Cost-Benefit Analysis of Two Identical Warm Standby System subject to Aerodynamics Effects of an Engine Failure in a multi-engine aircraft

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Abstract : In this paper we have taken aerodynamics effects of an engine failure in a multi-engine aircraft, due to this asymmetric thrust and drag cause effects on the aircraft’s axes of rotation (AAR). When the main unit fails then warm standby system becomes operative. When an engine failure in a multi-engine aircraft occurs drag cannot occur simultaneously in both the units and after failure the unit undergoes Type-I or Type-II or Type-III repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

Keyword: Engine failure, asymmetric thrust, drag, MTSF, Availability, Busy period, Benefit-Function analysis

INTRODUCTION

Aerodynamics Effects of an Engine Failure

When an engine failure occurs in a multi-engine aircraft, asymmetric thrust and drag cause the following effects on the aircraft’s axes of rotation (AAR):

Pitch Down (Lateral Axis)
Loss of accelerated slipstream over the horizontal stabilizer causes it to produce less negative lift, causing the aircraft to pitch down. To compensate for the pitch down effect, additional back pressure is required.

Roll Toward the Failed Engine (Longitudinal Axis)
The wing produces less lift on the side of the failed engine due to the loss of accelerated slipstream. Reduced lift causes a roll toward the failed engine and requires additional aileron deflection into the operating engine.

Yaw Toward the Dead Engine (Vertical Axis)
Loss of thrust and increased drag from the wind milling propeller cause the aircraft to yaw toward the failed engine. This requires additional rudder pressure on the side of the operating engine. “Dead foot, dead engine.”

Engine Inoperative Climb Performance

Climb performance depends on the excess power needed to overcome drag.

When a multi-engine airplane loses an engine, the airplane loses 50% of its available power. This power loss results in a loss of approximately 80% of the aircraft’s excess power and climb performance. Drag is a major factor relative to the amount of excess power available. An increase in drag (such as the loss of one engine) must be offset by additional power. This additional power is now taken from the excess power, making it unavailable to aid the aircraft in climb. When an engine is lost, maximize thrust (full power) and minimize drag (flaps and gear up, prop feathered, etc.) in order to achieve optimum single-engine climb performance.

Approximate Drag Factors

- Full flaps : -300 to -400 fpm vertical speed
- Wind milling prop : -400 to -500 fpm vertical speed
- Gear extended : -150 fpm vertical speed
- Control deflections : variable
Airspeeds for Max Single-Engine Performance

VXSE
The airspeed for the steepest angle of climb on single-engine.

VYSE
The airspeed for the best rate of climb on single engine. (or for the slowest loss of altitude on drift down.) Blue line is the marking on the airspeed indicator corresponding to Vyse at max weight.

Sideslip Versus Zero Sideslip

During flight with one engine inoperative, proper pilot technique is required to maximize aircraft performance. An important technique is to establish a Zero sideslip condition.

Sideslip Condition (Undesirable)
When an engine failure occurs, thrust from the operating engine yaws the aircraft. To maintain aircraft heading with the wings level, rudder must be applied toward the operating engine. This rudder force results in the sideslip condition by moving the nose of the aircraft in a direction resulting in the misalignment of the fuselage and the relative wind. This condition usually allows the pilot to maintain aircraft heading; however, it produces a high drag condition that significantly reduces aircraft performance.

Zero Sideslip Condition (Best Performance)
The solution to maintain aircraft heading and reducing drag to improve performance is the Zero Sideslip Condition. When the aircraft is banked into the operating engine (usually 2 – 5 deg) the bank angle creates a horizontal component of lift. The horizontal lift component aids in counteracting the turning moment of the operating engine, minimizing the rudder deflection required to align the longitudinal axis of the aircraft to the relative wind. In addition to banking into the operating engine, the appropriate amount of rudder required is indicated by the inclinometer ball being split towards the operating engine side. The Zero sideslip condition aligns the fuselage with the relative wind to minimize drag and must be flown for optimum aircraft performance.

