Cost-Benefit Analysis of Two Similar Warm Standby Aircraft System subject to failure due to heavy rain and thunderstorm

Ashok Kumar Saini

BLJS COLLEGE, TOSHAM (BHIWANI) HARYANA,, INDIA
Email : drashokksaini2009@gmail.com

Abstract- In this paper we have taken failure due to heavy rain and failure due to thunderstorm. When the main unit fails then warm standby system becomes operative. Failure due to thunderstorm cannot occur simultaneously in both the units and after failure the unit undergoes Type-I or Type-II or Type-III repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

Keywords: Warm Standby, failure due to heavy rain, failure due to thunderstorm, first come first serve, MTSF, Availability, Busy period, Benefit -Function.

INTRODUCTION

6 August 1997; Korean Air 747-300; flight 801 Agana, Guam USA: The aircraft crashed about three miles (4.8 km) short of the runway during a night time approach in heavy rain. Twenty one of the 23 crew members and 207 of the 231 passengers were killed. **American Airlines Flight 1420** was a flight from Dallas-Fort Worth International Airport to Little Rock National Airport in the USA. On June 1, 1999, the McDonnell Douglas MD-82 (registration number N215AA) operating for Flight 1420 overran the runway upon landing in Little Rock and crashed. Of the 145 people aboard, the captain and ten passengers were killed in the crash.

The pilots of Flight 1420 were Captain Richard Buschmann, 48, and First Officer Michael Origel, 35. Captain Buschmann was a very experienced chief pilot for American Airlines with 10,234 total flight hours, of which approximately half were accumulated flying the MD-80. First Officer Origel had been with the airline for less than a year. He had 4,292 hours of flying experience.

According to the National Transportation Safety Board (NTSB) accident report, they learned that the winds were changing direction and that a wind shear alert had sounded in the aircraft due to a thunderstorm nearby. Air traffic control originally told them to expect Runway 22L for landing, but after the wind direction changed rapidly, Captain Buschmann requested a change to Runway 4R.

As the aircraft approached Runway 4R, a severe thunderstorm arrived over the airport. The controller's last report, prior to the landing, stated that the winds were 330 degrees at 28 knots. That exceeded the MD-82's crosswind limit for landing in reduced visibility on a wet runway. With that information, plus two wind shear reports, the approach should have been abandoned at that point, but Captain Buschmann decided to continue his approach to Runway 4R.

During their rush to land as soon as possible, both pilots became overloaded with multiple necessary tasks. That led to errors and omissions, which proved to be the final links in the accident chain. Consequently they failed to arm the automatic ground spoiler system.

The pilots also failed to arm the auto braking system. Both automatic deployment of the ground spoilers and automatic engagement of the brakes are essential to ensure the plane's ability to stop within the confines of a wet runway, especially one that is being subjected to strong and gusting winds.

After landing, First Officer Origel stated, "We're down. We're sliding." This was followed by the captain saying "Oh No!" Neither pilot observed that the spoilers did not deploy, so there was no attempt to activate them manually. The result was almost no braking at all, since only about 15 percent of the airplane's weight was supported by the main landing gear.

Directional control was lost when Captain Buschmann applied too much reverse thrust, in contradiction to the limits stated in the flight manual.

The aircraft skidded off the far end of the runway at high speed, slammed into a steel walkway with the landing lights for runway 22L and finally came to a stop on the banks of the Arkansas River.

"After departing the end of the runway, the airplane struck several tubes extending outward from the left edge of the instrument landing system (ILS) localizer array, located 411 feet beyond the end of the runway; passed through a chain link security fence and over a rock embankment to a flood plain, located approximately 15 feet below the runway elevation; and collided with the structure supporting the runway 22L approach lighting system."

Such structures are usually frangible - i.e. designed to shear off on impact - but because the approach lights were located on the unstable river bank, they were firmly anchored and the impact destroyed the aircraft. It broke into three pieces and ignited.

Captain Buschmann was killed instantly, when the cockpit impacted a steel walkway attached to the approach lighting system for Runway 22L, and first officer Origel received serious injuries. Ten of the 139 passengers also died.

14-year-old Rachel Fuller, a passenger who sustained severe burns, died on June 16, following the amputation of her leg.

