



# Benefit-Function of Two- Identical Cold Standby System subject to Failure due to buckling or sun kink caused by high compressive stress

Ashok Kumar Saini

BLJS COLLEGE, TOSHAM (BHIWANI) HARYANA INDIA

Email : drashokksaini2009@gmail.com

**Abstract-** In science, **buckling** is a mathematical instability, leading to a failure mode. Theoretically, buckling is caused by a bifurcation in the solution to the equations of static equilibrium. At a certain stage under an increasing load, further load is able to be sustained in one of two states of equilibrium: an undeformed state or a laterally-deformed state.

In practice, buckling is characterized by a sudden failure of a structural member subjected to high compressive stress, where the actual compressive stress at the point of failure is less than the ultimate compressive stresses that the material is capable of withstanding. Mathematical analysis of buckling often makes use of an axial load eccentricity that introduces a secondary bending moment, which is not a part of the primary applied forces to which the member is subjected. As an applied load is increased on a member, such as column, it will ultimately become large enough to cause the member to become unstable and is said to have buckled. Further load will cause significant and somewhat unpredictable deformations, possibly leading to complete loss of the member's load-carrying capacity. If the deformations that follow buckling are not catastrophic the member will continue to carry the load that caused it to buckle. If the buckled member is part of a larger assemblage of components such as a building, any load applied to the structure beyond that which caused the member to buckle will be redistributed within the structure. In this paper we have taken **Failure due to buckling or sun kink caused by high compressive stress**. When the main unit **failure due to buckling caused by high compressive stress** then cold standby system becomes operative. **Failure due to buckling caused by high compressive stress** cannot occur simultaneously in both the units and after failure the unit undergoes very costly repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

**Keywords:** Cold Standby, Failure due to buckling or sun kink caused by high compressive stress ,first come

first serve, MTSF, Availability, Busy period, Benefit - Function.

## INTRODUCTION

If the load on a column is applied through the center of gravity (centroid) of its cross section, it is called an axial load. A load at any other point in the cross section is known as an eccentric load. A short column under the action of an axial load will fail by direct compression before it buckles, but a long column loaded in the same manner will fail by buckling (bending), the buckling effect being so large that the effect of the axial load may be neglected. The intermediate-length column will fail by a combination of direct compressive stress and bending.

In 1757, mathematician Leonhard Euler derived a formula that gives the maximum axial load that a long, slender, ideal column can carry without buckling. An ideal column is one that is perfectly straight, homogeneous, and free from initial stress. The maximum load, sometimes called the critical load, causes the column to be in a state of unstable equilibrium; that is, the introduction of the slightest lateral force will cause the column to fail by buckling. The formula derived by Euler for columns with no consideration for lateral forces is given below. However, if lateral forces are taken into consideration the value of critical load remains approximately the same.

Various forms of buckling

There are four basic forms of bifurcation associated with loss of structural stability or buckling in the case of structures with a single degree of freedom. These comprise two types of pitchfork bifurcation, one saddle-node bifurcation (often referred to as a limit point) and one transcritical bifurcation. The pitchfork bifurcations are the most commonly studied forms and include the buckling of columns and struts, sometimes known as Euler buckling; the buckling of plates, sometimes known as local buckling, which is well known to be relatively safe (both are supercritical phenomena) and the buckling of shells, which is well-known to be a highly dangerous (subcritical phenomenon). Using the

concept of potential energy, equilibrium is defined as a stationary point with respect to the degree(s) of freedom of the structure. We can then determine whether the equilibrium is stable, if the stationary point is a local minimum; or unstable, if it is a maximum, point of inflection or saddle point (for multiple-degree-of-freedom structures only)

### Bicycle wheels

A conventional bicycle wheel consists of a thin rim kept under high compressive stress by the (roughly normal) inward pull of a large number of spokes. It can be considered as a loaded column that has been bent into a circle. If spoke tension is increased beyond a safe level, the wheel spontaneously fails into a characteristic saddle shape (sometimes called a "taco" or a "pringle") like a three-dimensional Euler column. This is normally a purely elastic deformation and the rim will resume its proper plane shape if spoke tension is reduced slightly.