Single-Engine Service Ceiling

Single-engine service ceiling is the maximum density altitude at which the single-engine best rate of climb airspeed (Vyse) will produce a 50 FPM rate of climb with the critical engine inoperative.

Single-Engine Absolute Ceiling

Single-engine absolute ceiling is the maximum density altitude that an aircraft can attain or maintain with the critical engine inoperative. Vyse and Vxse are equal at
this altitude. The aircraft drifts down to this altitude when an engine fails.

**Climb Performance Depends on Four Factors**

- **Airspeed:** Too little or too much will decrease climb performance.

- **Drag:** Gear, Flaps, Cowl Flaps, Flight Control Deflection, Propeller, and Sideslip.

- **Power:** Amount available in excess of that needed for level flight.

  (engines may require leaning due to altitude for max engine performance)

- **Weight:** Passengers, baggage, and fuel load greatly affect climb performance.

**Critical Engine**

The critical engine is the engine that, when it fails, most adversely affects the performance and handling qualities of the airplane.

On most multi engine aircraft, both propellers rotate clockwise as viewed from the cockpit. By understanding the following factors when flying an aircraft that has both propellers rotating clockwise, it will be apparent that a left-engine failure makes the aircraft more difficult to fly than a right-engine failure. The clockwise rotation of the props contributes to the following factors that cause the left engine to be critical:

**P-Factor**

**Accelerated Slipstream**

**Spiraling Slipstream**

**Torque**

**P-Factor (Yaw)**

Both propellers turn clockwise as viewed from the cockpit. At low airspeed and high angles of attack, the descending blade produces more thrust than the ascending blade due to its increased angle of attack. Though both propellers produce the same overall thrust, the descending blade on the right engine has a longer arm from the CG than the descending blade on the left engine. The left engine produces the thrust closest to the center line. The yaw produced by the loss of the left engine will be greater than the yaw produced by the loss of the right engine, making the left engine critical.

**Accelerated Slipstream (Roll and Pitch)**

P-Factor causes more thrust to be produced on the right side of the propeller. This yields a center of lift that is closer to the aircraft’s longitudinal axis on the left engine and further from the longitudinal axis on the right engine and also results in less negative lift on the tail. Because of this, the roll produced by the loss of the left engine will be greater than the roll produced by the loss of the right engine, making the left engine critical.

**Spiraling Slipstream (Yaw)**

A spiraling slipstream from the left engine hits the vertical stabilizer from the left, helping to counteract the yaw produced by the loss of the right engine. However, with a left engine failure, slipstream from the right engine does not counteract the yaw toward the dead engine because it spirals away from the tail, making the left engine critical.

**Torque (Roll)**

For every action, there is an equal an opposite reaction. Since the propellers rotate clockwise, the aircraft will tend to roll counterclockwise. When the right engine is lost, the aircraft will roll to the right. The right rolling tendency, however, is reduced by the torque created by the left engine. When the left engine is lost, the aircraft will roll to the left, and the torque produced by the right engine.
will add to the left rolling tendency requiring more aileron input, which increases drag, making the left engine critical.

VMC

VMC is the minimum airspeed at which directional control can be maintained with the critical engine inoperative. VMC speed is marked on the airspeed indicator by a red radial line. Aircraft manufacturers determine VMC speed based on conditions set by the FAA under FAR 23.149.

1. Most Unfavorable Weight and Center of Gravity
2. Standard Day Conditions at Sea Level (Max Engine Power)
3. Maximum Power on the Operating Engine (Max Yaw)
4. Critical Engine Prop Wind milling (Max Drag)
5. Flaps Takeoff Position, Landing Gear Up (Least Stability)
6. Up to 5° of Bank into the Operating Engine

Any change to the previous conditions changes Vmc speed, possibly significantly. The following summarizes how Vmc may be affected by the above conditions:

1. Most Unfavorable Weight and Center of Gravity

The certification test allows up to 5° bank into the operating engine. In a given bank, the heavier the aircraft, the greater the horizontal component of lift that adds to the rudder force. As weight increases, the horizontal component of lift increases, which added to the rudder, decreases Vmc. As the center of gravity moves forward, the moment arm between the rudder and the CG is lengthened, increasing the leverage of the rudder. This increased leverage increases the rudder’s effectiveness and results in a lower Vmc speed.

