Gain-Function of Two Non-identical Warm Standby System with High Blood Pressure causing Heart, Kidney and Brain Damage

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Abstract -Blood pressure is the force of blood pushing against blood vessel walls as the heart pumps out blood, and high blood pressure, also called hypertension, is an increase in the amount of force that blood places on blood vessels as it moves through the body. Factors that can increase this force include higher blood volume due to extra fluid in the blood and blood vessels that are narrow, stiff, or clogged.

High blood pressure and Heart failure
If you have high blood pressure, this means that your heart has to work harder to push blood round your body. To cope with this extra effort, your heart becomes thicker and stiffer, which makes it less able to do its job.

If your heart is not able to pump as well as it should, this is called heart failure. Heart failure can cause extra fluid to build up in the body, and can also cause an irregular heartbeat. It does not mean that your heart is about to stop working, but it is a serious condition.

High blood pressure and Kidney failure
High blood pressure can damage blood vessels in the kidneys, reducing their ability to work properly. When the force of blood flow is high, blood vessels stretch so blood flows more easily. Eventually, this stretching scars and weakens blood vessels throughout the body, including those in the kidneys.

High blood pressure and Damage to your brain
Just like your heart, your brain depends on a nourishing blood supply to work properly and survive. But high blood pressure can cause several problems, including: Transient ischemic attack (TIA), Stroke, Dementia, Mild cognitive impairment.

The failure time distribution is taken as exponential and repair time distribution as general. Using Semi Markov regenerative point technique we have calculated different reliability characteristics such as MTSF, reliability of the system, availability analysis in steady state, busy period analysis of the system under repair, expected number of visits by the repairman in the long run and profit-function. Special case by taking repair as exponential has been derived and graphs are drawn.

Keyword: warm standby, High Blood Pressure causing Heart, Kidney failure and Brain damage, MTSF, Availability, busy period, Gain-function.

INTRODUCTION

High blood pressure and Heart failure
Hypertensive heart disease is the No. 1 cause of death associated with high blood pressure. It refers to a group of disorders that includes heart failure, ischemic heart disease, hypertensive heart disease, and left ventricular hypertrophy (excessive thickening of the heart muscle).

What Is Heart Failure?
Heart failure does not mean the heart has stopped working. Rather, it means that the heart's pumping power is weaker than normal or the heart has become less elastic. With heart failure, blood moves through the heart's pumping chambers less effectively, and pressure in the heart increases, robbing your body of oxygen and nutrients.

What are symptoms of heart failure?
Most of the symptoms of heart failure are felt due to the buildup of fluid in the body. Where the fluid builds up and the problems the buildup can cause depend on which side of the heart is affected.

If the left side of the heart is affected, not enough blood is pumped round the body, which can leave you feeling tired. More blood is entering the lungs than your heart can remove and, as the fluid builds up, you may cough up frothy phlegm or feel breathless when lying down.

If the right side of the heart is affected, fluid builds up in the body instead, causing swollen feet, ankles and legs called congestive heart failure. Again, you may feel more tired than normal.

Can heart failure be treated?
Heart failure cannot be cured, but there are treatments available. It is important to find the cause of heart failure so that your doctor is not just treating the symptoms.

Some blood pressure medicines can help treat heart failure, for example diuretics can help to reduce fluid
buildup. ACE inhibitors and angiotensin receptor blockers can also help. Certain medicines called beta-blockers are often used, but these are usually different from the types that are used to treat high blood pressure. Other medicines can also be used.

You can help improve the health of your heart by making lifestyle changes - stopping smoking, cutting down on alcohol, or being more active (it doesn’t need to be too energetic; walking every day will help your heart).

**High blood pressure and Kidney failure**

Kidneys work at the microscopic level. The kidney is not one large filter. Each kidney is made up of about a million filtering units called nephrons. Each nephron filters a small amount of blood. The nephron includes a filter, called the glomerulus, and a tubule. The nephrons work through a two-step process. The glomerulus lets fluid and waste products pass through it; however, it prevents blood cells and large molecules, mostly proteins, from passing. The filtered fluid then passes through the tubule, which sends needed minerals back to the bloodstream and removes wastes. The final product becomes urine.

