Cost-Benefit Analysis of Two Similar Warm Standby Satellite System subject to failure due to Premature Atmospheric Reentry, erosion and collisions caused by Space debris with different repair facilities

Ashok Kumar Saini

BLJS COLLEGE, TOSHAM (BHIWANI) HARYANA, INDIA
Email : drashokksaini2009@gmail.com

Abstract- In the present paper we have taken failure due to premature atmospheric reentry, erosion and collisions caused by Space debris with different repair facilities. When the main unit fails then warm standby system becomes operative. Failure due to erosion and collisions caused by Space debris cannot occur simultaneously in both the units and after failure the unit undergoes Type-I or Type-II or Type-III or Type IV repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

Keywords: Warm Standby, failure due to premature atmospheric reentry, erosion and collisions caused by Space debris, first come first serve, MTSF, Availability, Busy period, Benefit -Function.

INTRODUCTION

Premature Atmospheric Reentry:

If you visit the Space Track resource at the NASA Goddard Space Flight Center and examine the annual catalog of space debris and satellites that reenter the atmosphere each year, you will see that at least for satellites in LEO orbits, their lifetimes are eventually terminated by atmospheric drag. This is a severe problem for satellites with orbits below 500 km. The International Space Station, for example, is in a Space Shuttle-accessible orbit with an altitude of 219 miles (perigee 354 km). It requires re-boosting several times every year because atmospheric drag is constantly decreasing its altitude by a kilometer every 12 days. For example, in May 2000, the Space Shuttle used its thrusters to 're-boost' the ISS by adding a whopping 43 km (27 miles) to its current orbit.

This decay process has been extensively modeled by NASA and military scientists in an effort to predict when their LEO satellite systems will be entering the atmosphere. The reason this is crucial to know is that uncontrolled reentries are potentially very dangerous for larger satellites that may not fully burn up. Large fragments (like the pieces from Skylab in July, 1979) can reach the ground, and were this to happen over densely populated areas, property damage or even injury could result.

Space debris, also known as **orbital debris**, **space junk** and **space waste**, is the collection of defunct objects in orbit around Earth. This includes spent rocket stages, old satellites and fragments from disintegration, erosion and collisions. Since orbits overlap with new spacecraft, debris may collide with operational spacecraft.

Spacecraft (unmanned)

Spacecraft in a debris field are subject to wear as a result of impacts. Although critical areas are normally protected by Whipple shields, eliminating most damage, low-mass impacts affect the life of a space mission if the craft is powered by solar panels. These panels are difficult to protect since their face must be directly exposed to the Sun, and they are often punctured by debris. When struck, solar panels produce a cloud of gas-sized particles which does not present as much of a risk as debris does to other spacecraft. The gas, generally a plasma when created, is an electrical risk to the panels themselves.

The effect of impacts with smaller debris was notable on Mir, the Soviet space station, since it remained in space for long periods with its original module panels. Impacts with larger debris usually destroy a spacecraft, and several known (or suspected) impact events have occurred. The earliest on record was the loss of Kosmos 1275, which disappeared on 24 July 1981 (a month after launch). Tracking indicated that it had broken up, creating 300 new objects. Kosmos, which contained no volatiles, is assumed to have collided with a small object but a battery explosion is another possible cause. Kosmos 1484 broke up in a similar manner on 18 October 1993.

Debris impacts on Mir's solar panels degraded their performance. The damage is most noticeable on the panel on the right, which is facing the camera with a high degree of contrast. Extensive damage to the smaller panel below is due to impact with a Progress spacecraft.

Several confirmed impact events have occurred since. Olympus-1 was struck by a meteoroid on 11 August 1993, and left adrift. On 24 July 1996, the French microsatellite Cerise was hit by fragments of an Ariane-1 H-10 upper-stage booster which exploded in November 1986. On 29 March 2006, the Russian Ekspress AM11 communications satellite was struck by an unknown object and rendered inoperable; its engineers had sufficient time in contact with the spacecraft to send it to a parking orbit out of GEO.

The first major space-debris collision occurred on 10 February 2009 at 16:56 UTC. The deactivated 950 kg (2,090 lb) Kosmos 2251 and the operational 560 kg (1,230 lb) Iridium 33 collided, 500 mi (800 km) over northern Siberia. The relative speed of impact was about 11.7 km/s (7.3 mi/s), or about 42,120 km/h (26,170 mph). Both satellites were destroyed; the collision created a debris cloud, with accurate estimates of the number of pieces of debris unavailable. On 22 January 2013 BLITS (a Russian laser-ranging satellite) was struck by debris suspected to be from the 2007 Chinese anti-satellite missile test, changing its orbit and spin rate.

Space Shuttle missions (manned spacecraft)

Since the early Space Shuttle missions, NASA has used the NORAD database to monitor the Shuttle's orbital path for debris. During the 1980s, this used a substantial amount of the NORAD tracking system's capacity. The first Space Shuttle collision-avoidance maneuver occurred during STS-48 in September 1991, in which a seven-second reaction control system burn was performed to avoid debris from Kosmos 955. Similar manoeuvres followed on missions 53, 72 and 82.

One of the first events to publicize the debris problem occurred on Challenger's second flight, STS-7. A fleck of paint struck its front window, creating a pit over 1 mm (0.04 in) wide. Endeavour experienced a similar impact on STS-59 in 1994, pitting the window about half its depth. Post-flight examinations indicate an increase in the number of minor debris impacts since 1998.