Of the cabin crew: 3 received serious injuries,1 received minor injuries

Of the surviving passengers: 41 received serious injuries,64 received minor injuries

24 were uninjured

9 May 1976; Iranian Air Force 747-100; flight 48; near Madrid, Spain: The aircraft was operating as a military flight from Tehran, Iran to Madrid, Spain and encountered an area of thunderstorms near its destination. The aircraft was apparently struck by lightning, which ignited fuel vapors from a tank in the left wing. The subsequent explosion damaged the wing and eventually led to a major structural failure of the wing. All 10 crew members and seven passengers were killed.

In this paper we have taken failure due to heavy rain, and failure due to thunderstorm. When the main operative unit fails then warm standby system becomes operative. Failure due to thunderstorm cannot occur simultaneously in both the units. After failure the unit undergoes repair facility of Type- I or Type- II by ordinary repairman, Type III or Type IV by multispecialty repairman immediately when failure due to heavy rain and failure due to thunderstorm. The repair is done on the basis of first fail first repaired.

Assumptions

1. λ_1, λ_2 λ_3 are constant failure rates when failure due to heavy rain, failure due to thunderstorm respectively. The CDF of repair time distribution of Type I, Type II and multispecialty repairmen Type-III, IV are $G_1(t)$, $G_2(t)$ and $G_3(t)$, $G_4(t)$.
2. The failure due to thunderstorm is non-instantaneous and it cannot come simultaneously in both the units.
3. The repair starts immediately after failure due to heavy rain and failure due to thunderstorm and works on the principle of first fail first repaired basis. The repair facility does no damage to the units and after repair units are as good as new.
4. The switches are perfect and instantaneous.
5. All random variables are mutually independent.
6. When both the units fail, we give priority to operative unit for repair.
7. Repairs are perfect and failure of a unit is detected immediately and perfectly.
8. The system is down when both the units are non-operative.

Symbols for states of the System

Superscripts O, CS, HRF, TSAF,

Operative, Warm Standby, failure due to heavy rain, failure due to thunderstorm respectively

Subscripts nhrf, hrf, tsaf, ur, wr, uR

No failure due to heavy rain, failure due to heavy rain, failure due to thunderstorm, under repair, waiting for repair, under repair continued from previous state respectively

Up states – 0, 1, 2, 3, 10 ; Down states – 4, 5, 6, 7,8,9,11, regeneration point – 0,1,2, 3, 8, 9,10

States of the System

0(O_{nhrf}, CS_{nhrf}) One unit is operative and the other unit is warm standby and there is no failure due to heavy rain of both the units.

1($HRF_{hrf,\ urI}$, O_{nhrf}) The operating unit failure due to heavy rain is under repair immediately of Type- I and standby unit starts operating with no failure due to heavy rain

2($TSAF_{TSAF,\ urII}$, O_{nhrf}) The operative unit failure due to thunderstorm and undergoes repair of Type II and the standby unit becomes operative with no failure due to heavy rain

3($TSAF_{TSAF,\ urIII}$, O_{nhrf}) The first unit failure due to thunderstorm and under Type-III multispecialty repairman and the other unit is operative with no failure due to heavy rain

4($HRF_{hrf,uR1}$, $HRF_{hrf,wrI}$) The unit failed due to HRF resulting from failure due to heavy rain under repair of Type- I continued from state 1and the other unit failed due to HRF resulting from failure due to heavy rain is waiting for repair of Type-I.

5($HRF_{hrf,uR1}$, $TSAF_{TSAF,\ wrII}$) The unit failed due to HRF resulting from failure due to heavy rain is under repair of Type- I continued from state 1and the other unit fails due to failure due to thunderstorm is waiting for repair of Type- II.

6($TSAF_{tsaf,\ uRII}$, $HRF_{hrf\ ,wrI}$) The operative unit failed due to failure due to thunderstorm is under repair continues from state 2 of Type –II and the other unit failed due to HRF resulting from failure due to heavy rain is waiting under repair of Type-I.

7($TSAF_{tsaf,uRII}$, $HRF_{hrf,wrII}$) The one unit failed due to failure due to thunderstorm is continued to be under repair of Type II and the other unit failed due to HRF resulting from failure due to heavy rain is waiting for repair of Type-II.

8($HRF_{hrf,urIII}$, $TSAF_{tsaf,\ wrII}$) The one unit failure due to heavy rain is under multispecialty repair of Type-III and the other unit failed due to failure due to thunderstorm is waiting for repair of Type-II.