**Kidney failure.** High blood pressure is one of the most common causes of kidney failure. That's because it can damage both the large arteries leading to your kidneys and the tiny blood vessels (glomeruli) within the kidneys. Damage to either makes it so your kidneys can't effectively filter waste from your blood. As a result, dangerous levels of fluid and waste can accumulate. You might ultimately require dialysis or kidney transplantation.

**Kidney scarring (glomerulosclerosis).** Glomerulosclerosis (gloe-mer-ul-oe-skluh-ROE-sis) is a type of kidney damage caused by scarring of the glomeruli (gloe-MER-ul-i). The glomeruli are tiny clusters of blood vessels within your kidneys that filter fluid and waste from your blood. Glomerulosclerosis can leave your kidneys unable to filter waste effectively, leading to kidney failure.

**Kidney artery aneurysm.** An aneurysm is a bulge in the wall of a blood vessel. When it occurs in an artery leading to the kidney, it's known as a kidney (renal) artery aneurysm. One potential cause is atherosclerosis, which weakens and damages the artery wall. Over time, high blood pressure in a weakened artery can cause a section to enlarge and form a bulge — the aneurysm. Aneurysms can rupture and cause life-threatening internal bleeding.

If the kidneys’ blood vessels are damaged, they may stop removing wastes and extra fluid from the body. Extra fluid in the blood vessels may then raise blood pressure even more, creating a dangerous cycle.

High blood pressure is the second leading cause of kidney failure in the United States after diabetes; In addition, the rate of kidney failure due to high blood pressure increased 7.7 percent from 2000 to 2010.

**Damage to your kidneys**

Your kidneys filter excess fluid and waste from your blood — a process that depends on healthy blood vessels. High blood pressure can injure both the blood vessels in and leading to your kidneys, causing several types of kidney disease (nephropathy). Having diabetes in addition to high blood pressure can worsen the damage.

**High blood pressure and Damage to your brain**

**Transient ischemic attack (TIA).** Sometimes called a ministroke, a transient ischemic (is-KEE-mik) attack is a brief, temporary disruption of blood supply to your brain. It's often caused by atherosclerosis or a blood clot — both of which can arise from high blood pressure. A transient ischemic attack is often a warning that you're at risk of a full-blown stroke.

**Stroke.** A stroke occurs when part of your brain is deprived of oxygen and nutrients, causing brain cells to die. Uncontrolled high blood pressure can lead to stroke by damaging and weakening your brain's blood vessels, causing them to narrow, rupture or leak. High blood pressure can also cause blood clots to form in the arteries leading to your brain, blocking blood flow and potentially causing a stroke.

**Dementia.** Dementia is a brain disease resulting in problems with thinking, speaking, reasoning, memory, vision and movement. There are a number of causes of dementia. One cause, vascular dementia, can result from narrowing and blockage of the arteries that supply blood to the brain. It can also result from strokes caused by an interruption of blood flow to the brain. In either case, high blood pressure may be the culprit.

**Mild cognitive impairment.** Mild cognitive impairment is a transition stage between the changes in understanding and memory that come with aging and the more-serious problems caused by Alzheimer’s disease. Like dementia, it can result from blocked blood flow to the brain when high blood pressure damages arteries.

**Assumptions**

**Assumptions**

1. The failure time distribution is exponential whereas the repair time distribution is arbitrary of two non-identical units.

2. The repair facility is of four types:
   - Type I, II repair facility
   - when failure due to high blood pressure causing heart failure and high blood pressure causing kidney failure of first unit occurs respectively

Type III, IV repair facility
- when failure due to high blood pressure causing heart failure and failure due to high blood pressure causing brain damage of the second unit occurs respectively.