### Surface materials

Buckling is also a failure mode in pavement materials, primarily with concrete, since asphalt is more flexible. Radiant heat from the sun is absorbed in the road surface, causing it to expand, forcing adjacent pieces to push against each other. If the stress is great enough, the pavement can lift up and crack without warning. Going over a buckled section can be very jarring to automobile drivers, described as running over a speed hump at highway speeds.



Similarly, rail tracks also expand when heated, and can fail by buckling, a phenomenon called **sun kink**. It is more common for rails to move laterally, often pulling the underlain railroad ties (sleepers) along.

In this paper we have taken **Failure due to buckling or sun kink caused by high compressive stress**. When the main operative unit fails then cold standby system becomes operative. **Failure due to buckling caused by high compressive stress** cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of very high cost in case of Failure due to buckling caused by high compressive stress immediately. The repair is done on the basis of first fail first repaired.

### Assumptions

1.  $\lambda_1, \lambda_2$  are constant failure rates for **Failure due to buckling, sun kink caused by high compressive**

**stress** respectively. The CDF of repair time distribution of Type I and Type II are  $G_1(t)$  and  $G_2(t)$ .

2. The Failure due to buckling caused by high compressive stress is non-instantaneous and it cannot come simultaneously in both the units.
3. The repair starts immediately after Failure due to buckling or sun kink caused by high compressive stress and works on the principle of first fail first repaired basis.
4. The repair facility does no damage to the units and after repair units are as good as new.
5. The switches are perfect and instantaneous.
6. All random variables are mutually independent.
7. When both the units fail, we give priority to operative unit for repair.
8. Repairs are perfect and failure of a unit is detected immediately and perfectly.
9. The system is down when both the units are non-operative.

### Notations

$\lambda_1, \lambda_2$  - Failure due to buckling or sun kink caused by high compressive stress respectively.

$G_1(t), G_2(t)$  - repair time distribution Type-I or Type-II due to Failure due to buckling, sun kink caused by high compressive stress respectively.

$p, q$  - probability of Failure due to buckling, sun kink caused by high compressive stress respectively such that  $p+q=1$

$M_i(t)$  System having started from state  $i$  is up at time  $t$  without visiting any other regenerative state

$A_i(t)$  state is up state at instant  $t$

$R_i(t)$  System having started from state  $i$  is busy for repair at time  $t$  without visiting any other regenerative state.

$B_i(t)$  the server is busy for repair at time  $t$ .

$H_i(t)$  Expected number of visits by the server for repairing given that the system initially starts from regenerative state  $i$

### Symbols for states of the System

**Superscripts O, CS, BF, SKF,**

Operative, Cold Standby, Failure due to buckling, sun kink caused by high compressive stress respectively

**Subscripts nbf, bf, skf, ur, wr, uR**

No Failure due to buckling, Failure due to buckling, Failure due to sun kink caused by high compressive stress, under repair, waiting for repair, under repair continued from previous state respectively

Up states – 0, 1, 2, 7, 8 ;  
 Down states – 3, 4, 5, 6  
 regeneration point – 0,1,2, 7, 8

**States of the System**

**0(O<sub>nbf</sub>, CS<sub>nbf</sub>)**

One unit is operative and the other unit is cold standby and there is no Failure due to buckling in both the units.

**1(SKF<sub>skf,ur</sub>, O<sub>nbf</sub>)**

The operating unit fails due to Failure due to sun kink caused by high compressive stress and is under repair immediately of very costly Type- II and standby unit starts operating with no Failure due to buckling

**2(BF<sub>bf,ur</sub>, O<sub>nbf</sub>)**

The operative unit fails due to failure due to buckling and undergoes repair of type I and the standby unit becomes operative with no Failure due to buckling

**3(BF<sub>bf,uR</sub>, SKF<sub>skf,wr</sub>)**

The first unit fails due to Failure due to buckling and under very costly Type-I repair is continued from state 1 and the other unit fails due to SKF resulting from failure due to sun kink caused by high compressive stress and is waiting for repair of Type -II.