Damage from smaller debris has become a significant problem, with window chipping and minor damage to thermal protection system tiles (TPS) common by the 1990s. To mitigate its impact, when the Shuttle reached orbit it was flown tail-first to take as much of the debris load as possible on the engines and rear cargo bay (not used in orbit or during descent, and less critical for post-launch operation). When flying to the International Space Station, the Shuttle was placed where the station provided as much protection as possible.

Endeavour had a major impact on its radiator during STS-118. The entry hole is about ¼ inch, and the exit hole is twice as large.

The increase in debris led to a re-evaluation of the issue, with a catastrophic impact with large debris considered the primary threat to Shuttle operations on every mission. Mission planning required a thorough examination of debris risk, with an executive-level decision to proceed required if the risk of catastrophic impact is greater than 1 in 200. On a normal (low-orbit) mission to the ISS the estimated risk was 1 in 300, but the STS-125 mission to repair the Hubble Space Telescope at 350 mi (560 km) was initially calculated at a 1-in-185 risk (due to the 2009 satellite collision). A re-analysis with better debris numbers reduced the estimated risk to 1 in 221, and the mission was allowed to proceed.

Two serious debris incidents have occurred on recent Shuttle missions. In 2006, Atlantis was struck by a fragment of circuit board during STS-115 which bored a small hole through the radiator panels in the cargo bay. A similar incident occurred on STS-118 in 2007, when debris blew a bullet-like hole through Endeavour's radiator panel.

International Space Station

Although the International Space Station (ISS) uses Whipple shielding to protect itself from minor debris, portions (notably its solar panels) cannot be protected. In 1989 the ISS panels were predicted to experience about 0.23 percent degradation in four years, and they were overdesigned by one percent.

The primary protection for the ISS against larger debris, as for the Shuttle, is avoidance. A maneuver order is issued if ground controllers estimate that "there is a greater than one-in-10,000 chance of a debris strike." As of January 2014, there have been sixteen debris-maneuver firings in the fifteen years the ISS has been in orbit.

The crews were directed on three occasions to abandon work and take refuge in the Soyuz capsule due to late debris-proximity warnings. In addition to the sixteen firings and three Soyuz-capsule shelter orders, one attempted maneuver failed. A March 2009 close call involved debris believed to be a 10 cm (3.9 in) piece of the Kosmos 1275 satellite. In 2013 the ISS did not need to maneuver to avoid space debris, after a record four debris-related maneuver firings the previous year. In this paper we have taken failure due to premature atmospheric reentry, erosion and collisions caused by Space debris with different repair facilities. When the main operative unit fails then warm standby system becomes operative. Failure due to failure due to erosion and collisions caused by Space debris can’t occur simultaneously in both the units and after failure the unit undergoes repair facility of Type- II by ordinary repairman or Type III, Type IV by multispecialty repairman immediately when failure due to solar penal degradation caused by energetic particles from the sun and cosmic rays from elsewhere in space. The repair is done on the basis of first fail first repaired.

Assumptions

$\lambda_1, \lambda_2\ \lambda_3$ are constant failure rates when failure due to failure due to premature atmospheric reentry caused by Space debris and failure due to failure due to erosion and collisions caused by Space debris respectively. The CDF of repair time distribution of Type I, Type II and multispecialty repairmen Type-III, IV are $G_1(t)$, $G_2(t)$ and $G_3(t)$ $G_4(t)$.

1. The failure due to erosion and collisions caused by Space debris is non-instantaneous and it cannot come simultaneously in both the units.

2. The repair starts immediately after failure due to premature atmospheric reentry caused by Space debris and failure due to erosion and collisions caused by Space debris and works on the principle of first fail first repaired basis. The repair facility does no damage to the units and after repair units are as good as new.
3. The switches are perfect and instantaneous.
4. All random variables are mutually independent.
5. When both the units fail, we give priority to operative unit for repair.
6. Repairs are perfect and failure of a unit is detected immediately and perfectly.
7. The system is down when both the units are non-operative.

Symbols for states of the System

Superscripts O, WS, PARF, ECSDF,

Operative, Warm Standby, failure due to premature atmospheric reentry caused by Space debris and failure due to erosion and collisions caused by Space debris respectively

Subscripts nparf, parf, ecsdf, ur, wr, uR

No failure due to premature atmospheric reentry caused by Space debris, failure due to premature atmospheric reentry caused by Space debris, failure due to erosion and collisions caused by Space debris, under repair, waiting for repair, under repair continued from previous state respectively

Up states –0,1, 2, 3, 10 ; Down states – 4, 5, 6, 7,8,9,11

regeneration point – 0,1,2, 3, 8, 9,10

States of the System

0(O_{nparf}, WS_{nparf}) One unit is operative and the other unit is warm standby and there is no failure due to premature atmospheric reentry caused by Space debris of both the units.