3. The repair starts immediately upon failure of units and the repair discipline is FCFS.

4. The repairs are perfect and start immediately after failure due to high blood pressure causing heart failure, kidney failure and brain damage as soon as the operation of the system becomes normal.

5. The failure of a unit is detected immediately and perfectly.

6. The switches are perfect and instantaneous.

7. All random variables are mutually independent.

**Symbols for states of the System**

**Superscripts** O, WS, FHPHF, FHPKF, FHPBD

Operative, Warm Standby, High blood pressure causing heart failure, kidney failure and brain damage respectively

**Subscripts** nhp, hp, ur, wr, uR

no high blood pressure, high blood pressure under repair, waiting for repair, under repair continued respectively

Up states – 0,1,2,9; down states – 3,4,5,6,7,8,10,11

Regeneration point – 0,1,2,4,7,10

**States of the System**

0(O\text{nhp}, WS_{\text{nhp}})

One unit is operative and there is no high blood pressure and the other unit is warm standby with no high blood pressure.

1(SO\text{nhp}, O_{\text{nhp}})

The operation of the first unit stops automatically due to high blood pressure and warm standby unit starts operating with no high blood pressure.

2(FHPHF_{ur}, O_{\text{nhp}})

The first unit fails and undergoes repair after heart failure is over and the other unit continues to be operative with no high blood pressure.

3(FHPHF_{ut}, SO_{hp})

The repair of the first unit is continued from state 2 and the operation of second unit stops automatically due to high blood pressure.

4(FHPHF_{tr}, SO_{hp}) The first unit fails and undergoes repair after the heart failure is over and the other unit also stops automatically due to high blood pressure.

5(FHPHF_{ut} \text{, FHPKF}_{hp,wr}) The repair of the first unit is continued from state 4 and the other unit is failed due to high blood pressure caused by kidney failure & is waiting for repair.

6(O_{\text{nhp}}, FHPKF_{ur}) The first unit becomes operative with no high blood pressure.

and the second unit is failed due to high blood pressure caused by kidney failure is under repair.

7(SO_{hp}, FHBBD_{hp,ur})

The operation of the first unit stops automatically due to high blood pressure and the second unit fails due to high blood pressure causing brain damage and undergoes repair.

8(FHPHF_{hp,wr}, FHPBD_{ur,wr})

The repair of failed unit due to high blood pressure is continued from state 7 and the first unit is failed due to high blood pressure caused by heart failure is waiting for repair.

9(O_{\text{nhp}}, SO_{hp})

The first unit is operative with no high blood pressure and the operation of warm standby second unit is stopped automatically due to high blood pressure caused by brain damage.

10(SO_{hp}, FHPBD_{ur})

The operation of the first unit stops automatically due to high blood pressure and the second unit fails due to high blood pressure caused by brain damage and undergoes repair after the high blood pressure is over.

11(FHPHF_{hp,wr}, FHPBD_{ut})

The repair of the second unit is continued from state 10 and the first unit is failed due to high blood pressure caused by heart failure is waiting for repair.

**Transition Probabilities**

Simple probabilistic considerations yield the following expressions:

\begin{align*}
p_{01} &= \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3}, & p_{07} &= \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}, \\
p_{09} &= \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3}, & p_{12} &= \frac{\lambda_1 + \lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}, & p_{14} &= \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3}, \\
p_{20} &= G_1(\lambda_1), & p_{22} &= G_1(\lambda_1), & p_{23} &= G_2(\lambda_2), \\
p_{22} &= G_1(\lambda_1), & p_{28} &= P_8
\end{align*}

We can easily verify that

\begin{align*}
p_{01} + p_{07} + p_{09} &= 1, & p_{12} + p_{14} &= 1, & p_{20} + p_{23} &= 1 \\
p_{46} &= 1, & p_{50} &= 1, & p_{72} &= 1, & p_{82} &= 1, \\
p_{910} &= 1, & p_{1012} &= 1 \\
\text{And mean sojourn time is} & \quad \mu_0 = E(T) = \int_0^{\infty} P[T > t] \, dt \quad (2)
\end{align*}

