Benefit-Function of Two- Identical Cold Standby System subject to Failure due to engineering disasters or Failure due to miscommunication

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Abstract -Failure due to miscommunication

Engineering is a precise discipline and in order to be precise, communication among project developers is pertinent for a successful product. There are several forms of miscommunication that can lead to a flawed design in a system. There are various fields of engineering that have to intercommunicate when working toward a mutual goal. These fields include civil, electrical, mechanical, industrial, chemical, biological, and environmental engineering. When creating a modern automobile, electrical engineers, mechanical engineers, and environmental engineers are required to work together to produce a fuel-efficient, durable product for consumers. If engineers do not adequately communicate among one another, a potential design could have flaws and be unsafe for consumer purchase. Engineering disasters can be a result of such miscommunication. In this paper we have taken Failure due to engineering disasters or Failure due to miscommunication. When the main unit fails due to failure due to engineering disasters then cold standby system becomes operative. Failure due to engineering disasters cannot occur simultaneously in both the units and after failure the unit undergoes very costly repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

Keywords: Cold Standby, Failure due to engineering disasters or Failure due to miscommunication, first come first serve, MTSF, Availability, Busy period, Benefit - Function.

INTRODUCTION

Shortcuts in engineering design can lead to **engineering disasters**. Engineering is the science and technology used to meet the needs and demands of society. These demands include buildings, aircraft, vessels, and computer software. In order to meet society's demands, the creation of newer technology and infrastructure must be met efficiently and cost-effectively. To accomplish this, managers and engineers have to have a mutual approach to the specified demand at hand. This can lead to shortcuts in engineering design to reduce costs of construction and fabrication. Occasionally, these shortcuts can lead to unexpected design failures.

Importance of safety

In the field of engineering, the importance of safety is emphasized. Learning from past engineering failures and infamous disasters such as the Challenger explosion brings the sense of reality to what can happen when appropriate safety precautions are not taken. Safety tests such as tensile testing, finite element analysis (FEA), and failure theories help provide information to design engineers about what maximum forces and stresses can be applied to a certain region of a design. These precautionary measures help prevent failures due to overloading and deformation.

Background of failure

Failure occurs when a structure or device has been used past the limits of design that inhibits proper function. If a structure is designed to only support a certain amount of stress, strain, or loading and the user applies greater amounts, the structure will begin to deform and eventually fail. Several factors contribute to failure including a flawed design, improper use, financial costs, and miscommunication.

Failure due to miscommunication

Engineering is a precise discipline and in order to be precise, communication among project developers is pertinent for a successful product. There are several forms of miscommunication that can lead to a flawed design in a system. There are various fields of engineering that have to intercommunicate when working toward a mutual goal. These fields include civil, electrical, mechanical, industrial, chemical, biological, and environmental engineering. When creating a modern automobile, electrical engineers, mechanical engineers, and environmental engineers are required to work together to produce a fuel-efficient, durable product for consumers. If engineers do not adequately communicate among one another, a potential design could have flaws and be unsafe for consumer purchase. **Engineering disasters** can be a result of such miscommunication. Such disaster include the 2005 levee failures in Greater New Orleans, Louisiana during

Hurricane Katrina, the Space Shuttle Columbia disaster, and the Hyatt Regency walkway collapse.

Infamous disasters in engineering

Engineering products and inventions are utilized everyday including computers, microwaves, and elevators. A broken microwave can have limited consequences; however, when larger projects such as infrastructures and airplanes fail, multiple people can be affected which leads to an **engineering disaster**. A disaster is defined as a calamity that results in significant damage which may include the loss of life. Large scale engineering disasters are recorded in the history books. Just like any other mistake, these disasters become reminders and guidelines of how to improve and not repeat the same mistakes. In-depth observations and post-disaster analysis have been documented to a large extent to help prevent similar disasters to reoccur.