2. Standard Day Sea level

Standard day conditions yield high air density that allows the engine to develop maximum power. An increase in altitude or temperature (a decrease in air density) will result in reduced engine performance and prop efficiency. This decreases the adverse yaw effect. Vmc speed decreases as altitude increases.

3. Maximum Power On The Operating Engine

When the operating engine develops maximum power, adverse yaw is increased toward the inoperative engine. The pilot must overcome this yaw to maintain directional control. Any condition that increases power on the operating engine will increase Vmc speed. Any condition that decreases power on the operating engine (such as power reduction by the pilot, an increase in altitude, temperature, low density, or aging engine) will decrease Vmc.

4. Critical Engine Prop Wind milling

When the propeller is in a low pitch position (unfeathered), it presents a large area of resistance to the relative wind. This resistance causes the engine to windmill. The wind milling creates a large amount of drag and results in a yawing moment into the dead engine. When the propeller is feathered, the blades are in a high pitch position, which aligns them with the relative wind, minimizing drag. A feathered prop will decrease drag and lower Vmc.

5. Flaps Takeoff Position, Landing Gear Up

When the gear is extended, the gear and gear doors have keel effect, reducing the yawing tendency and decreasing the Vmc speed. Extending the landing gear also lowers the CG and may move the CG. Extended flaps have a stabilizing effect that may reduce Vmc speed.

6. Up to 5° Bank into the Operating Engine

When the wings are level, only the rudder is used to stop the yaw produced by the operating engine (sideslip condition). Banking into the operating engine creates a horizontal component of lift which aids the rudder force. With this horizontal component of lift and full rudder deflection, Vmc increases with decreasing bank by a factor of approximately 3 knots per degree of bank angle.

In this paper we have taken aerodynamics effects of an engine failure in a multi-engine aircraft, due to this asymmetric thrust and drag cause effects on the aircraft’s axes of rotation (AAR). When the main operative unit fails then warm standby system becomes operative. When an engine failure in a multi-engine aircraft occurs drag cannot occur simultaneously in both the units. After failure the unit undergoes repair facility of Type-I or Type-II by ordinary repairman, Type III or Type IV by multispecialty repairman immediately when an engine failure in a multi-engine aircraft occurs due to this asymmetric thrust and drag cause effects on the aircraft’s axes of rotation (AAR). The repair is done on the basis of first fail first repaired.

Assumptions

1. \( \lambda_1, \lambda_2, \lambda_3 \) are constant failure rates when failure of warm standby, an engine failure in a multi-engine aircraft occurs due to this asymmetric thrust (with failure rate \( \lambda_2 \)) and drag (with failure rate \( \lambda_3 \))
cause effects on the AAR respectively. The CDF of repair time distribution of Type I, Type II and multispecialty repairmen Type-III, IV are \(G_1(t), \ G_2(t)\) and \(G_3(t), \ G_4(t)\).

2. When an engine failure in a multi-engine aircraft occurs due to this drag cause effects on the AAR is non-instantaneous and it cannot come simultaneously in both the units.

3. The repair starts immediately after an engine failure in a multi-engine aircraft occurs due to this asymmetric thrust and drag cause effects on the AAR and works on the principle of first fail first repaired basis. The repair facility does no damage to the units and after repair units are as good as new.

4. The switches are perfect and instantaneous.

5. All random variables are mutually independent.

6. When both the units fail, we give priority to operative unit for repair.

7. Repairs are perfect and failure of a unit is detected immediately and perfectly.

8. The system is down when both the units are non-operative.

**Symbols for states of the System**

**Superscripts**  
0, WS, EATF, EDF,
Operative, Warm Standby, an engine failure in a multi-engine aircraft occurs due to this asymmetric thrust and drag cause effects on the AAR respectively.