9($HRF_{hrf,urIII}$, $TSAF_{tsaf,\ wrI}$) The one unit failure due to heavy rain is under multispecialty repair of Type-III and the other unit failed due to failure due to thunderstorm is waiting for repair of Type-I

10(O_{nhrf} $TSAF_{tsaf,\ urIV}$) The one unit is operative with no failure due to heavy rain and warm standby unit fails due to failure due to thunderstorm and undergoes repair of type IV.

11(O_{nhrf} $TSAF_{tsaf,\ uRIV}$) The one unit is operative with no failure due to heavy rain and warm standby unit fails due to failure due to thunderstorm and repair of type IV continues from state 10.

Transition Probabilities

Simple probabilistic considerations yield the following expressions:

$p_{01} = \lambda_1 / \lambda_1 + \lambda_2 + \lambda_3$,

$p_{02} = \lambda_2 / \lambda_1 + \lambda_2 + \lambda_3$,

$p_{0,10} = \lambda_3 / \lambda_1 + \lambda_2 + \lambda_3$

$p_{10} = pG_1^*(\lambda_1) + q\,G_2^*(\lambda_2)$,

$p_{14} = p - pG_1^*(\lambda_1) = p_{11}^{(4)}$,

$p_{15} = q - q\,G_1^*(\lambda_2) = p_{12}^{(5)}$,

$p_{23} = pG_2^*(\lambda_1) + q\,G_2^*(\lambda_2)$,

$p_{26} = p - pG_2^*(\lambda_1) = p_{29}^{(6)}$,

$p_{27} = q - qG_2^*(\lambda_2) = p_{28}^{(7)}$,

$p_{30} = p_{82} = p_{91} = 1$,

$p_{0,10} = pG_4^*(\lambda_1) + q\,G_4^*(\lambda_2)$,

$p_{10,1} = p - pG_4^*(\lambda_1) = p_{10,1}^{(11)}$,

$p_{10,2} = q - q\,G_4^*(\lambda_2) = p_{10,2}^{(11)}$ (1)

We can easily verify that

$p_{01} + p_{02} + p_{03} = 1$,

$p_{10} + p_{14} (=p_{11}^{(4)}) + p_{15} (=p_{12}^{(5)}) = 1$,

$p_{23} + p_{26} (=p_{29}^{(6)}) + p_{27} (=p_{28}^{(7)}) = 1 p_{30} = p_{82} = p_{91} = 1$

$p_{10,0} + p_{10,1}^{(11)} (=p_{10,1}) + p_{10,2}^{(12)} (=p_{10,2}) = 1$ (2)

And mean sojourn time is

$\mu_0 = E(T) = \int_0^\infty P[T > t]\,dt$

Mean Time To System Failure

$\varnothing_0(t) = Q_{01}(t)[s]\ \varnothing_1(t) + Q_{02}(t)[s]\ \varnothing_2(t) + Q_{0,10}(t)[s]\ \varnothing_{10}(t)$

$\varnothing_1(t) = Q_{10}(t)[s]\ \varnothing_0(t) + Q_{14}(t) + Q_{15}(t)$

$\varnothing_2(t) = Q_{23}(t)[s]\ \varnothing_3(t) + Q_{26}(t) + Q_{27}(t), \varnothing_3(t) = Q_{30}(t)[s]\ \varnothing_0(t)$,

$\varnothing_{10}(t) = Q_{10,0}(t)[s]\ \varnothing_{10}(t) + Q_{10,1}(t)[s]\varnothing_1(t) + Q_{10,2}(t)[s]\ \varnothing_2(t)$ (3-6)

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-6) and solving for

$\varnothing_0^*(s) = N_1(s) / D_1(s)$ (7)

where

$N_1(s) = \{Q_{01}^* + Q_{0,10}^*\ Q_{10,1}^*\}\ [\ Q_{14}^*(s) + Q_{15}^*(s)\ ] + \{Q_{02}^* + Q_{0,10}^*\ Q_{10,2}^*\}\ [\ Q_{26}^*(s) + Q_{27}^*(s)\ ]$