1($PARF_{parf, urI}$, O_{nparf}) The operating unit failure due to premature atmospheric reentry caused by Space debris is under repair immediately of Type- I and standby unit starts operating with no failure due to premature atmospheric reentry caused by Space debris

2($ECSDF_{ecsdf, urII}$, O_{nparf}) The operative unit failure due to erosion and collisions caused by Space debris and undergoes repair of type II and the standby unit becomes operative with no failure due to premature atmospheric reentry caused by Space debris

3($ECSDF_{ecsdf, urIII}$, O_{nparf}) The first unit failure due to erosion and collisions caused by Space debris and under Type-III multispecialty repairman and the other unit is operative with no failure due to premature atmospheric reentry caused by Space debris

4($PARF_{parf,uR1}$, $PARF_{parf,wrI}$) The unit failed due to PARF resulting from failure due to premature atmospheric reentry caused by Space debris under repair of Type- I continued from state 1and the other unit failed due to PARF resulting from failure due to premature atmospheric reentry caused by Space debris is waiting for repair of Type-I.

5($PARF_{parf,uR1}$, $ECSDF_{ecsdf,wrII}$) The unit failed due to PARF resulting from failure due to premature atmospheric reentry caused by Space debris is under repair of Type- I continued from state 1and the other unit failure due to erosion and collisions caused by Space debris is waiting for repair of Type- II.

6($ECSDF_{ecsdf, uRII}$, $PARF_{parf ,wrI}$) The operative unit failed due to erosion and collisions caused by Space debris is under repair continues from state 2 of Type –II and the other unit failed due to PARF resulting from failure due to premature atmospheric reentry caused by Space debris is waiting under repair of Type-I.

7($ECSDF_{ecsdf,uRII}$, $PARF_{parf,wrII}$) The one unit failure due to failure due to erosion and collisions caused by Space debris is continued to be under repair of Type II and the other unit failed due to PARF resulting from failure due to premature atmospheric reentry caused by Space debris is waiting for repair of Type-II.

8($PARF_{parf,urIII}$, $ECSDF_{ecsdf, wrII}$) The one unit failure due to premature atmospheric reentry caused by Space debris is under multispecialty repair of Type-III and the other unit failure due to erosion and collisions caused by Space debris is waiting for repair of Type-II.

9($PARF_{parf,urIII}$, $ECSDF_{ecsdf, wrI}$) The one unit failure due to premature atmospheric reentry caused by Space debris is under multispecialty repair of Type-III and the other unit failure due to erosion and collisions caused by Space debris waiting for repair of Type-I

10(O_{nparf} , $ECSDF_{ecsdf, urIV}$)

The one unit is operative with no failure due to premature atmospheric reentry caused by Space debris and warm standby unit failure due to erosion and collisions caused by Space debris and undergoes repair of type IV.

11(O_{nparf} , $ECSDF_{ecsdf, uRIV}$)

The one unit is operative with no failure due to premature atmospheric reentry caused by Space debris and warm standby unit failure due to erosion and collisions caused by Space debris and repair of type IV continues from state 10.

Transition Probabilities

Simple probabilistic considerations yield the following expressions:

$p_{01} = \lambda_1 / \lambda_1 + \lambda_2 + \lambda_3,$
$p_{02} = \lambda_2 / \lambda_1 + \lambda_2 + \lambda_3,$
$p_{0,10} = \lambda_3 / \lambda_1 + \lambda_2 + \lambda_3$
$p_{10} = pG_1^*(\lambda_1)+q\, G_2^*(\lambda_2)\,,$

$p_{14} = p - pG_1^{*}(\lambda_1) = p_{11}^{(4)}$,
$p_{15} = q - q\,G_1^{*}(\lambda_2) = p_{12}^{(5)}$,
$p_{23} = pG_2^{*}(\lambda_1) + q\,G_2^{*}(\lambda_2)$,
$p_{26} = p - pG_2^{*}(\lambda_1) = p_{29}^{(6)}$,
$p_{27} = q - qG_2^{*}(\lambda_2) = p_{28}^{(7)}$,
$p_{30} = p_{82} = p_{91} = 1$
$p_{0,10} = pG_4^{*}(\lambda_1) + q\,G_4^{*}(\lambda_2)$
$p_{10,1} = p - pG_4^{*}(\lambda_1) = p_{10,1}^{(11)}$
$p_{10,2} = q - q\,G_4^{*}(\lambda_2) = p_{10,2}^{(11)}$ (1)

We can easily verify that

$p_{01} + p_{02} + p_{03} = 1$,
$p_{10} + p_{14}\,(=p_{11}^{(4)}) + p_{15}\,(=p_{12}^{(5)}) = 1$,
$p_{23} + p_{26}\,(=p_{29}^{(6)}) + p_{27}\,(=p_{28}^{(7)}) = 1$ $p_{30} = p_{82} = p_{91} = 1$
$p_{10,0} + p_{10,1}^{(11)}\,(=p_{10,1}) + p_{10,2}^{(12)}\,(=p_{10,2}) = 1$ (2)

And mean sojourn time is

$$\mu_0 = E(T) = \int_0^{\infty} P[T > t]\,dt$$

Mean Time to System Failure

$$Ø_0(t) = Q_{01}(t)[s]\,Ø_1(t) + Q_{02}(t)[s]\,Ø_2(t) + Q_{0,10}(t)[s]\,Ø_{10}(t)$$

$$Ø_1(t) = Q_{10}(t)[s]\,Ø_0(t) + Q_{14}(t) + Q_{15}(t)$$

$$Ø_2(t) = Q_{23}(t)[s]\,Ø_3(t) + Q_{26}(t) + Q_{27}(t)$$

$$Ø_3(t) = Q_{30}(t)[s]\,Ø_0(t)$$

$$Ø_{10}(t) = Q_{10,0}(t)[s]\,Ø_{10}(t) + Q_{10,2}(t)[s]\,Ø_1(t) + Q_{10,2}(t)[s]\,Ø_2(t) \qquad (3\text{-}6)$$