Infrastructure

Tacoma Narrows Bridge collapse (1940)

Tacoma Narrows Bridge (1940)

Hyatt Regency Hotel walkway collapse (1981)

Hyatt Regency walkway collapse

On the night of July 17, 1981 in Kansas City, Missouri, United States, two suspended walkways of the Hyatt Regency Hotel collapsed, killing 114 people and injuring 200 more. During this calamity, the hotel was hosting a dance competition. There were numerous competition attendants and observers standing and dancing on the suspended walkways when connections supporting the ceiling rods that hoisted both the second and fourth floor walkways across the atrium failed and collapsed onto the crowded first floor atrium below. The catastrophe was deadliest structural failure in United States history until the collapse of the World Trade Centers on September 11, 2001.

During investigation after the walkway collapse, architectural engineer Wayne G. Lischka noticed a substantial alteration of the original design. The fabricator constructed a double-rod support system rather than the originally designed single-rod system without approval of the engineering design team. In doing so, the created support beams doubled the loading on the connector which resulted in the failure of the walkway. It was documented that even the single-rod system would have barely supported the expected load and would not have met Kansas City Building Code standards. The final analysis of the damage had several conclusions reported including:

- The maximum load capacity of the fourth floor walkway was only 53% the maximum load capacity of Kansas City Building Code standards
- The fabrication alterations from the original design doubled the load that was received by the fourth floor walkway
- The deformation and distortion of the fourth floor hanger rods support the notion that the collapse began at that point
- No evidence that the quality of construction or material selection played a role in the walkway collapse.
- Aeronautics

Columbia disaster (2003)

The crew of the STS-107 mission.

The Space Shuttle Columbia disaster occurred on February 1, 2003 while reentering Earth's atmosphere over Louisiana and Texas. The shuttle unexpectedly disintegrated, resulting in the death of all seven astronauts on board. The cause was later discovered to be the loss of a piece of foam insulation of an external tank during launch. It was the seventh known instance of this particular piece breaking free during launch. As the shuttle re-entered Earth's atmosphere at a speed of Mach 23 (23 times faster than the speed of sound), the wing experienced temperatures of 2800 °F. The missing piece of insulation lost during the launch proved fatal as the shuttle disintegrated during the mission return. NASA's investigation team found smelted aluminum on the thermal tiles and inside edges of the left wing of the spacecraft, supporting the notion that the Columbia's destruction was due to hot gases that penetrated the damaged spot on the wing.

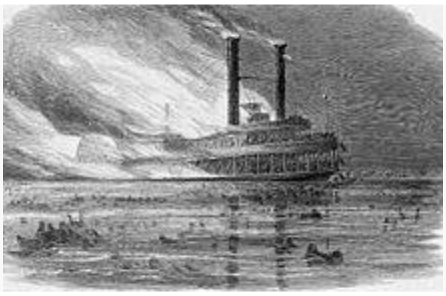
Roger L.M. Dunbar of New York University and Raghu Garud of Pennsylvania State University procured a case description of what missteps NASA had taken that led to the Columbia spacecraft catastrophe. Mission control deemed that foam shedding was a not a safety factor prior to launch, believed damage of the shuttle panels were not a significant issue which in-turn delayed analysis on damages as of January 17 of 2003, and denied mission action request between January 18 and 19. It was not until January 24, 2003 that mission control had classified the damage as a problematic issue. These missteps in communication between mission control and the debris assessment team inhibited a proper examination of the damages to the spacecraft.

Challenger explosion (1986)

Space Shuttle Challenger disaster

Vessels

SS Sultana (1865)



Depiction of the SS Sultana disaster

On the night of April 26, 1865, the passenger steamboat SS Sultana exploded on the Mississippi River seven miles north of Memphis, Tennessee. This maritime disaster is categorized as the worst in United States history. The explosion resulted in the loss of 1,547 lives, surpassing the total number of deaths caused by the sinking of the Titanic. The Sultana was overcrowded due to a soldier prisoner exchange towards the end of the United States Civil War. The overcrowding contributed significantly to the high death toll. Another reason for the high number of deaths is that the steamer was made mostly of wood, which was documented to have been completely engulfed in flames approximately seven minutes after the explosion. The explosion happened around midnight which was when the Mississippi River was at flood stage. It was documented that the single metal lifeboat on board the SS Sultana was thrown from the upper deck landing on several people swimming from the steamer resulting in further deaths.