**Mean Time To System Failure**

We can regard the failed state as absorbing
Taking Laplace-Stiltjes transform of eq. (3-5) and solving for
\[ \hat{Q}_0(s) = \frac{N_1(s)}{D_1(s)} \]  
where
\[ N_1(s) = \hat{Q}_{11}(s) \{ Q_{12}(s) \hat{Q}_2(3,s) + Q_{14}(s) \} + \hat{Q}_{05}(s) Q_{33}(s) \hat{Q}_2(7,s) \]
\[ D_1(s) = 1 + \hat{Q}_{11}(s) \hat{Q}_{12}(s) \hat{Q}_{22}(s) \]

Making use of relations (1) & (2) it can be shown that \( \theta_0(0) = 1 \), which implies that \( \theta_0(t) \) is a proper distribution.

\[ \text{MTSF} = E[T] = \lim_{s \to 0} \frac{\theta_0'(0)}{s} = \frac{D_1(0) - N_1(0)}{D_1(0)} \]

where
\[ \mu_0 = \mu_{01} + \mu_{07} + \mu_{09} \]
\[ \mu_1 = \mu_{12} + \mu_{14}, \mu_2 = \mu_{20} + \mu_{22}^{(3)} \]
\[ \mu_5 = \mu_{5,10} \]

### Availability analysis

Let \( M_0(t) \) be the probability of the system having started from state 1 is up at time \( t \) without making any other regenerative state. By probabilistic arguments, we have

The value of \( M_0(t), M_1(t), M_2(t), M_4(t) \) can be found easily.

The point wise availability \( A_i(t) \) have the following recursive relations

\[ A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{03}(t)[c]A_2(t) + q_{05}(t)[c]A_4(t) \]
\[ A_1(t) = M_1(t) + q_{12}(t)[c]A_2(t) + q_{14}(t)[c]A_4(t) \]
\[ A_2(t) = M_2(t) + q_{20}(t)[c]A_0(t) + q_{22}^{(3)}(t)[c]A_2(t) \]
\[ A_4(t) = q_{46}^{(3)}(t)[c]A_4(t) \]
\[ A_5(t) = q_{50}(t)[c]A_0(t) \]
\[ A_7(t) = q_{72}^{(3)}(t) [c]A_2(t) + q_{74}(t)[c]A_4(t) \]
\[ A_{10}(t) = q_{10,2}(t)[c]A_2(t) + q_{10,12}(t)[c]A_4(t) \]

Taking Laplace Transform of eq. (7-14) and solving for
\[ \hat{A}_0(s) = \frac{N_2(s)}{D_2(s)} \]

where
\[ \hat{N}_2(s) = (1 - \hat{Q}_2^{(3)}(s)) \{ \hat{M}_0(s) + \hat{q}_{01}(s) \hat{M}_1(s) + \hat{q}_{03}(s) \hat{M}_2(s) + \hat{q}_{05}(s) \hat{M}_4(s) + \hat{q}_{12}(s) \hat{A}_2(s) + \hat{q}_{14}(s) \hat{A}_4(s) + \hat{q}_{20}(s) \hat{A}_0(s) \}
\[ \hat{D}_2(s) = (1 - \hat{Q}_2^{(3)}(s)) [1 - \hat{q}_{46}^{(3)}(s) \hat{Q}_{06}(s) \hat{A}_0(s) \hat{A}_2(s) + \hat{Q}_{10}(s) \hat{A}_0(s) \hat{A}_4(s) + \hat{Q}_{12}(s) \hat{A}_2(s) + \hat{Q}_{14}(s) \hat{A}_4(s)] \]

The steady state availability
\[ A_0 = \lim_{s \to 0} [\hat{A}_0(s)] = \frac{N_2(0)}{D_2(0)} \]

