ANALYSIS OF ARRAY STEERING ALGORITHMS FOR DIELECTRIC LENS - ARRAY ANTENNAS

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Abstract: A cost effective smart antenna can be implemented by exciting the shaped dielectric lens by array antenna, by introducing dielectric lens the directivity is enhanced due to collimation of the rays in the specified direction, reduces interference and seals the array from environmental effects. However the major drawback with the use of a dielectric lens is compensation of the refractive effects of the lens in the receiving mode to obtain the angles of arrival to locate the desired signal. This paper dwells on the array signal processing algorithms to estimate the Direction Of Arrival (DOA) for array antennas using many DOA estimation methods, namely Bartlett, MVDR, MUSIC and Linear Prediction, with their pseudo spectrum equations were analyzed in Matlab considering resolution and varying SNR. It was observed that MVDR and MUSIC gives the best results among Classical and Sub-Space methods respectively. Analysis was continued for other conditions like resolution, different SNR values and presence of multiple sources with MUSIC and MVDR methods for linear array antenna. Ray tracing method was introduced to analyze DOA in presence of dielectric lens. This lead to determination of virtual DOA due to refraction of rays in dielectric lens the shape of dielectric chosen is ellipsoidal and 6-element linear array is chosen for analysis in this paper.

Index Terms — Adaptive Beam Forming, Direction of Arrival, Minimum Variance Distortionless Response, Multiple Signal Classification, Resolution, Signal-to-Noise ratio, Total Internal Reflection.

I. INTRODUCTION

Early smart antennas were designed for governmental use in military applications using directed beams to hide transmissions which was implemented by very large antenna structures and time-intensive processing. Use of smart antennas to reduce network interference caused by ever increasing simultaneous users helped in increasing total number of users a wireless network could handle in a given block of spectrum. But it is extremely difficult to perform complex calculations in the stringent time space available in the personal wireless communications.

Adaptive array smart antenna involves the array signal processing to manipulate the signals induced on various antenna elements in such way that the main beam is directed towards the desired user and nulls are formed towards the interferers. This is achieved by two in-built properties of the smart antenna namely, adaptive beamforming and DOA. The DOA algorithms namely Bartlett, MVDR, MUSIC and Linear Prediction are described in Constantine A. Balanis and Panayiotis I. Ioannides [2], Lal Chand Godara [3], Jeffrey Foutz, Andreas Spanias and Mahesh K. Banavar [4], Harry L. Van Trees [5], Monson H. Hayes [6] and Frank Gross [7], Godara.L.C [8],these authors provides a comprehensive study of the use of an antenna array to enhance the efficiency of mobile communication systems and provides details on the feasibility of antenna arrays for mobile communication applications.

A smart antenna also has to meet the contrasting requirement of compact form factor and high gain, to achieve this a shaped dielectric lens can be used along with the array antenna elements. The use of dielectric lens antennas to enhance the performance of wireless broadband communication systems by producing highly shaped beams is described by Carlos A. Fernandez et al. [10, 11]. The subject of lens antenna design has been treated in the past by many authors [17, 18, 19] in...
considerable detail. They point out the methods of design, types of structure and general problems involved in the use of lenses.

In this paper a shaped dielectric lens antenna is excited by linear array antenna by doing so high directivity, protection to array, cost of the system is greatly reduced. The major concern of this paper is to predict desired signal received in any direction. The complete analysis of this system to determine the DOA is carried out in two major steps firstly, only array antenna is analyzed to determine Direction Of Arrival (DOA) using different estimation methods, namely Bartlett, MVDR, MUSIC and Linear Prediction. Their pseudo spectrum equations were analyzed and simulated in Matlab considering resolution and varying SNR. It was observed that MVDR and MUSIC gives the best results among Classical and Sub-Space methods respectively. Analysis was continued for other conditions like resolution, different SNR values and presence of multiple sources with MUSIC and MVDR methods for linear antenna systems. Secondly, the array is now exciting lens antenna, analysis of this problem is carried out using ray tracing technique in which the shape of lens is first designed then the DOA of the signal is determined by fundamentals of ray theory.

The paper presents the following studies:
- Comparison of various DOA algorithms using a linear array.
- To simulate the DOA algorithms viz. MUSIC and MVDR for array antenna with ellipsoidal lens combination.
- Compensate for the change in the DOA due to the refractive effect of the lens using Ray Tracing technique.