$D_1(s) = 1 - \{Q_{01}^* + Q_{0,10}^*\ Q_{10,1}^*\}\ \ Q_{10}^* - \{Q_{02}^* + Q_{0,10}^*\ Q_{10,2}^*\}\ \ Q_{23}^*\ Q_{30}^* - Q_{0,10}^*\ Q_{10,0}^*$

Making use of relations (1) & (2) it can be shown that $\varnothing_0^*(0) = 1$, which implies that $\varnothing_0(t)$ is a proper distribution.

$$\text{MTSF} = E[T] = \frac{d}{ds}\,\varnothing_0^*(s)\Big|_{s=0}$$

$= (D_1'(0) - N_1'(0)) / D_1(0)$

$= (\ \mu_0 + \mu_1\ (\ p_{01} + p_{0,10}\ \ p_{10,1}) + (\ p_{02} + p_{0,10}\ \ p_{10,2})(\mu_2 + \mu_3) + \mu_{10}\ p_{0,10} / (1 - (p_{01} + p_{0,10}\ p_{10,1})\ p_{10} - (p_{02} + p_{0,10}\ p_{10,2})\ p_{23}) - p_{0,10}\ p_{10,0}$

where

$\mu_0 = \mu_{01} + \mu_{02} + \mu_{0,10}$,

$\mu_1 = \mu_{10} + \mu_{11}^{(4)} + \mu_{12}^{(5)}$,

$\mu_2 = \mu_{23} + \mu_{28}^{(7)} + \mu_{29}^{(6)}$,

$\mu_{10} = \mu_{10,0} + \mu_{10,1} + \mu_{10,2}$

Availability analysis

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

$M_0(t) = e^{-\lambda_1 t}\, e^{-\lambda_2 t}\, e^{-\lambda_3 t}$, , $M_1(t) = p\,\overline{G_1(t)}\ \ e^{-\lambda_1 t}$

$M_2(t) = q\,\overline{G_2(t)}\ e^{-\lambda_2 t}$, $M_3(t) = G_3(t)$, $M_{10}(t) = \overline{G_4(t)}\ e^{-\lambda_3 t}$

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t) + q_{0,10}(t)[c]A_{10}(t)$

$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{12}^{(5)}(t)[c]A_2(t) + q_{11}^{(4)}(t)[c]A_1(t)$,

$A_2(t) = M_2(t) + q_{23}(t)[c]A_3(t) + q_{28}^{(7)}(t)[c]\ \ A_8(t) + q_{29}^{(6)}(t)]\ [c]A_9(t)$

$A_3(t) = M_3(t) + q_{30}(t)[c]A_0(t)\ , A_8(t) = q_{82}(t)[c]A_2(t)$

$A_9(t) = q_{91}(t)[c]A_1(t)$, $A_{10}(t) = M_{10}(t) + q_{10,0}(t)[c]A_0(t) + q_{10,1}^{(11)}(t)[c]A_1(t) + q_{10,2}^{(11)}(t)[c]A_2(t)$ (8-15)

Taking Laplace Transform of eq. (8-15) and solving for $\hat{A}_0(s)$

$$\hat{A}_0(s) = N_2(s)/D_2(s) \qquad (16)$$

where

$N_2(s) = \{\hat{q}_{0,10}\widetilde{M}_{10} + \widetilde{M}_0\}[\{1 - \hat{q}_{11}^{(4)}\}\{1 - \hat{q}_{28}^{(7)}\hat{q}_{82}\} - \hat{q}_{12}^{(5)}\hat{q}_{29}^{(6)}\hat{q}_{91}] + \{\hat{q}_{01} + \hat{q}_{0,10}\hat{q}_{10,1}^{(11)}\}[\widetilde{M}_1\{1 - \hat{q}_{28}^{(7)}\hat{q}_{82}\} + \hat{q}_{12}^{(5)}\hat{q}_{23}\widetilde{M}_3 + \widetilde{M}_2] + \{\hat{q}_{02} + \hat{q}_{0,10}\hat{q}_{10,2}^{(11)}\}[\{\hat{q}_{23}\widetilde{M}_3\}\{1 - \hat{q}_{11}^{(4)}\} + \hat{q}_{29}^{(6)}\hat{q}_{91}\widetilde{M}_1]$