The disaster was believed to be the result of a repaired boiler explosion that led to the explosion of two of the three other boilers. The boiler had been previously found to have had a leak and was improperly repaired by boilermaker R.G. Taylor due to orders from Captain J. Cass Mason because of time constraints in Vicksburg, Mississippi. While chief engineer Nathan Wintringer approved the repaired boiler, Taylor stated that the boiler could not be considered safe since the boiler appeared to be burned from being worked on with too little water. Travelling along the Mississippi River, the boiler exploded causing to fire spreading throughout the steamer. The fire on board led to the collapse of both of the Sultana's smokestacks, killing many passengers.

Stochastic behavior of systems operating under changing environments has widely been studied. . Dhillon , B.S. and Natesan, J. (1983) studied an outdoor power systems in fluctuating environment . Kan Cheng (1985) has studied reliability analysis of a system in a randomly changing environment. Jinhua Cao (1989) has studied a man machine system operating under changing environment subject to a Markov process with two states. The change in operating conditions viz. fluctuations of voltage, corrosive atmosphere, very low gravity etc. may make a system completely inoperative. Severe environmental conditions can make the actual mission duration longer than the ideal mission duration. In this paper we have taken **Failure due to engineering disasters or Failure due to miscommunication.** When the main operative unit fails then cold standby system becomes operative. **Failure due to engineering disasters** cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of very high cost in case of **failure due to engineering disasters** immediately. The repair is done on the basis of first fail first repaired.

Assumptions

1. λ_1 , λ_2, are constant failure rates for **Failure due to engineering disasters, Failure due to miscommunication** respectively. The CDF of repair time distribution of Type I and Type II are $G_1(t)$ and $G_2(t)$.
2. The failure due to **failure due to engineering disasters** is non-instantaneous and it cannot come simultaneously in both the units.
3. The repair starts immediately after the failure due to **Failure due to engineering disasters or Failure due to miscommunication** works on the principle of first fail first repaired basis.
4. The repair facility does no damage to the units and after repair units are as good as new.
5. The switches are perfect and instantaneous.
6. All random variables are mutually independent.
7. When both the units fail, we give priority to operative unit for repair.
8. Repairs are perfect and failure of a unit is detected immediately and perfectly.
9. The system is down when both the units are non-operative.

Notations

λ_1, λ_2 are the **Failure due to engineering disasters, Failure due to miscommunication** respectively. $G_1(t)$, $G_2(t)$ – repair time distribution Type -I, Type-II due to **Failure due to engineering disasters, Failure due to miscommunication** respectively.

p, q - probability of **Failure due to engineering disasters, Failure due to miscommunication** respectively such that p+ q=1

$M_i(t)$ System having started from state i is up at time t without visiting any other regenerative state

A_i (t) state is up state as instant t

R_i (t) System having started from state i is busy for repair at time t without visiting any other regenerative state.

B_i (t) the server is busy for repair at time t.

$H_i(t)$ Expected number of visits by the server for repairing given that the system initially starts from regenerative state i

Symbols for states of the System

Superscripts O, CS, EDF, MCF

Operative, Cold Standby, **failure due to engineering disasters, Failure due to miscommunication** respectively

Subscripts nmcf, mcf, edf, ur, wr, uR

No Failure due to miscommunication, Failure due to miscommunication, engineering disasters, under repair, waiting for repair, under repair continued from previous state respectively

Up states – 0, 1, 2, 7, and 8;

Down states – 3, 4, 5, 6

regeneration point – 0,1,2, 7, 8

States of the System

$0(O_{nmcf}, CS_{nmcf})$

One unit is operative and the other unit is cold standby and there is no failure due to Failure due to miscommunication in both the units.