**4(SKF<sub>skf,uR</sub>, SKF<sub>skf,wr</sub>)**

The repair of the unit is failed due to SKF resulting from Failure due to sun kink caused by high compressive stress is continued from state I and the other unit failed due to SKF resulting from Failure due to sun kink caused by high compressive stress is waiting for repair of Type-II.

**5(BF<sub>bf,uR</sub>, BF<sub>bf,wr</sub>)**

The operating unit fails due to Failure due to buckling and under repair of Type – I continues from the state 2 and the other unit fails also due to Failure due to buckling is waiting for repair of Type- I.

**6(BF<sub>bf,uR</sub>, SKF<sub>skf,wr</sub>)**

The operative unit fails due to Failure due to buckling and under repair continues from state 2 of Type –I and the other unit is failed due to SKF resulting from Failure due to sun kink caused by high compressive stress and under very costly Type-II

**7(O<sub>nbf</sub>, SKF<sub>skf,ur</sub>)**

The one unit is operative with no Failure due to buckling and the other unit failed due to SKF resulting from Failure due to sun kink caused by high compressive stress is under repair of Type-II

**8(O<sub>nbf</sub>, BF<sub>bf,ur</sub>)**

The one unit is operative with no Failure due to buckling and the other unit is failed due to Failure due to buckling is under very costly repair of Type-I.

**Transition Probabilities**

Simple probabilistic considerations yield the following expressions:

$$\begin{aligned}
 p_{01} &= p, \quad p_{02} = q, \\
 p_{10} &= pG_1^*(\lambda_1) + qG_1^*(\lambda_2) = p_{70}, \\
 p_{20} &= pG_2^*(\lambda_1) + qG_2^*(\lambda_2) = p_{80}, \\
 p_{11}^{(3)} &= p(1 - G_1^*(\lambda_1)) = p_{14} = p_{71}^{(4)}, \quad p_{28}^{(5)} = q(1 - G_2^*(\lambda_2)) = p_{25} = p_{82}^{(5)} \quad (1)
 \end{aligned}$$

We can easily verify that

$$\begin{aligned}
 p_{01} + p_{02} &= 1, \\
 p_{10} + p_{17}^{(4)} (= p_{14}) + p_{18}^{(3)} (= p_{13}) &= 1, \\
 p_{80} + p_{82}^{(5)} + p_{87}^{(6)} &= 1 \quad (2)
 \end{aligned}$$

And mean sojourn time is

$$\mu_0 = E(T) = \int_0^\infty P[T > t] dt$$

**Mean Time To System Failure**

$$\begin{aligned}
 \emptyset_0(t) &= Q_{01}(t)[s] \emptyset_1(t) + Q_{02}(t)[s] \emptyset_2(t) \\
 \emptyset_1(t) &= Q_{10}(t)[s] \emptyset_0(t) + Q_{13}(t) + Q_{14}(t) \\
 \emptyset_2(t) &= Q_{20}(t)[s] \emptyset_0(t) + Q_{25}(t) + Q_{26}(t) \quad (3-5)
 \end{aligned}$$

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-5) and solving for

$$\emptyset_0^*(s) = N_1(s) / D_1(s) \quad (6)$$

where

$$\begin{aligned}
 N_1(s) &= Q_{01}^* [ Q_{13}^*(s) + Q_{14}^*(s) ] + \\
 &\quad Q_{02}^* [ Q_{25}^*(s) + Q_{26}^*(s) ] \\
 D_1(s) &= 1 - Q_{01}^* Q_{10}^* - Q_{02}^* Q_{20}^*
 \end{aligned}$$

Making use of relations (1) & (2) it can be shown that  $\emptyset_0^*(0) = 1$ , which implies that  $\emptyset_0(t)$  is a proper distribution.