$D_2(s) = \{1 - \hat{q}_{11}^{(4)}\}\{1 - \hat{q}_{28}^{(7)}\hat{q}_{82}\} - \hat{q}_{12}^{(5)}\hat{q}_{29}^{(6)}\hat{q}_{91} - \{\hat{q}_{01} + \hat{q}_{0,10}\hat{q}_{10,1}^{(11)}\}[\hat{q}_{10}\{1 - \hat{q}_{28}^{(7)}\hat{q}_{82}\} + \hat{q}_{12}^{(5)}\hat{q}_{23}\hat{q}_{30}] - \{\hat{q}_{02} + \hat{q}_{0,10}\hat{q}_{10,2}^{(11)}\}\{[\hat{q}_{23}\hat{q}_{30}\{1 - \hat{q}_{11}^{(4)}\} + \hat{q}_{29}^{(6)}\hat{q}_{91}\hat{q}_{10}]$

(Omitting the arguments s for brevity)

The steady state availability

$$A_0 = \lim_{t\to\infty}[A_0(t)] = \lim_{s\to 0}[s\hat{A}_0(s)] = \lim_{s\to 0}\frac{s N_2(s)}{D_2(s)}$$

Using L' Hospitals rule, we get

$$A_0 = \lim_{s\to 0}\frac{N_2(s) + s N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)} \qquad (17)$$

The expected up time of the system in (0,t] is

$$\lambda_u(t) = \int_0^\infty A_0(z)\,dz$$

So that $\bar{\lambda}_u(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(s)}{sD_2(s)}$ (18)

The expected down time of the system in (0,t] is

$$\lambda_d(t) = t - \lambda_u(t)$$

So that $\bar{\lambda}_d(s) = \frac{1}{s^2} - \bar{\lambda}_u(s)$ (19)

The expected busy period of the server when there is failure due to heavy rain, and failure due to thunderstorm in (0,t]-R_0

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t) + q_{0,10}(t)[c]R_{10}(t)$

$R_1(t) = S_1(t) + q_{10}(t)[c]R_0(t) + q_{12}^{(5)}(t)[c]R_2(t) + q_{11}^{(4)}(t)[c]R_1(t)$

$R_2(t) = S_2(t) + q_{23}(t)[c]R_3(t) + q_{28}^{(7)}(t)\ R_8(t) + q_{29}^{(6)}(t)][c]R_9(t)$

$R_3(t) = S_3(t) + q_{30}(t)[c]R_0(t)$

$R_8(t) = S_8(t) + q_{82}(t)[c]R_2(t)$

$R_9(t) = S_9(t) + q_{91}(t)[c]R_1(t)$

$R_{10}(t) = S_{10}(t) + q_{10,0}(t)[c]R_0(t) + q_{10,1}^{(11)}(t)[c]R_1(t) + q_{10,2}^{(11)}(t)[c]R_2(t)$ (19-25)

where

$S_1(t) = p\,G_1(t)\,e^{-\lambda_1 t}$, $S_2(t) = q\,G_2(t)\,e^{-\lambda_2 t}$

$S_3(t) = S_8(t) = S_9(t) = G_3(t)$

$S_{10}(t) = G_4(t)$ (26)

Taking Laplacė Transform of eq. (19-25) and solving for $\bar{R}_0(s)$

$$\bar{R}_0(s) = N_3(s) / D_2(s) \qquad (27)$$

where

$N_3(s) = \{\hat{q}_{01} + \hat{q}_{0,10}\hat{q}_{10,1}^{(11)}\}[\hat{S}_1(1 - \hat{q}_{28}^{(7)}\hat{q}_{82}\} + \hat{q}_{12}^{(5)}[\hat{S}_2 + \hat{q}_{23}\hat{S}3 + \hat{q}_{28}^{(7)}\hat{S}8 + \hat{q}_{29}^{(6)}\hat{S}9)]] + \{\hat{q}_{02} + \hat{q}_{0,10}\hat{q}_{10,2}^{(11)}\}[\{\hat{S}_2 + \hat{q}_{23}\hat{S}_3 + \hat{q}_{28}^{(7)}\hat{S}_8 + \hat{S}_9\hat{q}_{29}^{(6)})(1 - \hat{q}_{11}^{(4)}) + \hat{S}_1\hat{q}_{29}^{(6)}\hat{q}_{91}] + \hat{q}_{0,10}\hat{S}_{10}[\{1 - \hat{q}_{28}^{(7)}\hat{q}_{82}\}\{1 - \hat{q}_{11}^{(4)}\} - \hat{q}_{29}^{(6)}\hat{q}_{91}\hat{q}_{12}^{(5)}]$

and $D_2(s)$ is already defined.