$1(MCF_{mcf,\ ur}\ ,\ O_{nmcf})$

The operating unit fails due to Failure due to miscommunication and is under repair immediately of Type- II and standby unit starts operating with no Failure due to miscommunication

$2(EDF_{edf,\ ur}\ ,\ O_{nmcf})$

The operative unit fails due to EDF resulting from failure due to engineering disasters and undergoes repair of very costly Type I and the standby unit becomes operative with no Failure due to miscommunication .

$3(MCF_{mcf,uR}\ ,\ EDF_{edf,wr})$

The first unit fails due to Failure due to miscommunication and under Type-II repair is continued from state 1 and the other unit fails due to EDF resulting from failure due to engineering disasters and is waiting for repair of very costly Type -I.

$4(MCF_{mcf,uR}\ ,\ MCF_{mcf,wr})$

The repair of the unit is failed due to MCF resulting from failure due to Failure due to miscommunication is continued from state 1and the other unit failed due to MCF resulting from failure due to Failure due to miscommunication is waiting for repair of Type-II.

$5(EDF_{edf,\ uR}\ ,\ EDF_{edf,\ wr})$

The operating unit fails due to failure due to engineering disasters (EDF mode) and under repair of very costly Type - I continue from the state 2 and the other unit fails also due to failure due to engineering disasters is waiting for repair of very costly Type- I.

$6(EDF_{edf,uR}\ ,\ MCF_{mcf,wr})$

The operative unit fails due to EDF resulting from failure due to engineering disasters and under repair continues from state 2 of Type –I and the other unit is failed due to MCF resulting from failure due to Failure due to miscommunication and under Type-II

$7(O_{nmcf}\ ,\ MCF_{mcf,ur})$

The repair of the unit failed due to MCF resulting from failure due to Failure due to miscommunication failure is completed and there is no failure due to engineering disasters and the other unit is failed due to MCF resulting from failure due to Failure due to miscommunication is under repair of Type-II

$8(O_{nmcf}\ ,\ EDF_{edf,ur})$

The repair of the unit failed due to MCF resulting from failure due to Failure due to miscommunication failure is completed and there is no failure due to engineering disasters and the other unit is failed due to EDF resulting from failure due to engineering disasters is under repair of Type-I.

Transition Probabilities

Simple probabilistic considerations yield the following expressions:

$p_{01} = p,\quad p_{02} = q,$

$p_{10} = pG_1^*(\lambda_1)+q\,G_1^*(\lambda_2)= p_{70},$

$p_{20} = pG_2^*(\lambda_1)+q\,G_2^*(\lambda_2)= p_{80},$

$p_{11}^{(3)}= p(1- G_1^*(\lambda_1))= p_{14} = p_{71}^{(4)}\ p_{28}^{(5)}= q(1- G_2^*(\lambda_2))= p_{25} = p_{82}^{(5)}$ (1)

We can easily verify that

$p_{01} + p_{02} = 1,\ p_{10} + p_{17}^{(4)} (= p_{14}) + p_{18}^{(3)} (=p_{13}) = 1,$

$p_{80} + p_{82}^{(5)} + p_{87}^{(6)} = 1$ (2)

And mean sojourn time is

$\mu_0 = E(T) = \int_0^\infty P[T > t]dt$

Mean Time To System Failure

$Ø_0(t) = Q_{01}(t)[s]\ Ø_1(t) + Q_{02}(t)[s]\ Ø_2(t)$

$Ø_1(t) = Q_{10}(t)[s]\ Ø_0(t) + Q_{13}(t) + Q_{14}(t)$

$Ø_2(t) = Q_{20}(t)[s]\ Ø_0(t) + Q_{25}(t) + Q_{26}(t)$ (3-5)

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-5) and solving for

$ø_0^*(s) = N_1(s) / D_1(s)$ (6)

where

$N_1(s) = Q_{01}^*[Q_{13}^*(s) + Q_{14}^*(s)] + Q_{02}^*[Q_{25}^*(s) + Q_{26}^*(s)]$