The remainder of the paper is organized as follows: Section II describes the received data model of the array, section III highlights the DOA algorithms used viz. MUSIC and MVDR, section IV focuses on ray tracing through the lens, and in section V conclusions are drawn out from previous simulated results of the corrected DOA based on the estimated virtual DOA after refraction through the lens for the case of a linear array is presented using the new technique.

II. RECEIVED DATA MODEL

Figure 1 depicts a uniform planar array considered on the yz plane of the coordinate system. The array is centered on the origin. There are M elements along any row in the y direction and N elements along any column in the z direction. Let \( d_y \) and \( d_z \) be the inter element spacing along y axis and z axis respectively. In figure 1, K signals arrive from K uncorrelated sources in K directions. Each received signal \( x_k(t) \) includes additive white Gaussian noise. Under this model, the received signals can be expressed as a superposition of signals from all the sources and linearly added noise represented by [1, 3].

![Uniform planar array](image)

\[ x(t) = \sum_{k=1}^{K} a(\theta_k, \Phi_k) s_k(t) + n(t) \]  \hspace{1cm} (1)

where \( s_k(t) \) is the incoming plane wave from the \( k^{th} \) source at time \( t \) and arriving from the direction \( (\theta_k, \Phi_k) \) with \( \theta_k \) is the elevation angle in the range \( 0^\circ \leq \theta_k \leq 180^\circ \) and \( \Phi_k \) is the azimuth angle, in the range \( -90^\circ \leq \Phi_k \leq 90^\circ \). \( a(\theta_k, \Phi_k) \) is the array steering vector for the \( (\theta_k, \Phi_k) \) direction of arrival, and \( n(t) \) represents additive white Gaussian noise. A single observation \( x(t) \) from the array is often referred to as a snapshot. Using a matrix notation (1) can also be written as [1,3]

\[ x(t) = A(\Theta) s(t) + n(t) \]  \hspace{1cm} (2)

where \( A(\Theta) \) is the \((M*N) \times K\) matrix of array steering vectors. It is assumed that the arriving signals are uncorrelated and the number of arriving signals \( K < (M*N) \). The received signal covariance matrix of size \((M*N) \times (M*N)\) is given by [1, 2],

\[ R_{xx} = A(\Theta) R_{ss} A^H(\Theta) + \sigma_n^2 I \]  \hspace{1cm} (3)

where \( \sigma_n^2 \) is the noise variance and \( I \) is an identity matrix of size \((M*N) \times (M*N)\), \( R_{ss} \) is the source signal covariance matrix.

III. DIRECTION OF ARRIVAL ESTIMATION ALGORITHMS

The problem of localization of sources radiating energy by observing their signal received at the array antenna elements is of considerable importance occurring in many fields including radar, sonar, mobile communications, radio astronomy and seismology. In this chapter an estimation of the Direction Of Arrival (DOA) of narrowband sources of the same central frequency located in the far field of an array of antenna elements is considered and various DOA estimation methods are described. Data from an array of sensors are collected and the objective is to locate point sources assumed to be radiating energy that is detectable by the array elements.

Although most of the algorithms have been presented in the context of estimating a single angle per emitter (e.g. elevation only), generalizations to the elevation/azimuth case are relatively straightforward. Additional parameters such as frequency, polarization angle and range can also be incorporated provided that the response of the array is known as a function of these parameters. In

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general the DOA estimation algorithms can be categorized into two groups; the conventional algorithms and the subspace algorithms [2, 3, 4].

A. Conventional DOA estimation methods

Two methods are usually classified as conventional methods:

1) The Delay-and-sum method or Bartlett method.
2) Capon’s Minimum Variance method or Minimum Variance Distortionless Response method

1) Delay-and-sum method or Bartlett method

One of the earliest methods of spectral analysis is the Bartlett method. The idea is to scan across the angular region of interest (usually in discrete steps), and whichever direction produces the largest output power is the estimate of the desired signal’s direction. More specifically, as the look direction \( \theta \) is varied incrementally across the space of access the array response vector \( a(\theta) \) is calculated and the output power of the beamformer is measured by [2-7].

\[
P_{\text{DAS}} = \frac{a^H(\theta)R_{xx}a(\theta)}{a^H(\theta)a(\theta)}
\]

This quantity is also referred to as the spatial spectrum and the estimate of the true DOA is the angle \( \hat{\theta} \) that corresponds to the peak value of the output power spectrum.