**Subscripts**  
neatf, eatf, edf, ur, wr, uR
No engine failure in a multi-engine aircraft occurs due to this asymmetric thrust and drag cause no effects on the AAR, an engine failure in a multi-engine aircraft occurs due to this asymmetric thrust cause effects on the AAR, an engine failure in a multi-engine aircraft occurs due to this cause effects on the AAR, under repair, waiting for repair, under repair continued from previous state respectively.

Up states \(-0, 1, 2, 3, 10\); Down states \(-4, 5, 6, 7, 8, 9, 11\), regeneration point \(-0, 1, 2, 3, 8, 9, 10\)

**States of the System**

0(\(O_{neatf}, WS_{neatf}\)) One unit is operative and the other unit is warm standby and there is no engine failure in a multi-engine aircraft occurs due to this asymmetric thrust and drag cause no effects on the AAR of both the units.

1(EATF\(_{eatf, urI}\), \(O_{neatf}\)) An engine failure in a multi-engine aircraft occurs due to this asymmetric thrust cause effects on the AAR is under repair immediately of Type- I and standby unit starts operating with no engine failure in a multi-engine aircraft occurs due to this asymmetric thrust and drag cause no effects on the AAR.

2(EDF\(_{edf, wrI}\), \(O_{neatf}\)) An engine failure in a multi-engine aircraft occurs due to this drag cause effects on the AAR and undergoes repair of Type II and the standby unit becomes operative with no engine failure in a multi-engine aircraft occurs due to this asymmetric thrust and drag cause no effects on the AAR.

3(EDF\(_{edf, urII}\), \(O_{neatf}\)) An engine failure in a multi-engine aircraft occurs due to this drag cause effects on the AAR and under Type-III multispecialty repairman and the other unit is operative with no engine failure in a multi-engine aircraft occurs due to this asymmetric thrust and drag cause no effects on the AAR.

4(EATF\(_{eatf, urII}\), EATF\(_{eatf, wrI}\)) The unit failed due to EATF resulting from an engine failure in a multi-engine aircraft occurs due to this asymmetric thrust cause effects on the AAR under repair of Type-I. I continued from state land the other unit failed due to EATF resulting from an engine failure in a multi-engine aircraft occurs due to this asymmetric thrust cause effects on the AAR is waiting for repair of Type-I.

5(EATF\(_{eatf, urI}\), EDF\(_{edf, wrI}\)) The unit failed due to EATF resulting from an engine failure in a multi-engine aircraft occurs due to this asymmetric thrust cause effects on the AAR is under repair of Type- I continued from state land an engine failure in a multi-engine aircraft occurs due to this drag cause effects on the AAR is waiting for repair of Type- II.

6(EDF\(_{edf, urII}\), EATF\(_{eatf, wrI}\)) An engine failure in a multi-engine aircraft occurs due to this drag cause effects on the AAR is under repair continues from state 2 of Type –II and the other unit failed due to EATF resulting from an engine failure in a multi-engine aircraft occurs due to this asymmetric thrust cause effects on the AAR is under repair of Type-I.

7(EDF\(_{edf, urII}\), EATF\(_{eatf, wrII}\)) An engine failure in a multi-engine aircraft occurs due to this drag cause effects on the AAR is continued to be under repair of Type II and the other unit failure due to EATF resulting from an engine failure in a multi-engine aircraft occurs due to this asymmetric thrust cause effects on the AAR is waiting for repair of Type-II.

8(EATF\(_{eatf, urIII}\), EDF\(_{edf, wrII}\)) An engine failure in a multi-engine aircraft occurs due to this asymmetric thrust cause effects on the AAR is under multispecialty repair of Type-III and an engine failure in a multi-engine aircraft occurs due to this drag cause effects on the AAR is waiting for repair of Type-II.