$D_1(s) = 1 - Q_{01}^*\ Q_{10}^* - Q_{02}^*\ Q_{20}^*$

Making use of relations (1) & (2) it can be shown that $ø_0^*(0) = 1$, which implies that $ø_0(t)$ is a proper distribution.

$$\text{MTSF} = E[T] = \left.\frac{d}{ds}\,ø_0^*(s)\right|_{s=0}$$

$$= (D_1'(0) - N_1'(0))\ /\ D_1(0)$$

$$= (\mu_0 + p_{01}\,\mu_1 + p_{02}\,\mu_2)\ /\ (1 - p_{01}\,p_{10} - p_{02}\,p_{20})$$

where

$\mu_0 = \mu_{01} + \mu_{02}$,

$\mu_1 = \mu_{01} + \mu_{17}^{(4)} + \mu_{18}^{(3)}$,

$\mu_2 = \mu_{02} + \mu_{27}^{(6)} + \mu_{28}^{(5)}$

Availability analysis

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

$M_0(t) = e^{-\lambda_1 t} e^{-\lambda_2 t}$,

$M_1(t) = p\, \overline{G_1(t)}\, e^{-(\lambda_1 + \lambda_2)} = M_7(t)$

$M_2(t) = q\, \overline{G_2(t)}\, e^{-(\lambda_1 + \lambda_2)} = M_8(t)$

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) +$
$q_{02}(t)[c]A_2(t)$

$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) +$
$q_{18}^{(3)}(t)[c]A_8(t) + q_{17}^{(4)}(t)[c]A_7(t)$

$A_2(t) = M_2(t) + q_{20}(t)[c]A_0(t) +$
$[q_{28}^{(5)}(t)[c]\ A_8(t) + q_{27}^{(6)}(t)]\ [c]A_7(t)$

$A_7(t) = M_7(t) + q_{70}(t)[c]A_0(t) +$
$[q_{71}^{(4)}(t)[c]\ A_1(t) + q_{78}^{(3)}(t)]\ [c]A_8(t)$ $A_8(t) = M_8(t) + q_{80}(t)[c]A_0(t)$
$+[q_{82}^{(5)}(t)[c]\ A_2(t) + q_{87}^{(6)}(t)]\ [c]A_7(t)$ (7-11)

Taking Laplace Transform of eq. (7-11) and solving for $\hat{A}_0(s)$

$$\hat{A}_0(s) = N_2(s) / D_2(s) \qquad (12)$$

where

$N_2(s) = \widetilde{M}_0 (1 - \hat{q}_{78}^{(3)} - \hat{q}_{87}^{(6)}) - \hat{q}_{82}^{(5)}$
$(\hat{q}_{27}^{(6)} \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)} - \hat{q}_{71}^{(4)}$
$(\hat{q}_{17}^{(4)} + \hat{q}_{87}^{(6)} \hat{q}_{18}^{(3)}) + \hat{q}_{71}^{(4)} \hat{q}_{82}^{(5)}$
$(\hat{q}_{17}^{(4)} - \hat{q}_{27}^{(6)} \hat{q}_{18}^{(3)})] + \hat{q}_{01}[\widetilde{M}_1(1 -$
$\hat{q}_{78}^{(3)} \hat{q}_{87}^{(6)}) + \hat{q}_{71}^{(4)}(\widetilde{M}_7 + \hat{q}_{78}^{(3)}$
$\widetilde{M}_8) + \hat{q}_{18}^{(3)}(\widetilde{M}_7 \hat{q}_{87}^{(6)} - \widetilde{M}_8) -$
$\hat{q}_{82}^{(5)}(\widetilde{M}_1(\hat{q}_{27}^{(6)} \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) +$
$\hat{q}_{17}^{(4)} (- \widetilde{M}_2(\hat{q}_{78}^{(3)} + \widetilde{M}_7 \hat{q}_{28}^{(5)}) -$
$\hat{q}_{18}^{(3)}(\widetilde{M}_2 + \widetilde{M}_7 \hat{q}_{27}^{(6)})\}]\ \hat{q}_{02}[\widetilde{M}_2(1 - \hat{q}_{78}^{(3)} \hat{q}_{87}^{(6)}) + \hat{q}_{27}^{(6)}($
$\widetilde{M}_7 + \hat{q}_{78}^{(3)} \widetilde{M}_8) + \hat{q}_{28}^{(5)}(\widetilde{M}_7 \hat{q}_{87}^{(6)} + \widetilde{M}_8) - \hat{q}_{71}^{(4)}(\widetilde{M}_1(- \hat{q}_{27}^{(6)} - \hat{q}_{28}^{(5)} +$
$\hat{q}_{87}^{(6)}) + \hat{q}_{17}^{(4)} (\widetilde{M}_2 + \hat{q}_{28}^{(5)} \widetilde{M}_8) - \hat{q}_{18}^{(3)} (- \widetilde{M}_2 \hat{q}_{87}^{(6)} + \widetilde{M}_8 \hat{q}_{27}^{(6)})\}]$
$\hat{q}_{18}^{(3)}(\widetilde{M}_2 + \widetilde{M}_7 \hat{q}_{27}^{(6)})\}]$