$$\begin{aligned}
 \text{MTSF} = E[T] &= \left. \frac{d}{ds} \emptyset_0^*(s) \right|_{s=0} \\
 &= (D_1'(0) - N_1'(0)) / D_1(0) \\
 &= (\mu_0 + p_{01} \mu_1 + p_{02} \mu_2) / (1 - p_{01} p_{10} - p_{02} p_{20})
 \end{aligned}$$

where

$$\begin{aligned}
 \mu_0 &= \mu_{01} + \mu_{02}, \\
 \mu_1 &= \mu_{01} + \mu_{17}^{(4)} + \mu_{18}^{(3)}, \\
 \mu_2 &= \mu_{02} + \mu_{27}^{(6)} + \mu_{28}^{(5)}
 \end{aligned}$$

**Availability analysis**

Let  $M_i(t)$  be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

$$\begin{aligned}
 M_0(t) &= e^{-\lambda_1 t} e^{-\lambda_2 t}, \quad M_1(t) = p G_1(t) e^{-(\lambda_1 + \lambda_2)t} = M_7(t) \\
 M_2(t) &= q G_2(t) e^{-(\lambda_1 + \lambda_2)t} = M_8(t)
 \end{aligned}$$

The point wise availability  $A_i(t)$  have the following recursive relations

$$\begin{aligned}
 A_0(t) &= M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t) \\
 A_1(t) &= M_1(t) + q_{10}(t)[c]A_0(t) +
 \end{aligned}$$

$$q_{18}^{(3)}(t)[c]A_8(t) + q_{17}^{(4)}(t)[c]A_7(t),$$

$$A_2(t) = M_2(t) + q_{20}(t)[c]A_0(t) + [q_{28}^{(5)}(t)[c] A_8(t) + q_{27}^{(6)}(t) [c]A_7(t)$$

$$A_7(t) = M_7(t) + q_{70}(t)[c]A_0(t) + [q_{71}^{(4)}(t)[c] A_1(t) + q_{78}^{(3)}(t) [c]A_8(t) \quad A_8(t) = M_8(t) + q_{80}(t)[c]A_0(t) + [q_{82}^{(5)}(t)[c] A_2(t) + q_{87}^{(6)}(t) [c]A_7(t) \quad (7-11)$$