2) Minimum Variance Distortionless Response method

The MVDR method is similar to the delay-and-sum technique described, in that it measures the power of the received signal in all possible directions. In this method, the output power is minimized with the constraint that the gain in the desired direction remains unity. Solving this constraint optimization problem for the weight vector we obtain [2]

\[
W = \frac{R^{-1}_{xx}a(\theta)}{a^H(\theta)R^{-1}_{xx}a(\theta)}
\]

which gives the MVDR spectrum [2-7]

\[
P_{\text{MVDR}} = W^H R_{xx} W = \frac{1}{a^H(\theta)R^{-1}_{xx}a(\theta)}
\]

Again, the estimate of the true direction of arrival is the angle \( \hat{\theta} \) that corresponds to the peak value in this spectrum. The MVDR only requires an additional matrix inversion compared to the delay-and-sum method and exhibits greater resolution in most cases.

B. Subspace approach to DOA estimation

The other main group of DOA estimation algorithms is called the subspace methods. Geometrically, the received signal vectors form the received signal vector space whose vector dimension is equal to the number of array elements \( N \). The received signal space can be separated into two parts: the signal subspace and the noise subspace. The signal subspace is the subspace spanned by the columns of \( A(\Theta) \) [2, 22] and the subspace orthogonal to the signal subspace is known as the noise subspace. The subspace algorithms exploit this orthogonality to estimate the signals’ DOAs.

1) The MUSIC algorithm

Within the class of the so-called signal subspace algorithms, MUSIC has been the most widely examined. MUSIC stands for Multiple Signal Classification. The MUSIC algorithm was developed by Schmidt [22] by noting that the desired signal array response is orthogonal to the noise subspace. The signal and noise subspaces are first identified using Eigen decomposition of the received signal covariance matrix. Following, the MUSIC spatial spectrum is computed from which the DOAs are estimated. Inside the algorithm the general array manifold is defined to be the set

\[
A = \{a(\theta_i) : \theta_i \in \Theta\}
\]

The subspace identified are typically achieved by Eigen decomposition of the autocovariance matrix of the received data \( R_{xx} \). Using the model and assuming spatial whiteness, i.e., \( \mathbb{E}[n(t)n^H(t)] = \sigma^2 I \), the Eigen decomposition of \( R_{xx} \) will give the Eigen values \( \lambda_n \) such that \( \lambda_1 > \lambda_2 > \ldots > \lambda_K > \lambda_{K+1} = \lambda_{K+2} = \ldots = \lambda_N = \sigma^2_n \). Furthermore, it is easily shown that \( R_{xx} \) can be written in the following form [2, 4]

\[
R_{xx} = \sum_{i=1}^{N} \lambda_i e_i e_i^H = E \Lambda E^H = E_s \Lambda_s E_s^H + E_n \Lambda_n E_n^H
\]

where \( E = [e_1, e_2, \ldots, e_K] \), \( E_s = [e_1, e_2, \ldots, e_K] \), \( E_n = [e_{K+1}, e_{K+2}, \ldots, e_N] \), \( \Lambda = \text{diag} \{ \lambda_1, \lambda_2, \ldots, \lambda_N \} \), \( \Lambda_s = \text{diag} \{ \lambda_1, \lambda_2, \ldots, \lambda_K \} \), \( \Lambda_n = \text{diag} \{ \lambda_{K+1}, \lambda_{K+2}, \ldots, \lambda_N \} \) and \( \Lambda_s = \Lambda_s - \sigma^2_n I \). The
Eigen vectors $E = [E_S, E_N]$ can be assumed to form an orthonormal basis (i.e., $EE^H = E^HE = I$). The span of the $K$ vectors $E_S$ defines the signal subspace and the orthogonal complement spanned by $E_N$ defines the noise subspace. For a detailed analysis of the Eigen structure properties of the signal autocovariance matrices $R_{xx}$ and $R_{ss}$ the reader is referred to [22]. Once the subspaces are determined the DOAs of the desired signals can be estimated by calculating the MUSIC spatial spectrum over the region of interest [2-7]

$$P_{MUSIC}(\theta) = \frac{a^H(\theta)a(\theta)}{a^H(\theta)E_n^HE_n^Hn(\theta)}$$

(10)

Note that the $a(\theta)$ is the array response vectors calculated for all angles $\theta$ within the range of interest. Because the desired array response vectors $A(\Theta)$ are orthogonal to the noise subspace, the peaks in the MUSIC spatial spectrum represent the DOA estimates for the desired signals.