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-6) and solving for

$$ø_0^{*}(s) = N_1(s) \,/\, D_1(s) \qquad (7)$$

where

$N_1(s) = \{Q_{01}^{*} + Q_{0,10}^{*}\,Q_{10,1}^{*}\}\,[\,Q_{14}^{*}(s) + Q_{15}^{*}(s)\,] + \{Q_{02}^{*} + Q_{0,10}^{*}\,Q_{10,2}^{*}\}\,[\,Q_{26}^{*}(s) + Q_{27}^{*}(s)\,]$

$D_1(s) = 1 - \{Q_{01}^{*} + Q_{0,10}^{*}\,Q_{10,1}^{*}\}\;Q_{10}^{*} - \{Q_{02}^{*} + Q_{0,10}^{*}\,Q_{10,2}^{*}\}\;Q_{23}^{*}\,Q_{30}^{*} - Q_{0,10}^{*}\,Q_{10,0}^{*}$

Making use of relations (1) & (2) it can be shown that $ø_0^{*}(0) = 1$, which implies that $ø_0(t)$ is a proper distribution.

$$\text{MTSF} = E[T] = \left.\frac{d}{ds}\,ø_0^{*}(s)\right|_{s=0}$$

$= (D_1'(0) - N_1'(0)) \,/\, D_1(0)$
$= (\mu_0 + \mu_1\,(p_{01} + p_{0,10}\,p_{10,1}) + (p_{02} + p_{0,10}\,p_{10,2})(\mu_2 + \mu_3) + \mu_{10}\,p_{0,10} \,/\, (1 - (p_{01} + p_{0,10}\,p_{10,1})\,p_{10} - (p_{02} + p_{0,10}\,p_{10,2})\,p_{23}) - p_{0,10}\,p_{10,0}$

where

$\mu_0 = \mu_{01} + \mu_{02} + \mu_{0,10}$,
$\mu_1 = \mu_{10} + \mu_{11}^{(4)} + \mu_{12}^{(5)}$,
$\mu_2 = \mu_{23} + \mu_{28}^{(7)} + \mu_{29}^{(6)}$,
$\mu_{10} = \mu_{10,0} + \mu_{10,1} + \mu_{10,2}$

Availability analysis

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

$M_0(t) = e^{-\lambda_1 t}\,e^{-\lambda_2 t}\,e^{-\lambda_3 t}$

, $M_1(t) = p\,G_1\overline{(t)}\;e^{-\lambda_1 t}$

$M_2(t) = q\,G_2\overline{(t)\,e}^{\,-\lambda_2 t}$,

$M_3(t) = G_3\overline{(t),}\;M_{10}(t) = G_4(t)\,e^{-\lambda_3 t}\,\overline{\ }$

The point wise availability $A_i(t)$ have the following recursive relations

$$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t) + q_{0,10}(t)[c]A_{10}(t)$$

$$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{12}^{(5)}(t)[c]A_2(t) + q_{11}^{(4)}(t)[c]A_1(t),$$

$$A_2(t) = M_2(t) + q_{23}(t)[c]A_3(t) + q_{28}^{(7)}(t)[c]\,A_8(t) + q_{29}^{(6)}(t)]\,[c]A_9(t)$$

$$A_3(t) = M_3(t) + q_{30}(t)[c]A_0(t)$$

$$A_8(t) = q_{82}(t)[c]A_2(t)$$

$$A_9(t) = q_{91}(t)[c]A_1(t)$$

$$A_{10}(t) = M_{10}(t) + q_{10,0}(t)[c]A_0(t) + q_{10,1}^{(11)}(t)[c]A_1(t) + q_{10,2}^{(11)}(t)[c]A_2(t) \qquad (8\text{-}14)$$

Taking Laplace Transform of eq. (8-14) and solving for $\hat{A}_0(s)$

$$\hat{A}_0(s) = N_2(s) \,/\, D_2(s) \qquad (15)$$

where

$N_2(s) = \{\hat{q}_{0,10}\,\widetilde{M}_{10} + \widetilde{M}_0\}\,[\{1 - \hat{q}_{11}^{(4)}\}\{1 - \hat{q}_{28}^{(7}\,\hat{q}_{82}\} - \hat{q}_{12}^{(5)}\,\hat{q}_{29}^{(6)}\,\hat{q}_{91}] + \{\hat{q}_{01} + \hat{q}_{0,10}\,\hat{q}_{10,1}^{(11)}\}[\widetilde{M}_1\{1 - \hat{q}_{28}^{(7)}\,\hat{q}_{82}\} + \hat{q}_{12}^{(5)}\,\hat{q}_{23}\,\widetilde{M}_3 + \widetilde{M}_2] + \{\hat{q}_{02} + \hat{q}_{0,10}\,\hat{q}_{10,2}^{(11)}\}\,[\{\hat{q}_{23}\,\widetilde{M}_3\}\{1 - \hat{q}_{11}^{(4)}\} + \hat{q}_{29}^{(6)}\,\hat{q}_{91}\,\widetilde{M}_1]$

