Identification of crack parameters in a cantilever beam using experimental and wavelet analysis

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Abstract—Crack obstructs the optimum performance of a machine. Material fatigue is the cause of most of the failures faced by the machine. Crack changes the stiffness of the structural element which affects the dynamic behavior of the whole structure. This change in dynamic behavior helps to identify the crack parameters i.e. crack location and size. In this paper the crack is modeled as a rotational spring and equation for non-dimensional spring stiffness is developed. By evaluating first three natural frequencies using vibration measurements, curves of crack equivalent stiffness are plotted and the intersection of the three curves indicates the crack location and size. Experiments are performed on cantilever beams with single crack (each at different locations and having varying sizes) using FFT set up to obtain natural frequencies which are compared with those obtained by ANSYS package. The time-amplitude data obtained is further used in the wavelet analysis to obtain time-frequency data. This data is important to find out the effect of small crack parameters on the dynamic properties of the system.

Keywords - Crack identification, cantilever beams, natural frequency, wavelet analysis.

I. INTRODUCTION

The physical discontinuity occurred in the geometry of the structures or machine components is termed as 'Crack or Damage'. In the past few years a significant amount of effort has been dedicated to the development of non-destructive testing techniques for detection of crack induced damage in a structure or machine component. In this view the methods based on vibration for detection of cracks offer the advantage that they enable determination of location and size of crack from the vibration data collected from a single point or multiple points of the component [1]. Vibration parameters of a component are changed due to development of crack in the component i.e it results in reduction in stiffness and increase in the damping. As these changes are mode dependent, it becomes possible to estimate location and size of crack by measuring changes in vibration parameters. The vibration parameters can be structural parameters such as mass, stiffness and flexibility or can be modal parameters such as natural frequencies, modal damping values and mode shapes. Crack detection using vibration methods require any one or of these parameters as a basis for crack detection [2].

The proposed method is based on the analysis of changes observed in the modal parameters of dynamic system because of crack. Modal frequency parameter is selected for analysis because modal frequencies are properties of whole component, so only one test is required to assess the integrity of complete component. Crack induces local flexibility in the structures at crack location as a result modal frequencies are reduced [3]. The reduction in modal frequencies depends on crack depth, crack location and number of cracks. Hence crack or damage identification is an important concern today using effective, in-expensive and non-destructive methods is a real challenge.

Various researchers have conducted studies on the identification of crack location and size through vibration measurements in changes of the modal parameters of the system. R. Y. Liang et al. [1] developed the theoretical relationships between eigen-frequency changes and magnitudes and the locations of crack-induced damage for beam structures with either simply supported or cantilever boundary conditions. A characteristic equation is derived as a base for the development of the relationship. Zheng Li et al. [2] proposed a damage detection method based on a continuous wavelet transform is and applied to analyze flexural wave in a cracked beam. For flexural waves obtained from FEM or experiments, some useful characters of the incident, reflected and transmitted waves at a certain frequency can be extracted by the Gabor wavelet to exactly identify the damage location and its extent.

II. THEORETICAL ANALYSIS

2.1 Derivation Of Characteristic Equation For Cantilever Beam:
The physical model of a beam with a discrete crack can be depicted as in fig. in which the local flexibility introduced by a crack is represented by a massless torsional spring, with a spring constant $K_t$. From the basic vibration theory, the modes of harmonic vibration of the two sides of the spring can be written as,

$$u_1(R) = A_1 \sin \lambda \beta + A_2 \cos \lambda \beta + A_3 \sinh \lambda \beta + A_4 \cosh \lambda \beta$$

(1)

and

$$u_2(R) = B_1 \sin \lambda \beta + B_2 \cos \lambda \beta + B_3 \sinh \lambda \beta + B_4 \cosh \lambda \beta$$

(2)

where $u_1$ and $u_2$ are the displacement on left and right side of the crack respectively, are functions of $\beta$, and defined as $\beta = x/L$; $R = L_n/L$ is dimensionless crack position; $A_i$ and $B_i$ ($i=1,2,3,4$) are the constants to be determined from the boundary conditions; and

$$\lambda^2 = \frac{\omega^2 E L^4}{EI}$$

in which $\omega$ is the natural vibration angular frequency (measured one), $\rho$ is the material density, and $A$ is the cross-sectional area of the beam.