Taking Laplace Transform of eq. (7-11) and solving for  $\hat{A}_0(s)$

$$\hat{A}_0(s) = N_2(s) / D_2(s) \quad (12)$$

where

$$N_2(s) = \bar{M}_0(1 - \hat{q}_{78}^{(3)} - \hat{q}_{87}^{(6)}) - \hat{q}_{82}^{(5)}(\hat{q}_{27}^{(6)}\hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)} - \hat{q}_{71}^{(4)}) - (\hat{q}_{17}^{(4)} + \hat{q}_{87}^{(6)}\hat{q}_{18}^{(3)}) + \hat{q}_{71}^{(4)}\hat{q}_{82}^{(5)}(\hat{q}_{17}^{(4)} - \hat{q}_{27}^{(6)}\hat{q}_{18}^{(3)}) + \hat{q}_{01}[\bar{M}_1(1 - \hat{q}_{78}^{(3)}\hat{q}_{87}^{(6)}) + \hat{q}_{71}^{(4)}(\bar{M}_7 + \hat{q}_{78}^{(3)}\bar{M}_8) + \hat{q}_{18}^{(3)}(\bar{M}_7\hat{q}_{87}^{(6)} - \bar{M}_8) - \hat{q}_{82}^{(5)}(\bar{M}_1(\hat{q}_{27}^{(6)}\hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) + \hat{q}_{17}^{(4)}(-\bar{M}_2(\hat{q}_{78}^{(3)} + \bar{M}_7\hat{q}_{28}^{(5)}) - \hat{q}_{18}^{(3)}(\bar{M}_2 + \bar{M}_7\hat{q}_{27}^{(6)}))] \hat{q}_{02}[\bar{M}_2(1 - \hat{q}_{78}^{(3)}\hat{q}_{87}^{(6)}) + \hat{q}_{27}^{(6)}(\bar{M}_7 + \hat{q}_{78}^{(3)}\bar{M}_8) + \hat{q}_{28}^{(5)}(\bar{M}_7\hat{q}_{87}^{(6)} + \bar{M}_8) - \hat{q}_{71}^{(4)}(\bar{M}_1 - \hat{q}_{27}^{(6)}\hat{q}_{18}^{(3)}) + \hat{q}_{28}^{(5)} + \hat{q}_{87}^{(6)} + \hat{q}_{17}^{(4)}(\bar{M}_2 + \hat{q}_{28}^{(5)}\bar{M}_8) - \hat{q}_{18}^{(3)}(-\bar{M}_2\hat{q}_{87}^{(6)} + \bar{M}_8\hat{q}_{27}^{(6)})] \hat{q}_{18}^{(3)}(\bar{M}_2 + \bar{M}_7\hat{q}_{27}^{(6)})] D_2(s) = (1 - \hat{q}_{78}^{(3)} - \hat{q}_{87}^{(6)}) - \hat{q}_{82}^{(5)}(\hat{q}_{27}^{(6)}\hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) - \hat{q}_{71}^{(4)}(\hat{q}_{17}^{(4)} + \hat{q}_{87}^{(6)}\hat{q}_{18}^{(3)}) + \hat{q}_{71}^{(4)}\hat{q}_{82}^{(5)}(\hat{q}_{17}^{(4)}\hat{q}_{28}^{(5)} - \hat{q}_{18}^{(3)}) + \hat{q}_{01}[-\hat{q}_{10}(1 - \hat{q}_{78}^{(3)}\hat{q}_{87}^{(6)}) - \hat{q}_{71}^{(4)}(\hat{q}_{70} + \hat{q}_{78}^{(3)}\hat{q}_{80}) - \hat{q}_{18}^{(3)}(\hat{q}_{70}\hat{q}_{87}^{(6)} - \hat{q}_{80}) - \hat{q}_{82}^{(5)}(-\hat{q}_{10}(\hat{q}_{27}^{(6)}\hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) + \hat{q}_{17}^{(4)}(\hat{q}_{20}(\hat{q}_{78}^{(3)} - \hat{q}_{70}\hat{q}_{28}^{(5)}) + \hat{q}_{18}^{(3)}(\hat{q}_{20} + \hat{q}_{70}\hat{q}_{27}^{(6)}))] \hat{q}_{02}[-\hat{q}_{20}(1 - \hat{q}_{78}^{(3)}\hat{q}_{87}^{(6)}) - \hat{q}_{27}^{(6)}(\hat{q}_{70} + \hat{q}_{78}^{(3)}\hat{q}_{80})]$$

$$\hat{q}_{80}) - \hat{q}_{28}^{(5)}(\hat{q}_{70}\hat{q}_{87}^{(6)} + \hat{q}_{80}) - \hat{q}_{71}^{(4)}(\hat{q}_{10}(\hat{q}_{27}^{(6)} + \hat{q}_{28}^{(5)}\hat{q}_{87}^{(6)}) - \hat{q}_{17}^{(4)}(\hat{q}_{20} - \hat{q}_{28}^{(5)}\hat{q}_{80}) - \hat{q}_{18}^{(3)}(\hat{q}_{20}\hat{q}_{87}^{(6)} + \hat{q}_{80}\hat{q}_{27}^{(6)}))]$$

(Omitting the arguments s for brevity)

The steady state availability

$$A_0 = \lim_{t \rightarrow \infty} [A_0(t)] = \lim_{s \rightarrow 0} [s \hat{A}_0(s)] = \lim_{s \rightarrow 0} \frac{s N_2(s)}{D_2(s)}$$

Using L' Hospital's rule, we get

$$A_0 = \lim_{s \rightarrow 0} \frac{N_2(s) + s N_2'(s)}{D_2(s)} = \frac{N_2(0)}{D_2(0)} \quad (13)$$

The expected up time of the system in (0,t] is

$$\lambda_u(t) = \int_0^t A_0(z) dz$$

$$\text{So that } \bar{\lambda}_u(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(s)}{s D_2(s)} \quad (14)$$

The expected down time of the system in (0,t] is

$$\lambda_d(t) = t - \lambda_u(t)$$

$$\text{So that } \bar{\lambda}_d(s) = \frac{1}{s^2} - \bar{\lambda}_u(s) \quad (15)$$

