the use of same weighting matrices for LQ control and the performances of each controller can be improved by choosing the adequate weighting matrices.
Moreover from (b), (d), (f) and (h) in Fig. 5, the probabilities of the packet mark $p(t)$ become random. The dynamical behavior is quite different from the result in Case 1 from (b) of Fig. 5 and this congestion controller is not suitable for data transfer.

V. CONCLUSION

In this paper, congestion controllers for TCP/AQM networks are designed based on state predictive control and LQI control. Moreover a condition about the effective equilibrium point is derived. In simulation results, it is demonstrated that the designed state predictive LQI congestion controllers stabilize the queue sizes of the bottleneck routers in computer networks by using SIMULINK and the ns-2 simulator.

REFERENCES


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