(Omitting the arguments s for brevity)

In the long run, $R_0 = \frac{N_3(0)}{D_2'(0)}$ (28)

Where

$N_3(0) = \{p_{01} + p_{0,10}\,p_{10,1}^{(11)}\}[\hat{S}_1(1 - p_{28}^{(7)}\} + p_{12}^{(5)}[\hat{S}_2 + p_{23}\hat{S}3 + p_{28}^{(7)}\hat{S}8 + p_{29}^{(6)}\hat{S}9)]] + \{p_{02} + p_{0,10}\,p_{10,2}^{(11)}\}[\{\hat{S}_2 + p_{23}\hat{S}_3 + p_{28}^{(7)}\hat{S}_8 + \hat{S}_9\ p_{29}^{(6)})(1 - p_{11}^{(4)}) + \hat{S}_1 p_{29}^{(6)}] + p_{0,10}\hat{S}_{10}[\{1 - p_{28}^{(7)}\}\{1 - p_{11}^{(4)}\} - p_{29}^{(6)}p_{12}^{(5)}]$

and $D_2'(0)$ is already defined.

The expected busy period of the server when there is failure due to heavy rain and failure due to thunderstorm in (0,t]

$\lambda_{rv}(t) = \int_0^\infty R_0(z)\,dz$ So that $\bar{\lambda}_{rv}(s) = \frac{\hat{R}_0(s)}{s}$

The expected number of visits by the repairman Type-I or Type-II for repairing the identical units in (0,t]-H_0

$H_0(t) = Q_{01}(t)[s][1 + H_1(t)] + Q_{02}(t)[s][1 + H_2(t)] + Q_{0,10}(t)[s]\,H_{10}(t)]$

$H_1(t) = Q_{10}(t)[s]H_0(t)] + Q_{12}^{(5)}(t)[s]\ H_2(t) + Q_{11}^{(4)}(t)][s]H_1(t)$,

$H_2(t) = Q_{23}(t)[s]H_3(t) + Q_{28}^{(7)}(t)\ [s]\ H_8(t) + Q_{29}^{(6)}(t)][c]H_9(t)$

$H_3(t) = Q_{30}(t)[s]H_0(t)$

$H_8(t) = Q_{82}(t)[s]H_2(t)$

$H_9(t) = Q_{91}(t)[s]H_1(t)$

$H_{10}(t) = Q_{10,0}(t)[s]H_{10}(t)] + Q_{10,1}^{(11)}(t)[s]H_1(t)]+Q_{10,2}^{(11)}(t)[s]\ H_2(t)]$ (29-35)

Taking Laplace Transform of eq. (29-35) and solving for $H_0^*(s)$

$$H_0^*(s) = N_4(s)\ /\ D_3(s) \qquad (36)$$

$N_4(s) = \{ Q_{01}^* + Q_{02}^*\}[\{ 1 - Q_{11}^{(4)*}\}\{1-Q_{28}^{(7)*}\ Q_{82}^*\} - Q_{12}^{(5)*}\ Q_{29}^{(6)*}\ Q_{91}^*\ ]$

And

$D_3(s) = \{1 - Q_{11}^{(4)*}\}\ \{ 1- Q_{28}^{(7)*}\ Q_{82}^*\} - Q_{12}^{(5)*}\ Q_{29}^{(6)*}\ Q_{91}^*](1- Q_{0,10}^*\ Q_{10,0}^*)-\{ Q_{01}^*+ Q_{0,10}^*\ Q_{10,1}^{(11)*}\}[Q_{10}^*\{ 1 - Q_{28}^{(7)*}\ Q_{82}^*\ \}+\ Q_{12}^{(5)*}\ Q_{23}^*\ Q_{30}^*] - \{Q_{02}^* + Q_{0,10}^*\ Q_{10,2}^{(11)*}\}[Q_{23}^*\ Q_{30}^*\{1 - Q_{11}^{(4)*}\}+ Q_{29}^{(6)*}\ Q_{91}^*\ Q_{10}^*]$

(Omitting the arguments s for brevity)