**The expected busy period of the server when there is Failure due to buckling failure or failure due to sun kink caused by high compressive stress in (0,t]**

$$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t) \\ R_1(t) = S_1(t) + q_{10}(t)[c]R_0(t) + q_{18}^{(3)}(t)[c] R_8(t) + q_{17}^{(4)}(t)[c]R_7(t) \\ R_2(t) = S_2(t) + q_{20}(t)[c]R_0(t) + q_{28}^{(5)}(t) R_8(t) + q_{27}^{(6)}(t)[c]R_7(t) \\ R_7(t) = S_7(t) + q_{70}(t)[c]R_0(t) + Q_{71}^{(4)}(t) R_1(t) + q_{78}^{(3)}(t)[c]R_8(t) \\ R_8(t) = S_8(t) + q_{80}(t)[c]R_0(t) + Q_{82}^{(5)}(t) R_2(t) + q_{87}^{(6)}(t)[c]R_7(t) \quad (16-20)$$

Taking Laplace Transform of eq. (16-20) and solving for  $\bar{R}_0(s)$

$$\bar{R}_0(s) = N_3(s) / D_2(s) \quad (21)$$

where

$$N_3(s) = \hat{q}_{01}[\hat{S}_1(1 - \hat{q}_{78}^{(3)}\hat{q}_{87}^{(6)}) + \hat{q}_{71}^{(4)}(\hat{S}_7 + \hat{q}_{78}^{(3)}\hat{S}_8) + \hat{q}_{18}^{(3)}(\hat{S}_7\hat{q}_{87}^{(6)} - \hat{S}_8) - \hat{q}_{01}\hat{q}_{82}^{(5)}(\hat{S}_1\hat{q}_{27}^{(6)}\hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) + \hat{q}_{17}^{(4)}(\hat{S}_2\hat{q}_{78}^{(3)} + \hat{S}_7\hat{q}_{28}^{(5)}) - \hat{q}_{18}^{(3)}(\hat{S}_2 + \hat{S}_7\hat{q}_{27}^{(6)})] + \hat{q}_{02}[\hat{S}_2(1 - \hat{q}_{78}^{(3)}\hat{q}_{87}^{(6)}) + \hat{q}_{27}^{(6)}(\hat{S}_7 + \hat{q}_{78}^{(3)}\hat{S}_8) + \hat{q}_{28}^{(5)}(\hat{S}_7\hat{q}_{87}^{(6)} + \hat{S}_8) - \hat{q}_{02}\hat{q}_{71}^{(4)}(\hat{S}_1 - \hat{q}_{27}^{(6)}\hat{q}_{28}^{(5)}\hat{q}_{87}^{(6)}\hat{q}_{17}^{(4)}(\hat{S}_2 + \hat{q}_{28}^{(5)}\hat{S}_8) - \hat{q}_{18}^{(3)}(-\hat{S}_2\hat{q}_{87}^{(6)} + \hat{S}_8\hat{q}_{27}^{(6)})]$$

and

$D_2(s)$  is already defined.

(Omitting the arguments  $s$  for brevity)

$$\text{In the long run, } R_0 = \frac{N_s(0)}{D_2(0)} \quad (22)$$

The expected period of the system under Failure due to buckling failure or failure due to sun kink caused by high compressive stress is

$$\lambda_{rv}(t) = \int_0^\infty R_0(z) dz \quad \text{So that } \tilde{\lambda}_{rv}(s) = \frac{\tilde{R}_0(s)}{s}$$