9(EATF\(_{eatf, urIII}\), EDF\(_{edf, wrII}\)) An engine failure in a multi-engine aircraft occurs due to this asymmetric thrust cause effects on the AAR is under multispecialty repair of Type-III and an engine failure in a multi-engine aircraft occurs due to this drag cause effects on the AAR is waiting for repair of Type-I.

10(\(O_{neatf}, EDF_{edf, wrIV}\)) The one unit is operative with no engine failure in a multi-engine aircraft occurs due to this asymmetric thrust cause effects on the AAR and
an engine failure in a multi-engine aircraft occurs due to this drag cause effects on the AAR and undergoes repair of type IV.

\[ 11 \text{(Oleaf, EDF, ext, utilv)} \] The one unit is operative with an engine failure in a multi-engine aircraft occurs due to this asymmetric thrust cause effects on the AAR and an engine failure in a multi-engine aircraft occurs due to this drag cause effects on the AAR and repair of type IV continues from state 10.

**Transition Probabilities**

Simple probabilistic considerations yield the following expressions:

\[ p_{01} = \lambda_1 / \lambda_1 + \lambda_2 + \lambda_3, \quad p_{02} = \lambda_2 / \lambda_1 + \lambda_2 + \lambda_3, \]
\[ p_{010} = \lambda_3 / \lambda_1 + \lambda_2 + \lambda_3, \quad p_{010} = p_{G_1} (\lambda_1) + q_{G_2} (\lambda_2), \]
\[ p_{14} = p - p_{G_1} (\lambda_1) + p_{15} \quad q = q_{G_1} (\lambda_2) = p_{25} (5), \]
\[ p_{23} = p_{G_2} (\lambda_1) + q_{G_2} (\lambda_2), \quad p_{26} = p - p_{G_2} (\lambda_2) = p_{29} (6), \]
\[ p_{27} = q - q_{G_2} (\lambda_2) = p_{27} (7), \quad p_{30} = q_{p_2} = q_{p_1} = 1. \]
\[ p_{010} = p_{G_4} (\lambda_1) + q_{G_4} (\lambda_2), \quad p_{011} = p - p_{G_4} (\lambda_1) = p_{011} (11), \]
\[ p_{012} = q - q_{G_4} (\lambda_2) = p_{012} (11) (1) \]

We can easily verify that

\[ p_{01} + p_{02} + p_{03} = 1, \]
\[ p_{10} + p_{12} = p_{11} (4) + p_{15} = p_{12} (5) = 1, \]
\[ p_{23} + p_{26} = p_{25} (6) + p_{27} = p_{28} (7), \]
\[ p_{30} = p_{32}, p_{30} = 1. \]
\[ p_{010} + p_{011} + p_{012} = 1 (2) \]

And mean sojourn time is

\[ \mu_i = E(T) = \int_0^\infty P[T > t]dt \]

**3. Mean Time To System Failure**

\[ \phi_0(t) = Q_{01}(t) [\dot{\lambda}_1 + \dot{\lambda}_2 + \dot{\lambda}_3] + Q_{010}(t) [\dot{\lambda}_1 + \dot{\lambda}_2 + \dot{\lambda}_3] + Q_{0101}(t) \]
\[ \phi_1(t) = Q_{01}(t) [\lambda_1 + \lambda_2 + \lambda_3] + Q_{14}(t) + Q_{15}(t) \]
\[ \phi_2(t) = Q_{23}(t) [\lambda_1 + \lambda_2 + \lambda_3] + Q_{26}(t) + Q_{27}(t) \]
\[ \phi_3(t) = Q_{00}(t) [\lambda_1 + \lambda_2 + \lambda_3] + Q_{10}(t) [\lambda_1 + \lambda_2 + \lambda_3] \]
\[ \phi_10(t) = Q_{010}(t) [\dot{\lambda}_1 + \dot{\lambda}_2 + \dot{\lambda}_3] + Q_{012}(t) [\dot{\lambda}_1 + \dot{\lambda}_2 + \dot{\lambda}_3] \]

We can regard the failed state as absorbing.