C. Flowchart to obtain varying SNR and resolution plots

The DOA is estimated for 6-element linear array antenna with MVDR and MUSIC algorithms. Pseudo-spectrum equation is used to obtain the plots. The flowchart of DOA for linear array antenna is given below.

Flowchart of DOA for linear array antenna

Assume 'D' sources which are coherent and uncorrelated, obtain diagonal correlation matrix.

Signals are steered by certain predefined angle for simulation purpose.

Consider effect of Noise by adding $\text{input} \times \text{Noise correlation}$ Define angular range for simulation.

Is iteration for defined range of angle completed?

YES

ENO

NO

Calculate power at each and every angle.

Figure 2: Flowchart of DOA estimation for linear array antenna

Assume ‘D’ sources which are coherent and uncorrelated, obtain its diagonal correlation matrix. The signals are steered at specific predefined angle. Simulation with the help of pseudo spectrum equation of each method gives peak value at angle same as that of steered signal. This angle is the estimated DOA of the system.

D. Comparison of Algorithms

It is necessary to understand and compare the algorithms available for further procedure. Algorithms are analyzed on the basis of resolution and varying SNR values.

Figure 3 (a) & (b): Comparison of various algorithms for two different resolution and SNR value cases

It was observed that Bartlett method had poor resolution compared with other algorithms. It was also seen that MUSIC gave the best results during the comparison. So, it was decided to proceed the procedure with two algorithms, one from classical method namely MVDR and other from subspace algorithm namely MUSIC.

E. Pseudo-spectrum plots for MVDR and MUSIC

1) Minimum Variance Distortionless Response method results

To estimate the true DOA, the MVDR method was executed for linear 6-array antenna.

Figure 4: Varying SNR plot for far angles(-30 and 30 degrees)

MVDR algorithm is applied for 6 elements linear antenna for different values of SNR for the system when signals are steered at -30 and 30 degrees. The values of SNR are
10dB, 20dB, 30dB and 40dB. It is been observed that the better peaks are obtained for higher SNR values.

Figure 5: Varying SNR plot for near angles(-3 and 3 degrees)

MVDR algorithm is applied for 6 elements linear antenna for different values of SNR for the system when signals are steered at -3 and 3 degrees. The values of SNR are 10dB, 20dB, 30dB and 40dB. It is been observed that the resolutions of angles gets better with higher SNR.

2) MUSIC method results

To estimate the true DOA, the MUSIC method was executed for linear 6-array antenna.

Figure 6: Varying SNR plot for far angles(-20 and 20 degrees)

MUSIC algorithm is applied for 6 elements linear antenna for different values of SNR for the system. The values of SNR are 5dB, 15Db and 25dB. It is been observed that the better peaks are obtained for higher SNR. The values of SNR required for better peaks are less than those of MVDR.

Figure 7: Varying SNR plot for near angles(-1 and 1 degree)

MUSIC algorithm is applied for 6 elements linear antenna for different values of SNR for the system. The values of SNR are 5dB, 15Db and 25dB. It is been observed that the resolutions of angles gets better for higher SNR.

IV. RAY TRACING THROUGH A DIELECTRIC LENS

Figure 8 shows the cross section of a rotationally symmetric dielectric lens with its general contours S1 and S2 represented by (x1, y1) and (x2, y2) respectively [4]. The lens has dielectric constant εr. The distance between the origin and the first surface of the lens is f, called the focal length of the lens. Let T be the central thickness of the lens and D be the diameter of the lens aperture.

Figure 8: Geometry for dielectric lens design

The most important condition to be imposed in the derivation of the lens profiles is the path length constraint. Mathematically the path length condition is given by [4],

\[
(x_1^2 + y_1^2)^{1/2} + n[(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2} - x_2 = (n - 1)T
\]

(11)

where the central ray has been used as a path length reference and n=√ εr is the refractive index of the lens, which is greater than unity for real dielectrics. By taking the differential of y1 with respect to x1 in (11) the slope of the lens at (x1, y1) is obtained as [4],

\[
\frac{dy_1}{dx_1} = \frac{nL_1(x_2 - x_1) - L_2x_1}{L_2y_1 - nL_1(y_2 - y_1)}
\]

(12)

where

\[
L_1 = (x_1^2 + y_1^2)^{1/2}, \quad L_2 = [(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2}
\]

(13)