**The expected number of visits by the repairman for repairing the identical units in (0,t]**

$$H_0(t) = Q_{01}(t)[s][1+H_1(t)] + Q_{02}(t)[s][1+H_2(t)]$$

$$H_1(t) = Q_{10}(t)[s]H_0(t) + Q_{18}^{(3)}(t)[s]$$

$$H_8(t) + Q_{17}^{(4)}(t) [s]H_7(t) ,$$

$$H_2(t) = Q_{20}(t)[s]H_0(t) + Q_{28}^{(5)}(t) [s]$$

$$H_8(t) + Q_{27}^{(6)}(t) [c]H_7(t)$$

$$H_7(t) = Q_{70}(t)[s]H_0(t) + Q_{71}^{(4)}(t) [s]$$

$$H_1(t) + Q_{78}^{(3)}(t) [c]H_8(t)$$

$$H_8(t) = Q_{80}(t)[s]H_0(t) + Q_{82}^{(5)}(t) [s]$$

$$H_2(t) + Q_{87}^{(6)}(t) [c]H_7(t) \quad (23-27)$$

Taking Laplace Transform of eq. (23-27) and solving for  $H_0^*(s)$

$$H_0^*(s) = N_4(s) / D_3(s) \quad (28)$$

In the long run ,

$$H_0 = N_4(0) / D_3(0) \quad (29)$$

**Benefit- Function Analysis**

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under Failure due to buckling failure or failure due to sun kink caused by high compressive stress, expected number of visits by the repairman for unit failure.

The expected total Benefit-Function incurred in (0,t] is

$$C(t) = \text{Expected total revenue in } (0,t]$$

- expected busy period of the system under failure due to Failure due to buckling failure or failure due to sun kink caused by high compressive stress for repairing the units in (0,t ]

- expected number of visits by the repairman for repairing of identical the units in (0,t]

The expected total cost per unit time in steady state is

$$C = \lim_{t \rightarrow \infty} (C(t)/t)$$

$$= \lim_{s \rightarrow 0} (s^2 C(s))$$

$$= K_1 A_0 - K_2 R_0 - K_3 H_0$$

where

$K_1$  - revenue per unit up-time,

$K_2$  - cost per unit time for which the system is under repair of type- I or type- II

$K_3$  - cost per visit by the repairman for units repair.

**CONCLUSION**

After studying the system, we have analyzed graphically that when the failure rate due to buckling or failure rate due to sun kink caused by high compressive stress increases, the MTSF and steady state availability decreases and the Benefit-function decreased as the failure increases.

**REFERENCES**

- [1] Dhillon, B.S. and Natesen, J, Stochastic Anaysis of outdoor Power Systems in fluctuating environment, Microelectron. Reliab. ,1983; 23, 867-881.
- [2] Kan, Cheng, Reliability analysis of a system in a randomly changing environment, Acta Math. Appl. Sin. 1985, 2, pp.219-228.
- [3] Cao, Jinhua, Stochastic Behaviour of a Man Machine System operating under changing environment subject to a Markov Process with two states, Microelectron. Reliab. ,1989; 28, pp. 373-378.
- [4] Barlow, R.E. and Proschan, F., Mathematical theory of Reliability, 1965; John Wiley, New York.
- [5] Gnedanke, B.V., Belyayar, Yu.K. and Soloyer , A.D. , Mathematical Methods of Reliability Theory, 1969 ; Academic Press, New York.

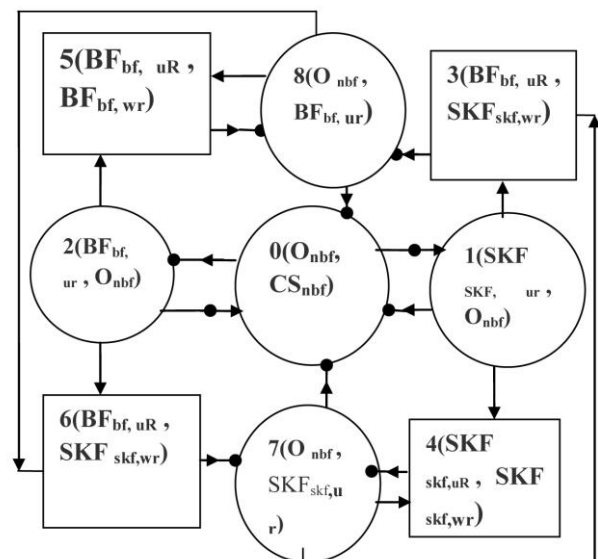


Fig. The State Space Diagram  
 ○ up state □ down state  
 • Regeneration point