$D_2(s) = \{1 - \hat{q}_{11}^{(4)}\}\{1 - \hat{q}_{28}^{(7}\,\hat{q}_{82}\} - \hat{q}_{12}^{(5)}\,\hat{q}_{29}^{(6)}\,\hat{q}_{91} - \{\hat{q}_{01} + \hat{q}_{0,10}\,\hat{q}_{10,1}^{(11)}\}[\hat{q}_{10}\{1 - \hat{q}_{28}^{(7)}\,\hat{q}_{82}\} + \hat{q}_{12}^{(5)}\,\hat{q}_{23}\,\hat{q}_{30}] - \{\hat{q}_{02} + \hat{q}_{0,10}\,\hat{q}_{10,2}^{(11)}\}\{[\hat{q}_{23}\,\hat{q}_{30}\,\{1 - \hat{q}_{11}^{(4)}\} + \hat{q}_{29}^{(6)}\,\hat{q}_{91}\,\hat{q}_{10}]$

(Omitting the arguments s for brevity)

The steady state availability

$$A_0 = \lim_{t\to\infty}[A_0(t)]$$
$$= \lim_{s\to 0}[s\,\hat{A}_0(s)] = \lim_{s\to 0}\frac{s\,N_2(s)}{D_2(s)}$$

Using L’ Hospitals rule, we get

$$A_0 = \lim_{s\to 0} \frac{N_2(s)+s\,N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)} \quad (16)$$

Where

$N_2(0) = \{p_{0,10}\,\tilde{M}_{10}(0) + \tilde{M}_0(0)\}\,[\{1 - p_{11}^{(4)}\}\{1 - p_{28}^{(7)}\} - p_{12}^{(5)}\,p_{29}^{(6)}] + \{p_{01} + p_{0,10}\,p_{10,1}^{(11)}\}[\tilde{M}_1(0)\{1 - p_{28}^{(7)}\} + p_{12}^{(5)}\,p_{23}\,\tilde{M}_3(0) + \tilde{M}_2(0)] + \{p_{02} + p_{0,10}\,p_{10,2}^{(11)}\}\,[\{p_{23}\,\tilde{M}_3(0) + \tilde{M}_2(0)\}\{1 - p_{11}^{(4)}\} + p_{29}^{(6)}\,\tilde{M}_1(0)]$

$D_2'(0) = \mu_0[p_{10}(1 - p_{28}^{(7)}\} + p_{12}^{(5)}\,p_{23}] + \mu_1[p_{29}^{(6)}{}_{+}\,p_{01}\,p_{23} - p_{0,10}\{p_{10,0}\{1 - p_{28}^{(7)}\} + p_{23}\,p_{10,2}^{(11)}\,p_{23}\}] + \mu_2[(1 - p_{11}^{(4)}) - p_{01}\,p_{10} - p_{0,10}\,(p_{10} - p_{10}\,p_{10,2}^{(11)} + p_{12}^{(5)}\,p_{10,0})]\} + \mu_3\,[p_{23}[p_{12}^{(5)}\{p_{01} + p_{0,10}\,p_{10,1}^{(11)}\} + (1 - p_{11}^{(4)})\{p_{02} + p_{0,10}\,p_{10,2}^{(11)}\}] + \mu_8\,[p_{28}^{(7)}(1 - p_{0,10}\,p_{10,0} - p_{10}\{p_{01} + p_{0,10}\,p_{10,1}^{(11)}\})] + \mu_9\,[p_{29}^{(6)}\{p_{12}^{(5)}(1 - p_{0,10}\,p_{10,0} + (p_{02} + p_{0,10}\,p_{10,2}^{(11)}\})] + \mu_{10}\,[p_{29}^{(6)}\{p_{12}^{(5)}(1 - p_{0,10}\,p_{10,0} + (p_{02} + p_{0,10}\,p_{10,2}^{(11)}\})]$

and

$\mu_3 = \mu_{30}$, $\mu_9 = \mu_{91}$, $\mu_8 = \mu_{81}$

The expected up time of the system in (0,t] is

$$\lambda_u(t) = \int_0^{\infty} A_0(z)\,dz$$

So that $\bar{\lambda}_u(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)}$ (17)

The expected down time of the system in (0,t] is

$$\lambda_d(t) = t - \lambda_u(t)$$

So that $\bar{\lambda}_d(s) = \frac{1}{s^2} - \bar{\lambda}_u(s)$ (18)

The expected busy period of the server when there is failure due to failure due to premature atmospheric reentry caused by Space debris and failure due to erosion and collisions caused by Space debris in (0,t]-R_0

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t) + q_{0,10}(t)[c]R_{10}(t)$

$R_1(t) = S_1(t) + q_{10}(t)[c]R_0(t) + q_{12}^{(5)}(t)[c]R_2(t) + q_{11}^{(4)}(t)[c]R_1(t)$

$R_2(t) = S_2(t) + q_{23}(t)[c]R_3(t) + q_{28}^{(7)}(t)\,R_8(t) + q_{29}^{(6)}(t)][c]R_9(t)$

$R_3(t) = S_3(t) + q_{30}(t)[c]R_0(t)$

$R_8(t) = S_8(t) + q_{82}(t)[c]R_2(t)$

$R_9(t) = S_9(t) + q_{91}(t)[c]R_1(t)$

$R_{10}(t) = S_{10}(t) + q_{10,0}(t)[c]R_0(t) + q_{10,1}^{(11)}(t)[c]R_1(t) + q_{10,2}^{(11)}(t)[c]R_2(t)$ (19-25)

where

$S_1(t) = p\,G_1\overline{(t)}\;e^{-\lambda_1 t}$,

$S_2(t) = q\,G_2\overline{(t)}\;e^{-\lambda_2 t}$

$S_3(t) = S_8(t) = S_9(t) = G_3(t)$

$S_{10}(t) = G_4\overline{(t)}$ (26)