Taking Laplace-Stiljes transform of eq. (3-6) and solving for

\[ \phi_0(s) = N_1(s) / D_1(s) \]

where

\[ N_1(s) = [Q_{01} + Q_{010} + Q_{0101}] [Q_{14} + Q_{15} + Q_{16}] + [Q_{02} + Q_{010} + Q_{0101}] [Q_{26} + Q_{27}] \]

\[ D_1(s) = 1 - (Q_{01} + Q_{010} + Q_{0101}) \]

Making use of relations (1) & (2) it can be shown that \( \phi_0 (0) = 1 \), which implies that \( \phi_0 (t) \) is a proper distribution.

\[ \text{MTSF} = E[T] = \frac{d}{ds} g_0 (s) = 0 \]

\[ \text{MTSF} = E[T] = \frac{d}{ds} g_0 (s) = 0 \]

\[ = (D_1(0) - N_1(0)) / D_1(0) \]

\[ = (\mu_1 + \mu_2 (p_{01} + p_{010} + p_{011}) + (p_{01} + p_{010} + p_{012})(\mu_2 + \mu_2) + p_{010} + (p_{01} + p_{010} + p_{012}) (1 - (p_{01} + p_{010} + p_{012}) p_{01} + (p_{01} + p_{010} + p_{012}) p_{010}) \]

4. Availability analysis

Let \( M(t) \) be the probability of the system having started from state 1 is up at time \( t \) without making any other regenerative state. By probabilistic arguments, we have

\[ M_0(t) = e^{-\lambda_1} t e^{-\lambda_2} t e^{-\lambda_3} t \]

\[ M(t) = e^{-\lambda_1} t e^{-\lambda_2} t \]

\[ M_2(t) = e^{-\lambda_1} t e^{-\lambda_2} t \]

\[ M_10(t) = e^{-\lambda_1} t e^{-\lambda_2} t \]

The point wise availability \( A_i(t) \) have the following recursive relations

\[ A_0(t) = M_0(t) + q_{01}(t) c [A_1(t)] + q_{02}(t) [A_1(t) + q_{01}(t)] c [A_1(t)] \]

\[ A_1(t) = M_1(t) + q_{10}(t) c [A_1(t)] \]

\[ A_2(t) = M_2(t) + q_{20}(t) c [A_1(t)] \]

\[ A_3(t) = M_3(t) + q_{30}(t) c [A_1(t)] \]

\[ A_4(t) = M_4(t) + q_{40}(t) c [A_1(t)] \]

\[ A_5(t) = M_5(t) + q_{50}(t) c [A_1(t)] \]

\[ A_6(t) = M_6(t) + q_{60}(t) c [A_1(t)] \]

\[ A_7(t) = M_7(t) + q_{70}(t) c [A_1(t)] \]

\[ A_8(t) = M_8(t) + q_{80}(t) c [A_1(t)] \]

\[ A_9(t) = M_9(t) + q_{90}(t) c [A_1(t)] \]

\[ A_{10}(t) = M_{10}(t) + q_{10}(t) c [A_1(t)] + q_{10}(t) c [A_1(t)] + q_{10}(t) c [A_1(t)] + q_{10}(t) c [A_1(t)] \]

Taking Laplace Transform of eq. (8-15) and solving for

\[ \hat{A}_0(s) \]

\[ \hat{A}_0(s) = N_2(s) / D_2(s) \]