$D_2(s) = (1 - \hat{q}_{78}^{(3)} - \hat{q}_{87}^{(6)}) - \hat{q}_{82}^{(5)}($
$\hat{q}_{27}^{(6)} \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) - \hat{q}_{71}^{(4)}$
$(\hat{q}_{17}^{(4)} + \hat{q}_{87}^{(6)} \hat{q}_{18}^{(3)}) + \hat{q}_{71}^{(4)} \hat{q}_{82}^{(5)} (\hat{q}_{17}^{(4)} \hat{q}_{28}^{(5)} - \hat{q}_{18}^{(3)})] + \hat{q}_{01}[- \hat{q}_{10} (1 -$
$\hat{q}_{78}^{(3)} \hat{q}_{87}^{(6)}) - \hat{q}_{71}^{(4)}(\hat{q}_{70} + \hat{q}_{78}^{(3)}$
$\hat{q}_{80}) - \hat{q}_{18}^{(3)}(\hat{q}_{70} \hat{q}_{87}^{(6)} - \hat{q}_{80}) -$
$\hat{q}_{82}^{(5)}(- \hat{q}_{10}(\hat{q}_{27}^{(6)} \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) +$
$\hat{q}_{17}^{(4)} (\hat{q}_{20} (\hat{q}_{78}^{(3)} - \hat{q}_{70} \hat{q}_{28}^{(5)}) +$
$\hat{q}_{18}^{(3)}(\hat{q}_{20} + \hat{q}_{70} \hat{q}_{27}^{(6)})\}]\ \hat{q}_{02}[- \hat{q}_{20}(1 - \hat{q}_{78}^{(3)} \hat{q}_{87}^{(6)}) - \hat{q}_{27}^{(6)}(\hat{q}_{70} + \hat{q}_{78}^{(3)} \hat{q}_{80}) - \hat{q}_{28}^{(5)}(\hat{q}_{70} \hat{q}_{87}^{(6)} + \hat{q}_{80}) - \hat{q}_{71}^{(4)}($
$\hat{q}_{10} (\hat{q}_{27}^{(6)} + \hat{q}_{28}^{(5)} \hat{q}_{87}^{(6)}) - \hat{q}_{17}^{(4)} (\hat{q}_{20} -$
$\hat{q}_{28}^{(5)} \hat{q}_{80}) - \hat{q}_{18}^{(3)} (\hat{q}_{20} \hat{q}_{87}^{(6)} + \hat{q}_{80}$
$\hat{q}_{27}^{(6)})\}]$

(Omitting the arguments s for brevity)

The steady state availability

$$A_0 = \lim_{t \to \infty} [A_0(t)]$$

$$= \lim_{s \to 0} [s\, \hat{A}_0(s)] \quad = \lim_{s \to 0} \frac{s\, N_2(s)}{D_2(s)}$$