Taking Laplace Transform of eq. (19-25) and solving for $\bar{R}_0(s)$

$$\bar{R}_0(s) = N_3(s)\,/\,D_2(s) \quad (27)$$

where

$N_3(s) = \{\hat{q}_{01} + \hat{q}_{0,10}\,\hat{q}_{10,1}^{(11)}\}[\hat{S}_1(1 - \hat{q}_{28}^{(7)}\,\hat{q}_{82}\} + \hat{q}_{12}^{(5)}[\hat{S}_2 + \hat{q}_{23}\hat{S}3 + \hat{q}_{28}^{(7)}\hat{S}8 + \hat{q}_{29}^{(6)}\hat{S}9)]] + \{\hat{q}_{02} + \hat{q}_{0,10}\,\hat{q}_{10,2}^{(11)}\}[\{\hat{S}_2 + \hat{q}_{23}\hat{S}_3 + \hat{q}_{28}^{(7)}\hat{S}_8 + \hat{S}_9\hat{q}_{29}^{(6)})(1 - \hat{q}_{11}^{(4)}) + \hat{S}_1\hat{q}_{29}^{(6)}\hat{q}_{91}] + \hat{q}_{0,10}\,\hat{S}_{10}[\{1 - \hat{q}_{28}^{(7)}\hat{q}_{82}\}\{1 - \hat{q}_{11}^{(4)}\} - \hat{q}_{29}^{(6)}\,\hat{q}_{91}\,\hat{q}_{12}^{(5)}]$

and $D_2(s)$ is already defined.

(Omitting the arguments s for brevity)

In the long run, $R_0 = \frac{N_3(0)}{D_2'(0)}$ (28)

Where

$N_3(0) = \{p_{01} + p_{0,10}\,p_{10,1}^{(11)}\}[\hat{S}_1(1 - p_{28}^{(7)}\} + p_{12}^{(5)}[\hat{S}_2 + p_{23}\hat{S}3 + p_{28}^{(7)}\hat{S}8 + p_{29}^{(6)}\hat{S}9)]] + \{p_{02} + p_{0,10}\,p_{10,2}^{(11)}\}[\{\hat{S}_2 + p_{23}\hat{S}_3 + p_{28}^{(7)}\hat{S}_8 + \hat{S}_9\,p_{29}^{(6)})(1 - p_{11}^{(4)}) + \hat{S}_1 p_{29}^{(6)}] + p_{0,10}\,\hat{S}_{10}[\{1 - p_{28}^{(7)}\}\{1 - p_{11}^{(4)}\} - p_{29}^{(6)}\,p_{12}^{(5)}]$

and $D_2'(0)$ is already defined.

The expected busy period of the server when there is failure due to failure due to premature atmospheric reentry caused by Space debris and failure due to erosion and collisions caused by Space debris in (0,t] is

$\lambda_{rv}(t) = \int_0^{\infty} R_0(z)\,dz$ So that $\bar{\lambda}_{rv}(s) = \frac{\hat{R}_0(s)}{s}$

The expected number of visits by the repairman Type-I or Type-II for repairing the identical units in (0,t]-H_0

$H_0(t) = Q_{01}(t)[s][1 + H_1(t)] + Q_{02}(t)[s][1 + H_2(t)] + Q_{0,10}(t)[s]\,H_{10}(t)]$

$H_1(t) = Q_{10}(t)[s]H_0(t)] + Q_{12}^{(5)}(t)[s]\,H_2(t) + Q_{11}^{(4)}(t)]\,[s]H_1(t)$,

$H_2(t) = Q_{23}(t)[s]H_3(t) + Q_{28}^{(7)}(t)\,[s]\,H_8(t) + Q_{29}^{(6)}(t)]\,[c]H_9(t)$

$H_3(t) = Q_{30}(t)[s]H_0(t)$

$H_8(t) = Q_{82}(t)[s]H_2(t)$

$H_9(t) = Q_{91}(t)[s]H_1(t)$

$H_{10}(t) = Q_{10,0}(t)[s]H_{10}(t)] + Q_{10,1}^{(11)}(t)[s]H_1(t)] + Q_{10,2}^{(11)}(t)[s]\,H_2(t)]$ (29-35)

Taking Laplace Transform of eq. (29-35) and solving for $H_0^*(s)$

$$H_0^*(s) = N_4(s)\,/\,D_3(s) \quad (36)$$

$N_4(s) = \{Q_{01}^{*} + Q_{02}^{*}\}[\{1 - Q_{11}^{(4)*}\}\{1 - Q_{28}^{(7)*}\,Q_{82}^{*}\} - Q_{12}^{(5)*}\,Q_{29}^{(6)*}\,Q_{91}^{*}]$