Similarly, the slope of the contour at (x2, y2) is [4],

\[
\frac{dy_2}{dx_2} = \frac{L_2 - n(x_2 - x_1)}{n(y_2 - y_1)}
\]

(14)

In this paper we first examine a ellipsoidal lens lens with a flat surface on S2 as show in figure 9. This lens transforms a spherical wavefront into a plane wave, or conversely focuses a beam from infinity onto focal point of the lens. The conditions imposed to derive S1 are x2 = f + T, the slope on S2 being infinity, and the equal path length constraint. It can be readily shown that S1 is a hyperbolic surface defined by [4],
\[ y_2 = \left[ \frac{x + (n-1)(F+T)}{n} \right]^2 - x^2 \right]^{1/2} \quad (15) \]

**Figure 9: Ellipsoidal dielectric lens**

**F. Ray Tracing Procedure**

**Backward Ray Tracing for Ellipsoidal Lens**

Consider an incident ray with slope \( m \), whose equation is given by \( y_1 = mx_1 + c_1 \).

Find the intersection point of incident ray at surface1 and apply Snell’s law to obtain slope refracted ray.

Obtain the intersection point of refracted ray with the surface2.

Applying Snell’s law, obtain the slope of final ray coming out of the lens and plot it till the point under consideration.

**G. Ray Tracing Simulation Outputs**

**Figure 10: Ray tracing for 6-elements linear array antenna in presence of double ellipsoidal lens showing internal reflection**

Internal reflection was observed in the ray tracing for 6-elements linear array antenna in presence of double ellipsoidal lens as shown in figure 10. Multiple internal refractions can also be plotted.

**Figure 11: Ray tracing for 6-element linear array antenna in presence of double ellipsoidal lens with three signal emitting source**

Ray tracing for 6-element linear array antenna in presence of ellipsoidal lens with three sources emitting parallel signals with DOA 150°, 155° and -150° is shown in figure 11. It is observed that at-150° actual DOA, no signals were able to reach the array antenna while other signals with 150° and 155° actual DOA are able to reach the array antenna in presence of the dielectric lens.

**V. CONCLUSION**

The objective of the paper is to simulate the different DOA algorithms viz. BARTLET, MVDR, Linear Prediction and MUSIC performance based on varying SNR and angular resolution without the presence of dielectric lens. Due to better performance, MUSIC and MVDR were used for linear array with ellipsoidal dielectric lens combination, and compensate for the change in the DOA due to the refractive effect of the lens using ray tracing technique.

Recognizing that the smartness of an array antenna depends on the underlying spatial signal processing algorithms viz. DOA and ABF, a study of the DOA algorithms has been done initially without lens. The simulation results show that the performance of MVDR and MUSIC improves with increasing SNR of signals compared to DELAY-AND-SUM and Linear Prediction methods. MVDR and MUSIC showed lower rms error with increasing SNR, as compared to DELAY-AND-SUM and Linear Prediction methods.

The results of DOA estimation in two directions viz. elevation angle and azimuth angle using MUSIC and MVDR algorithms in the presence of shaped dielectric lens for 6-element linear array has been shown. Lens has the advantages like collimation of signals, inherent high bandwidth, and castiness in fabrication and cost effectiveness, which led the paper to adapt for the array antenna. The lens also causes refraction of the rays that result in the measured angle of arrival to be different (virtual DOA) from the actual one (correct DOA).

**VI. FUTURE SCOPE**

A smart antenna is a technology that promises to reshape the future of our communication industry in the wireless domain. It has emerged as an important solution to the problems of transmission fidelity and spectral efficiency, faced by most in the communication industry. Although there have been tremendous developments in this field, the use of a lens based smart antenna definitely gives scope for future developments.
There are a few drawbacks that we encounter while using dielectric lens antennas such as blind scan effect and edge diffraction. So, there is a need for modification of array steering algorithms when dielectric lens is placed in front of the array antenna.

The array elements progressively become blind due to more elements suffering from total internal reflection at steep scan angles, causing blind scan effect. The number of array elements that receive the signal is dependent on the angle of arrival. Larger wide scan angles could be obtained by increasing the number of antenna elements or by using a larger lens diameter.

Ray tracing technique can give us only virtual DOA estimation. We need other techniques to obtain the actual DOA. Space sampling is one such technique.

Total Internal Reflection may occur in ray tracing method which is not desirable while analyzing. Space sampling can again give better results with TIR case.

REFERENCES