Using L' Hospitals rule, we get

$$A_0 = \lim_{s \to 0} \frac{N_2(s) + s\ N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)} \qquad (13)$$

The expected up time of the system in (0,t] is

$\lambda_u(t) = \int_0^{\infty} A_0(z)\,dz$ So that

$$\widetilde{\lambda_u}(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)} \qquad (14)$$

The expected down time of the system in (0,t] is

$\lambda_d(t) = t - \lambda_u(t)$ So that

$$\widetilde{\lambda_d}(s) = \frac{1}{s^2} - \widetilde{\lambda_u}(s) \qquad (15)$$

The expected busy period of the server when there is EDF - Failure due to engineering disasters or MCF-failure due to Failure due to Failure due to miscommunication in (0,t]

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t)$

$R_1(t) = S_1(t) + q_{10}(t)[c]R_0(t) +$
$q_{18}^{(3)}(t)[c]\ R_8(t) + q_{17}^{(4)}(t)[c]R_7(t)$

$R_2(t) = S_2(t) + q_{20}(t)[c]R_0(t) + q_{28}^{(5)}(t)$

$R_8(t) + q_{27}^{(6)}(t)][c]R_7(t)$

$R_7(t) = S_7(t) + q_{70}(t)[c]R_0(t) + Q_{71}^{(4)}(t)$

$R_1(t) + q_{78}^{(3)}(t)][c]R_8(t)$

$R_8(t) = S_8(t) + q_{80}(t)[c]R_0(t) + Q_{82}^{(5)}(t)$

$R_2(t) + q_{87}^{(6)}(t)][c]R_7(t)$ (16-20)

Taking Laplace Transform of eq. (16-20) and solving for $\overline{R_0}(s)$

$$\overline{R_0}(s) = N_3(s) / D_2(s) \quad (21)$$

where

$N_3(s) = \hat{q}_{01}[\hat{S}_1(1 - \hat{q}_{78}^{(3)} \hat{q}_{87}^{(6)}) +$

$\hat{q}_{71}^{(4)}(\hat{S}_7 + \hat{q}_{78}^{(3)} \hat{S}_8) + \hat{q}_{18}^{(3)}(\hat{S}_7$

$\hat{q}_{87}^{(6)} - \hat{S}_8)] - \hat{q}_{01} \hat{q}_{82}^{(5)}(\hat{S}_1 \hat{q}_{27}^{(6)} \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) + \hat{q}_{17}^{(4)}$
$(\hat{S}_2 \hat{q}_{78}^{(3)} + \hat{S}_7 \hat{q}_{28}^{(5)}) -$

$\hat{q}_{18}^{(3)}(\hat{S}_2 + \hat{S}_7 \hat{q}_{27}^{(6)})] + \hat{q}_{02}[\hat{S}_2(1 -$

$\hat{q}_{78}^{(3)} \hat{q}_{87}^{(6)}) + \hat{q}_{27}^{(6)}(\hat{S}_7 + \hat{q}_{78}^{(3)} \hat{S}_8) + \hat{q}_{28}^{(5)}(\hat{S}_7 \hat{q}$
${}_{87}^{(6)} + \hat{S}_8) - \hat{q}_{02} \hat{q}_{71}^{(4)}(\hat{S}_1(-$

$\hat{q}_{27}^{(6)} - \hat{q}_{28}^{(5)} \hat{q}_{87}^{(6)} \hat{q}_{17}^{(4)}(\hat{S}_2 + \hat{q}_{28}^{(5)} \hat{S}_8) - \hat{q}_{18}^{(3)}(-\hat{S}_2 \hat{q}$
${}_{87}^{(6)} + \hat{S}_8 \hat{q}_{27}^{(6)})]$

and

$D_2(s)$ is already defined.