In the long run,

$H_0 = N_4(0)\ /\ D_3'(0)$ (37)

where

$N_4(0) =\{1-p_{0,10}\}[\{1- p_{11}^{(4)}\}\ \{1- p_{28}^{(7)}\}-p_{12}^{(5)}\ p_{29}^{(6)}]$

The expected number of visits by the multispecialty repairman Type-III for repairing the identical units in (0,t]-W_0

$W_0(t)=Q_{01}(t)[s]W_1(t)+\ Q_{02}(t)[s]\ W_2(t) + Q_{10,0}(t)[s]\ W_{10}(t)$

$W_1(t) = Q_{10}(t)[s]W_0(t)] + Q_{12}^{(5)}(t)[s]\ W_2(t) +\ Q_{11}^{(4)}(t)][s]W_1(t)$,

$W_2(t) = Q_{23}(t)[s]W_3(t) + Q_{28}^{(7)}(t)\ [s]\ W_8(t) +Q_{29}^{(6)}(t)][c]W_9(t)$

$W_3(t) = Q_{30}(t)[s][1+W_0(t)\ ]$

$W_8(t) = Q_{82}(t)[s][1+W_2(t)\ ]$

$W_9(t) = Q_{91}(t)[s][1+W_1(t)\ ]$

$W_{10}(t)=Q_{10,0}(t)[s]W_0(t)+\ Q_{10,1}^{(11)}(t)[s]\ W_1(t) + Q_{10,2}^{(12)}(t)[s]\ W_2(t)$ (38-44)

Taking Laplace Transform of eq. (33-39) and solving for $H_0^*(s)$

$$H_0^*(s) = N_5(s)\ /\ D_3(s) \qquad (45)$$

$N_5(s) = \{Q_{01}^*+ Q_{0,10}^*Q_{0,10}^{(11)*}\ \}[Q_{12}^{(5)*}\ [\ Q_{23}^*\ Q_{30}^* + Q_{28}^{(5)*}\ Q_{82}^* + Q_{29}^{(6)*}\ Q_{91}^*\ ] + \{Q_{02}^* + Q_{0,10}^*Q_{10,2}^{(11)*}\}[\ [\ Q_{23}^*\ Q_{30}^* + Q_{28}^{(5)*}\ Q_{82}^* + Q_{29}^{(6)*}\ Q_{91}^*\ \{1 - Q_{11}^{(4)*}\}]$

(Omitting the arguments s for brevity)

In the long run,

$W_0 = N_5(0)\ /\ D_3'(0)$ (46)

where $N_5(0) = \{p_{01}+ p_{0,10}\ p_{10,1}^{(11)}\ \}p_{12}^{(5)} + \{ p_{02}+ p_{0,10}\ p_{10,2}^{(11)}\}\ \{1 - p_{11}^{(4)}\}]$

The expected number of visits by the multispecialty repairman Type-IV for repairing the identical units in (0,t]-Y_0

$Y_0(t)=Q_{01}(t)[s]Y_1(t)+\ Q_{02}(t)[s]\ Y_2(t) + Q_{0,10}(t)[s]\ [1+Y_{10}(t)]$

$Y_1(t) = Q_{10}(t)[s]Y_0(t) + Q_{12}^{(5)}(t)[s]Y_2(t) +\ Q_{11}^{(4)}(t)][s]Y_1(t)$,

$Y_2(t) = Q_{23}(t)[s]Y_3(t) + Q_{28}^{(7)}(t)\ [s]Y_8(t) +Q_{29}^{(6)}(t)][c]Y_9(t)$

$Y_3(t) = Q_{30}(t)[s][1+Y_0(t)\ ]$

$Y_8(t) = Q_{82}(t)[s]Y_2(t)$

$Y_9(t) = Q_{91}(t)[s]Y_1(t)$

$Y_{10}(t)=Q_{10,0}(t)[s]Y_0(t)+ Q_{10,1}^{(11)}(t)[s]\ Y_1(t) + Q_{10,2}^{(12)}(t)[s]\ Y_2(t)$ (47-53)