The boundary conditions for a cantilever beam are as follows:

At $\beta = 0$, $u_1(R) = 0$ and $u_1'(R) = 0$ (3)

At $\beta = 1$ $u_2(R) = 0$ and $u_2'(R) = 0$ (4)

The continuity equation at the crack requires,

$$u_1(R) = u_2(R)$$

$$u_1''(R) = u_2''(R)$$

(5)

The compatibility condition due to rotational flexibility at crack is,

$$u_1' + \frac{EI}{K_r} u_1''(R) = u_2' + \frac{EI}{K_r} u_2''(R)$$

(6)

Where $K_r = K_{rot}$ and $K_t$ is stiffness of massless torsional (rotational) spring of infinitesimal length.

Applying Equations (3) to (6) to Equations (1) and (2) we get characteristic equation for cantilever beam. Symbolically, the characteristic equation can be written as:

$$|\Delta| = 0$$

(7)

Or that can be written as,

$$\left(\frac{K}{K_t}\right) |\Delta| + |\Delta_2| = 0$$

(8)

Equation (8) can be written as,

$$K = -3L \frac{|\Delta_2|}{|\Delta_1|}$$

(9)

Where $\Delta_1$ and $\Delta_2$ are matrices that are solved and substituted in equation (9) to obtain the spring stiffness $K$. Based on fracture mechanics principal, the following equation provides the relationship between the depth ‘$a$’ of the crack and the values of $K_r$ and $h$ is the height of the crack.

$$K_r = \frac{K_{EI}}{L} = \frac{1}{C}$$

(10)

$$C = \frac{5.1346EI}{I(a/h)}$$

(11)

Where $C$ is compliance and $I(a/h)$ is dimensionless local compliance function given by,

$$I(a/h) = 1.8624(a/h) - 3.95(a/h) + 16.375(a/h)^2 - 37.226(a/h)^3 + 76.81(a/h)^4 - 126.9(a/h)^5 + 172(a/h)^6 - 143.97(a/h)^7 + 66.65(a/h)^8$$

(12)

By solving Eq. (9) the non-dimensional spring stiffness ($K$) is calculated for different crack locations ($R$) and having various crack sizes ($\beta$) and are tabulated as shown below:

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>R</th>
<th>$\beta$</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>For $f_1$</td>
</tr>
<tr>
<td>1</td>
<td>0.0</td>
<td>-</td>
<td>29.638</td>
</tr>
<tr>
<td>2</td>
<td>0.084</td>
<td>0.25</td>
<td>4.0286</td>
</tr>
<tr>
<td>3</td>
<td>0.084</td>
<td>0.5</td>
<td>0.7995</td>
</tr>
<tr>
<td>4</td>
<td>0.084</td>
<td>0.75</td>
<td>0.1413</td>
</tr>
<tr>
<td>5</td>
<td>0.167</td>
<td>0.25</td>
<td>3.9498</td>
</tr>
<tr>
<td>6</td>
<td>0.167</td>
<td>0.5</td>
<td>0.7956</td>
</tr>
<tr>
<td>7</td>
<td>0.167</td>
<td>0.75</td>
<td>0.1410</td>
</tr>
<tr>
<td>8</td>
<td>0.25</td>
<td>0.25</td>
<td>3.8464</td>
</tr>
<tr>
<td>9</td>
<td>0.25</td>
<td>0.5</td>
<td>0.7915</td>
</tr>
<tr>
<td>10</td>
<td>0.25</td>
<td>0.75</td>
<td>0.1408</td>
</tr>
<tr>
<td>11</td>
<td>0.3</td>
<td>0.25</td>
<td>4.2030</td>
</tr>
<tr>
<td>12</td>
<td>0.3</td>
<td>0.5</td>
<td>0.8976</td>
</tr>
<tr>
<td>13</td>
<td>0.3</td>
<td>0.75</td>
<td>0.1628</td>
</tr>
</tbody>
</table>

Table (I): Spring Stiffness for various crack locations and varying crack sizes.

Now for measured natural frequencies $\omega$, the right hand side of the Eq. (9) is known for all possible location of the crack characterized by ‘R’. Thus three curves of $K$ vs $R$ are obtained by Eq. (9), for measured first three natural frequencies. The point of intersection of the three curves gives the crack location. The crack size is then computed using the standard relation between stiffness and crack size given by Eq. (12). Thus as shown in Fig.
(2), (3) and (4) the intersection of the curves of first three natural frequencies gives the crack location R.