(Omitting the arguments s for brevity)

In the long run, $R_0 = \dfrac{N_3(0)}{D_2'(0)}$ (22)

The expected period of the system under EDF - failure due to engineering disasters or MCF- Failure due to Failure due to miscommunication in (0,t] is

$\lambda_{rv}(t) = \int_0^\infty R_0(z)dz$ So that $\overline{\lambda_{rv}}(s) = \dfrac{\hat{R}_0(s)}{s}$

The expected number of visits by the repairman for repairing the identical units in (0,t]

$H_0(t) = Q_{01}(t)[s][1 + H_1(t)] + Q_{02}(t)[s][1 + H_2(t)]$

$H_1(t) = Q_{10}(t)[s]H_0(t)] + Q_{18}^{(3)}(t)[s]$

$H_8(t) + Q_{17}^{(4)}(t)] [s]H_7(t)$,

$H_2(t) = Q_{20}(t)[s]H_0(t) + Q_{28}^{(5)}(t) [s]$

$H_8(t) + Q_{27}^{(6)}(t)] [c]H_7(t)$

$H_7(t) = Q_{70}(t)[s]H_0(t) + Q_{71}^{(4)}(t) [s]$

$H_1(t) + Q_{78}^{(3)}(t)] [c]H_8(t)$

$H_8(t) = Q_{80}(t)[s]H_0(t) + Q_{82}^{(5)}(t) [s]$

$H_2(t) + Q_{87}^{(6)}(t)] [c]H_7(t)$ (23-27)

Taking Laplace Transform of eq. (23-27) and solving for $H_0^*(s)$

$$H_0^*(s) = N_4(s) / D_3(s) \quad (28)$$

In the long run , $H_0 = N_4(0) / D_3'(0)$ (29)

Benefit- Function Analysis

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to Failure due to miscommunication or failure due to engineering disasters, expected number of visits by the repairman for unit failure.

The expected total Benefit-Function incurred in (0,t] is

C(t) = Expected total revenue in (0,t]

- expected busy period of the system under failure due to Failure due to miscommunication or failure due to engineering disasters for repairing the units in (0,t]

- expected number of visits by the repairman for repairing of identical the units in (0,t]

The expected total cost per unit time in steady state is

$$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2 C(s))$$

$$= K_1A_0 - K_2R_0 - K_3H_0$$

where

K_1 - revenue per unit up-time,

K_2 - cost per unit time for which the system is under repair of type- I or type- II

K_3 - cost per visit by the repairman for units repair.

Conclusion

After studying the system , we have analyzed graphically that when the failure rate due to Failure due to miscommunication or failure due to engineering disasters increases, the MTSF and steady state availability decreases and the Benefit-function decreased as the failure increases.

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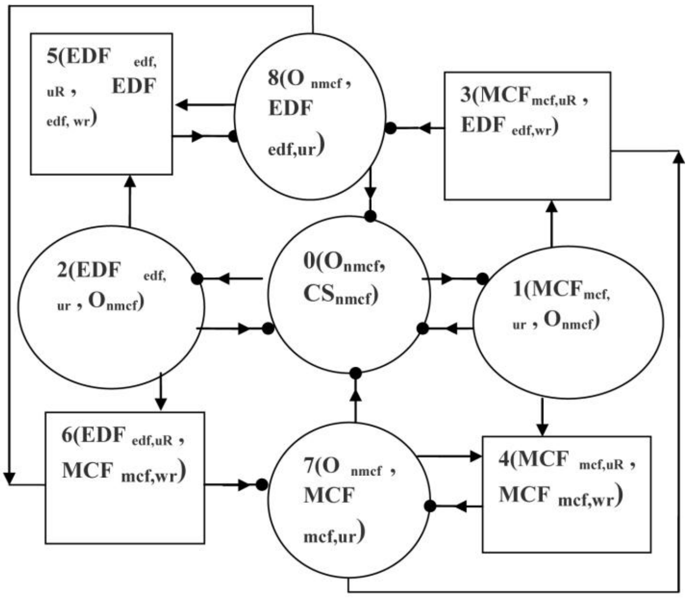

Fig. The State Space Diagram

◯ Up state ☐ down state

• Regeration point