where
N(s) = \{ \hat{\varphi}_{0,10} M_{10} + \hat{\varphi}_{0,1} \} \{ \{ 1 - \hat{\varphi}_{11}^{(4)} \} \{ 1 - \hat{\varphi}_{28}^{(7)} \hat{\varphi}_{82} \} \hat{\varphi}_{12}^{(5)} \hat{\varphi}_{29}^{(6)} \hat{\varphi}_{91} \} + \{ \hat{\varphi}_{0,11}^{(4)} \} \hat{\varphi}_{10,11}^{(11)} + \{ \hat{\varphi}_{0,10} + \hat{\varphi}_{0,10} \} \hat{\varphi}_{10,11}^{(11)} + \{ \hat{\varphi}_{0,10} \} \hat{\varphi}_{12}^{(5)} \hat{\varphi}_{23}^{(6)} \hat{\varphi}_{91} \hat{\varphi}_{10}^{(11)} \hat{\varphi}_{30} \{ 1 - \hat{\varphi}_{11}^{(4)} \} + \hat{\varphi}_{29}^{(6)} \hat{\varphi}_{91} \hat{\varphi}_{10}^{(11)} \hat{\varphi}_{30} \} + \hat{\varphi}_{10,11}^{(11)} \hat{\varphi}_{12}^{(5)} \hat{\varphi}_{23}^{(6)} \hat{\varphi}_{91} \hat{\varphi}_{10}^{(11)} \hat{\varphi}_{30} \{ 1 - \hat{\varphi}_{11}^{(4)} \} + \hat{\varphi}_{29}^{(6)} \hat{\varphi}_{91} \hat{\varphi}_{10}^{(11)} \hat{\varphi}_{30} \}

Using L’Hospital’s rule, we get

A_0 = \lim_{s \to \infty} \{ s \hat{A}_0(s) \} = \lim_{s \to 0} \frac{sN_0(s)}{D_0(s)}

The expected up time of the system in (0,t) is \( \lambda_u(t) = \int_0^t A_0(z) \, dz \)

So that \( \lambda_u(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_0(s)}{sD_0(s)} \) 

The expected down time of the system in (0,t) is \( \lambda_d(t) = t - \lambda_u(t) \)

So that \( \lambda_d(s) = \frac{1}{s} - \lambda_u(s) \)

Similarly, we can find out

1. The expected busy period of the server when there is an engine failure in a multi-engine aircraft occurs due to this asymmetric thrust cause effects on the AAR, and an engine failure in a multi-engine aircraft occurs due to this drag cause effects on the AAR in (0,t)-R_0.

2. The expected number of visits by the repairman Type-I or Type-II for repairing the identical units in (0,t)-H_0.

3. The expected number of visits by the multispecialty repairman Type-III or Type-IV for repairing the identical units in (0,t)-W_0, Y_0.

**Benefit-Function**

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under an engine failure in a multi-engine aircraft occurs due to this asymmetric thrust cause effects on the AAR, and an engine failure in a multi-engine aircraft occurs due to drag cause effects on the AAR, expected number of visits by the repairman for unit failure. The expected total Benefit-Function incurred in (0,t) is

\[ C = \lim_{t \to \infty} \frac{(C(t)/t)}{C(t)} = \lim_{t \to 0} \frac{s^2 C(s)}{s^2 C(s)} = K_1A_0 - K_2R_0 - K_3H_0 - K_4W_0 - K_5Y_0 \]

where

K_1 - revenue per unit up-time,
K_2 - cost per unit time for which the system is busy under repairing,
K_3 - cost per visit by the repairman type- I or type- II for units repair,
K_4 - cost per visit by the multispecialty repairman Type- III for units repair,
K_5 - cost per visit by the multispecialty repairman Type- IV for units repair

**CONCLUSION**

After studying the system, we have analyzed graphically that when the failure rate of an engine failure in a multi-engine aircraft occurs due to this asymmetric thrust cause effects on the AAR and an engine failure in a multi-engine aircraft occurs due to this drag cause effects on the AAR increases, the MTSF, steady state availability decreases and the Profit-function decreased as the failure increases.

**REFERENCES**


Fig. The State Transition Diagram

- Up-State
- Down-State
- regeneration point

Diagram shows transitions between states with labels such as 'EATF_eff, uRII, EDF_eff, wrI,' and 'O_pestf - WS_pestf.'