Taking Laplace Transform of eq. (47-53) and solving for$Y_0^*(s)$,we get

$$Y_0^*(s) = N_6(s)\ /\ D_3(s) \qquad (54)$$

$N_6(s) = Q_{0,10}^*\ [\{1 - Q_{11}^{(4)*}\}(1- Q_{28}^{(5)*}\ Q_{82}^*\ \} - Q_{12}^{(5)*}Q_{29}^{(6)*}\ Q_{91}^*\{1- Q_{0,10}^*Q_{,10,0}^*\ \}+\{Q_{02}^* + Q_{0,10}^*Q_{10,2}^{(11)*}\}[\ [\ Q_{23}^*\ Q_{30}^*\ \{1 - Q_{11}^{(4)*}\}+ Q_{10}^*\ Q_{29}^{(6)*}\ Q_{91}^*\ ]$

(Omitting the arguments s for brevity)

In the long run,

$W_0 = N_6(0)\ /\ D_3'(0)$ (55)

where $N_6(0) = p_{0,10}[\{1-p_{11}^{(4)}\}\{1- p_{28}^{(7)}\}- p_{12}^{(5)}\ p_{29}^{(6)}]$

$p_{12}^{(5)} + \{ p_{02}+ p_{0,10}\ p_{10,2}^{(11}\}\ \{1 - p_{11}^{(4)}\}]$

Benefit-Function

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to heavy rain and failure due to thunderstorm, expected number of visits by the repairman for unit failure. The expected total Benefit-Function incurred in (0,t] is

$$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2C(s))$$

$$= K_1A_0 - K_2R_0 - K_3H_0 - K_4W_0 - K_5Y_0$$

where

K_1 - revenue per unit up-time,

K_2 - cost per unit time for which the system is busy under repairing,

K_3 - cost per visit by the repairman type- I or type- II for units repair,

K_4 - cost per visit by the multispecialty repairman Type- III for units repair,

K_5 - cost per visit by the multispecialty repairman Type- IV for units repair

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to heavy rain and due to failure due to thunderstorm increases, the MTSF, steady state availability decreases and the Profit-function decreased as the failure increases.

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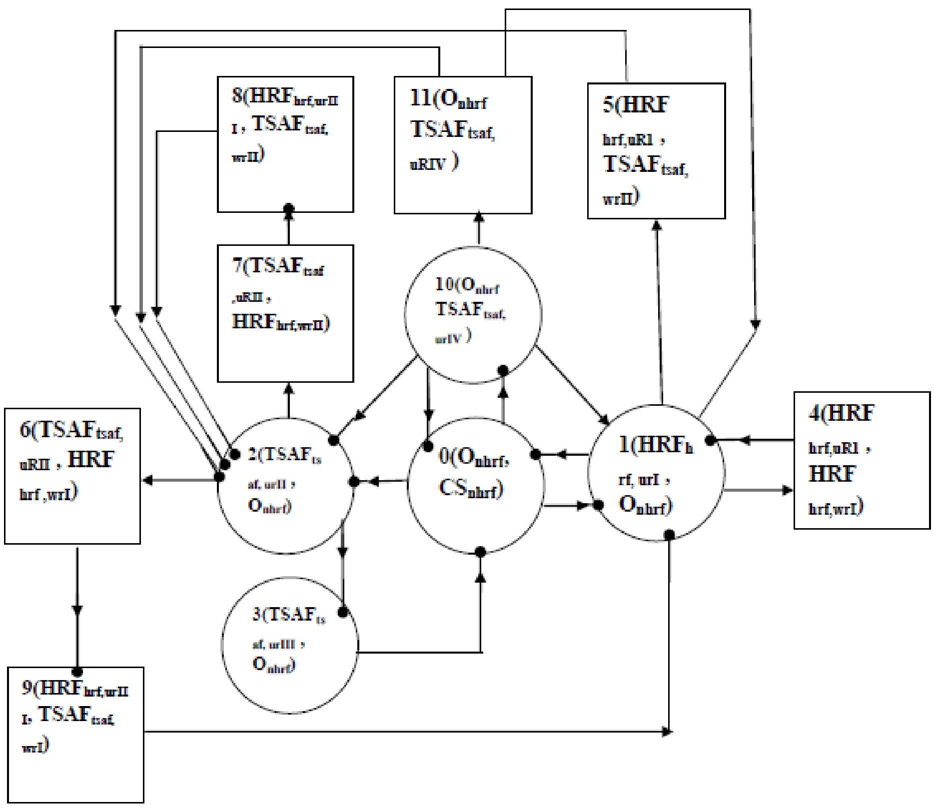

Fig. The State Transition Diagram

Up-State down-state

regeneration point

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