And

$D_3(s) = \{1 - Q_{11}^{(4)*}\}\{1- Q_{28}^{(7)*} Q_{82}^{*}\} - Q_{12}^{(5)*} Q_{29}^{(6)*} Q_{91}^{*}](1- Q_{0,10}^{*} Q_{10,0}^{*})-\{Q_{01}^{*}+ Q_{0,10}^{*} Q_{10,1}^{(11)*}\}[Q_{10}^{*}\{1 - Q_{28}^{(7)*} Q_{82}^{*}\}+ Q_{12}^{(5)*} Q_{23}^{*} Q_{30}^{*}] - \{Q_{02}^{*} + Q_{0,10}^{*} Q_{10,2}^{(11)*}\}[Q_{23}^{*} Q_{30}^{*}\{1 - Q_{11}^{(4)*}\}+ Q_{29}^{(6)*} Q_{91}^{*} Q_{10}^{*}]$

(Omitting the arguments s for brevity)

In the long run,

$H_0 = N_4(0) / D_3^{'}(0)$ (37)

where

$N_4(0) =\{1 - p_{0,10}\}[\{1 - p_{11}^{(4)}\}\{1-$

$p_{28}^{(7)}\} - p_{12}^{(5)} p_{29}^{(6)}]$

The expected number of visits by the multispecialty repairman Type-III for repairing the identical units in (0,t]-W_0

$W_0(t)=Q_{01}(t)[s]W_1(t)+ Q_{02}(t)[s] W_2(t) + Q_{10,0}(t)[s] W_{10}(t)$

$W_1(t) = Q_{10}(t)[s]W_0(t)] + Q_{12}^{(5)}(t)[s]$

$W_2(t) + Q_{11}^{(4)}(t)] [s]W_1(t)$,

$W_2(t) = Q_{23}(t)[s]W_3(t) + Q_{28}^{(7)}(t) [s]$

$W_8(t) +Q_{29}^{(6)}(t)] [c]W_9(t)$

$W_3(t) = Q_{30}(t)[s][1+W_0(t)]$

$W_8(t) = Q_{82}(t)[s][1+W_2(t)]$

$W_9(t) = Q_{91}(t)[s][1+W_1(t)]$

$W_{10}(t)=Q_{10,0}(t)[s]W_0(t)+$

$Q_{10,1}^{(11)}(t)[s] W_1(t) + Q_{10,2}^{(12)}(t)[s]$

$W_2(t)$ (38-44)

Taking Laplace Transform of eq. (33-39) and solving for $H_0^*(s)$

$H_0^*(s) = N_5(s) / D_3(s)$ (45)

$N_5(s) = \{Q_{01}^{*}+ Q_{0,10}^{*}Q_{0,10}^{(11)*}\}[Q_{12}^{(5)*} [Q_{23}^{*} Q_{30}^{*} + Q_{28}^{(5)*} Q_{82}^{*} + Q_{29}^{(6)*} Q_{91}^{*}] + \{Q_{02}^{*} + Q_{0,10}^{*}Q_{10,2}^{(11)*}\}[[Q_{23}^{*} Q_{30}^{*} + Q_{28}^{(5)*} Q_{82}^{*} + Q_{29}^{(6)*} Q_{91}^{*} \{1 - Q_{11}^{(4)*}\}]$

(Omitting the arguments s for brevity)

In the long run,

$W_0 = N_5(0) / D_3^{'}(0)$ (46)

where $N_5(0) = \{p_{01}+ p_{0,10}\, p_{10,1}^{(11)}\}$

$p_{12}^{(5)} + \{p_{02}+ p_{0,10}\, p_{10,2}^{(11}\}\{1 - p_{11}^{(4)}\}]$

The expected number of visits by the multispecialty repairman Type-III for repairing the identical units in (0,t]-Y_0

$Y_0(t)=Q_{01}(t)[s]Y_1(t)+Q_{02}(t)[s] Y_2(t) + Q_{0,10}(t)[s] [1+Y_{10}(t)]$

$Y_1(t) = Q_{10}(t)[s]Y_0(t) + Q_{12}^{(5)}(t)[s]$

$Y_2(t) + Q_{11}^{(4)}(t)] [s]Y_1(t)$,

$Y_2(t) = Q_{23}(t)[s]Y_3(t) + Q_{28}^{(7)}(t) [s]$

$Y_8(t) +Q_{29}^{(6)}(t)] [c]Y_9(t)$

$Y_3(t) = Q_{30}(t)[s][1+Y_0(t)]$

$Y_8(t) = Q_{82}(t)[s]Y_2(t)$

$Y_9(t) = Q_{91}(t)[s]Y_1(t)$

$Y_{10}(t)=Q_{10,0}(t)[s]Y_0(t)+ Q_{10,1}^{(11)}(t)[s] Y_1(t) + Q_{10,2}^{(12)}(t)[s] Y_2(t)$ (47-53)

Taking Laplace Transform of eq. (47-53) and solving for$Y_0^{*}(s)$,we get

$Y_0^{*}(s) = N_6(s) / D_3(s)$ (54)

$N_6(s) = Q_{0,10}^{*} [\{1 - Q_{11}^{(4)*}\}(1- Q_{28}^{(5)*} Q_{82}^{*}\} - Q_{12}^{(5)*}Q_{29}^{(6)*} Q_{91}^{*}\{1- Q_{0,10}^{*}Q_{,10,0}^{*}\}+\{Q_{02}^{*} + Q_{0,10}^{*}Q_{10,2}^{(11)*}\}[[Q_{23}^{*} Q_{30}^{*} \{1 - Q_{11}^{(4)*}\}+ Q_{10}^{*} Q_{29}^{(6)*} Q_{91}^{*}]$

(Omitting the arguments s for brevity)