III. EXPERIMENTAL METHOD

The purpose behind experimentation is to find out the change in the vibration signature (frequency change) of an actual structure due to presence of a crack. Again, it is proposed to relate the inverse problem of crack assessment to these signatures only.

Experimentation is performed only to measure the natural frequencies of a uniform cantilever beam for various combinations of crack parameters by modal analysis technique. The crack in a structure is quantified through the measurements of natural frequencies at two or more stages of its life. The basic requirement of the method is to determine reproducibly the first three natural frequencies of the structure under consideration. It does not matter whether the boundary conditions are exactly same as those assumed in the dynamic analysis, if they are reproducible and the mode shapes are not greatly altered from those predicted by the theoretical model.

In order to achieve this, experimental modal analysis is carried out which enables the determination of dynamic properties such as frequency, damping, and mode shapes.

3.1 Specimen specifications

Specimen selected for experimentation has same material and geometrical properties as those selected in the numerical analysis by FEM, which are as follows:

- Total length of the beam = 0.36 m
- Length of the free span of the beam (L) = 0.3 m
- Depth of the beam (h) = 0.03 m
- Thickness of the beam (t) = 0.01 m
- Area of cross section (A) = 3 × 10⁻⁴ m²
- Moment of Inertia (I) = 2.25 × 10⁻⁸ m⁴
- Material of the beam : Mild Steel
- Modulus of elasticity (E) = 2 × 10¹¹ N / mm²
- Material density (ρ) = 7800 Kg / m³.

3.2 Experimental Set up

![Fig. (5) Block diagram of experimental set up](image)

The experimental set up consists of the test instruments as shown in block diagram, the test specimens and a clamping fixture. The entire experimentation is based on the cantilever condition of the rectangular cross section beam under investigation. The requirement of the cantilever condition is that the deflection as well as slope at the fixed end should be zero. A T-slotted machine bed is used as a foundation for the clamping fixture to have very low frequencies.

Modal analysis involves fixing the accelerometer at one location and impacting the structure at one point and moving the accelerometer to other points of interest. The hammer impulse consists of a nearly constant force over a broad frequency range and is therefore capable of exciting all resonances in that range. The resulting Frequency Response Function (FRF) obtained imparts the modal parameters.
Some of the FFT spectrums (Frequency vs. Acceleration) are shown below:

![Fig. (6) for R=0.25, β=0.25](image)

![Fig. (7) for R=0.25, β=0.5](image)

![Fig. (8) for R=0.3, β=0.75](image)

First three natural frequencies for each beam (un-cracked and cracked) are obtained from these FFT spectrums and are compared with those obtained using FEM method. The comparison shows that the % error between the values of the two methods is very less and varies in the range of 2% to 5%.

IV. WAVELET ANALYSIS

When the time localization of the spectral components is needed, a transform giving the time-frequency representation of the signal is required. Thus Wavelet transform is capable of providing the time and frequency information simultaneously, giving a time-frequency representation of the signal. As far as crack identification in beams is concerned this representation is necessary to efficiently identify the small crack parameters as we know that small crack parameters have minor effects on the natural frequencies.

Wavelets methods combine the structural dynamic parameters such as modal frequencies, modal shape and modal damp, etc. with the determined exponent to detect the damage location and depth. In this study Continuous Wavelet Transform is performed using time-amplitude information obtained from the experimental results. To plot the obtained results Morlet wavelet is used as the mother wavelet. In order to perform wavelet analysis MATLAB 7.8 is used and the plots for each beam (un-cracked and cracked) are plotted.

Some of the wavelet plots are shown below with crack location R and crack size β:

![Fig. (9) For R=25mm, β=0.25](image)

![Fig. (10) For R=50mm, β=0.75](image)

![Fig. (11) For R=75mm, β=0.25](image)

In the obtained wavelet plots at low scale values we get higher frequencies and at high scale values we get lower frequencies. These plots also reveal what frequency component exists at particular time interval accurately.

Thus efficient time-frequency representation is shown by these plots.

V. CONCLUSION

In this paper, damage detection in cracked cantilever beams are studied by using various methods. Both FEM and experimental results show that the methods used have adequate accuracy for moderate cracks and high sensitivity for small cracks. As the crack depth increases, the estimated error of the crack location deceases. The method provides a simple, fast and effective nondestructive detection technique by using...
the CWT as the signal analysis tool. This method can be extended for damage detection of complex structures.

REFERENCES