In the long run,

$W_0 = N_6(0) / D_3^{'}(0)$ (55)

where $N_6(0) = p_{0,10}[\{1-p_{11}^{(4)}\}\{1- p_{28}^{(7)}\}- p_{12}^{(5)} p_{29}^{(6)}]$

$p_{12}^{(5)} + \{p_{02}+ p_{0,10}\, p_{10,2}^{(11}\}\{1 - p_{11}^{(4)}\}]$

Benefit- Function Analysis

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to premature atmospheric reentry caused by Space debris and failure due to erosion and collisions caused by Space debris, expected number of visits by the repairman for unit failure. The expected total Benefit-Function incurred in (0,t] is

C(t) = Expected total revenue in (0,t]

- expected busy period of the server when there is failure due to failure due to premature atmospheric reentry caused by Space debris and failure due to erosion and collisions caused by Space debris in (0,t]

- expected number of visits by the repairman Type- I or Type- II for repairing of identical the units in (0,t]

- expected number of visits by the multispecialty repairman Type- III for repairing of identical the units in (0,t]

- expected number of visits by the multispecialty repairman Type- IV for repairing of identical the units in (0,t]

$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2 C(s))$

$= K_1A_0 - K_2R_0 - K_3H_0 - K_4W_0 - K_5Y_0$

where

K_1 - revenue per unit up-time,

K_2 - cost per unit time for which the system is busy under repairing,

K_3 - cost per visit by the repairman type- I or type- II for units repair,

K_4 - cost per visit by the multispecialty repairman Type- III for units repair

K_5 - cost per visit by the multispecialty repairman Type- IV for units repair

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to premature atmospheric reentry caused by Space debris and failure due to erosion and collisions caused by Space debris increases, the MTSF, steady state availability decreases and the Profit-function decreased as the failure increases.

REFERENCES

[1] Dhillon, B.S. and Natesen, J, Stochastic Analysis of outdoor Power Systems in fluctuating environment, Microelectron. Reliab. ,1983; 23, 867-881.

[2] Kan, Cheng, Reliability analysis of a system in a randomly changing environment, Acta Math. Appl. Sin. 1985, 2, pp.219-228.

[3] Cao, Jinhua, Stochastic Behaviour of a Man Machine System operating under changing environment subject to a Markov Process with two states, Microelectron. Reliab. , 1989; 28, pp. 373-378.

[4] Barlow, R.E. and Proschan, F., Mathematical theory of Reliability, 1965; John Wiley, New York.

[5] Gnedanke, B.V., Belyayar, Yu.K. and Soloyer , A.D. , Mathematical Methods of Reliability Theory, 1969 ; Academic Press, New York.

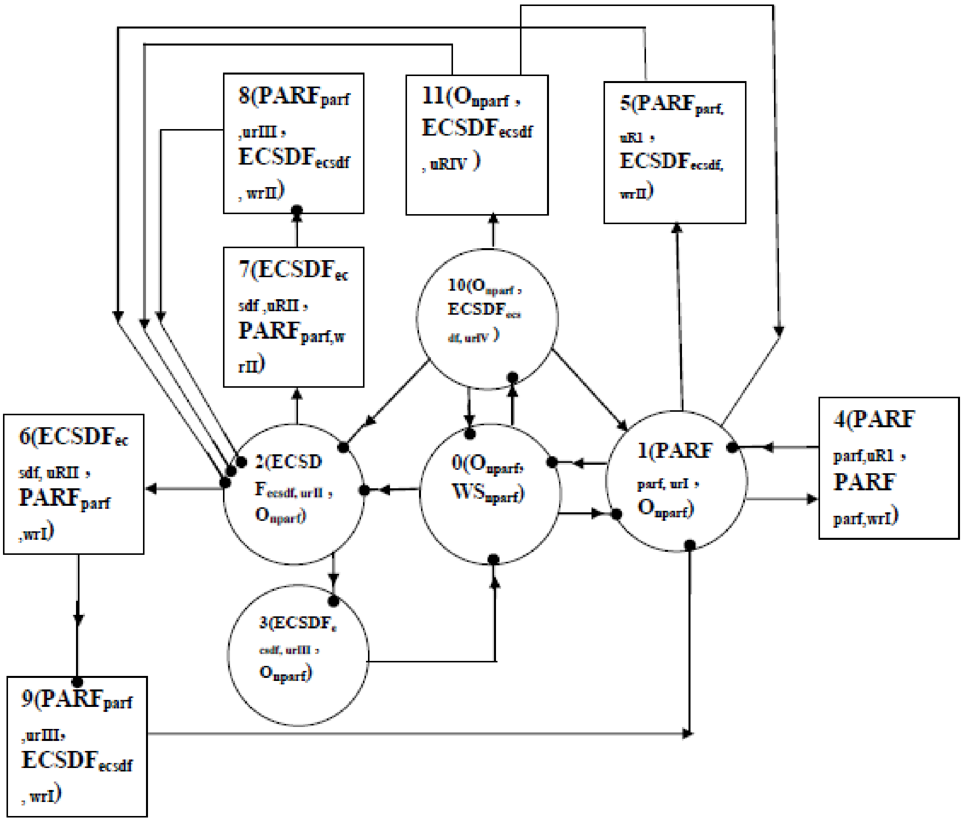

Fig. The State Transition Diagram

○ up state ☐ Down-State

● regeneration point

◆ ◆ ◆