Using L’ Hospitals rule, we get
\[ A_0 = \lim_{s \to 0} [N_2(s) + N_2(s)] = \frac{N_2(0)}{D_2(0)} \]

where
\[ N_2(0) = p_{20}(\hat{M}_{00}(0) + p_{01}\hat{M}_{10}(0) + p_{03}\hat{M}_{20}(0) + p_{05}\hat{M}_{40}(0)) + \hat{M}_{20}(0) \]
\[ D_2(0) = p_{20}(\hat{M}_{00}(0) + \hat{M}_{10}(0) + p_{01}p_{12} + p_{03}p_{32} + p_{05}p_{52} + p_{12}p_{20}) \]
\[ \hat{M}_0(s) = \frac{1}{s^2}, \hat{M}_1(s) = \frac{1}{s} \frac{1}{s^2} \]
\[ \hat{M}_2(s) = \frac{1}{s^2}, \hat{M}_4(s) = \frac{1}{s^2} \]

The expected up time of the system in \( (0,1] \) is
\[ \lambda_{u}(t) = \frac{1}{\hat{A}_0(s)} d\hat{A}_0(s) \]

So that
\[ \lambda_{u}(0) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(0)}{D_2(0)} \]

The expected down time of the system in \( (0,1] \) is
\[ \lambda_{d}(t) = 1 - \lambda_{u}(t) \]

So that
\[ \lambda_{d}(0) = \frac{1}{s} - \frac{\hat{A}_0(s)}{s} \]

The expected busy period of the server for repairing the failed unit due to high blood pressure causing heart failure in \( (0,1] \)

\[ \lambda_{u}(t) = S_0(t) + q_{01}(t)[c]R_1(t) + q_{03}(t)[c]R_3(t) + q_{05}(t)[c]R_5(t) \]

\[ \lambda_{d}(t) = S_1(t) + q_{12}(t)[c]R_2(t) + q_{14}(t)[c]R_4(t) \]

\[ \lambda_{u}(t) = q_{20}(t)[c]R_0(t) + q_{22}^{(3)}(t)[c]R_3(t) \]

\[ \lambda_{d}(t) = q_{46}^{(3)}(t)[c]R_4(t) \]

\[ \lambda_{u}(t) = q_{50}(t)[c]R_0(t) + q_{72}^{(3)}(t)[c]R_2(t) + q_{74}(t)[c]R_4(t) \]

\[ \lambda_{d}(t) = q_{10,2}(t)[c]R_1(t) + q_{10,12}(t)[c]R_3(t) + q_{10,12}(t)[c]R_4(t) \]

Taking Laplace Transform of eq. (19-26) and solving for
\[ \hat{R}_{0}(s) = \frac{N_3(s)}{D_2(s)} \]

Where
The expected busy period of the system under high blood pressure causing kidney failure in (0,t] is
\[ \lambda_{r_2}(t) = \int_0^t R_0(z) \, dz \]
So that \[ \overline{\lambda}_{r_2}(s) = \frac{R_0(z)}{s} \]

The expected Busy period of the server for repair of dissimilar units by the repairman due to high blood pressure causing kidney failure in (0,t]

Taking Laplace Transform of eq. (40) and solving for
\[ \overline{P_0}(s) = N_d(s) / D_2(s) \]
where
\[ N_3(0) = p_{20} p_{07} \hat{L}_2(0) \]
and \[ D_2(0) \] is already defined.

The expected busy period of the server for repair due to high blood pressure causing brain damage in (0,t] is
\[ \lambda_{r_2}(t) = \int_0^t P_0(z) \, dz \]
So that \[ \overline{\lambda}_{r_2}(s) = \frac{P_0(z)}{s} \]

The expected number of visits by the repairman for repairing the non-identical units in (0,t]

Taking Laplace Transform of eq. (51) and solving for
\[ H_0(s) = N_3(s) / D_3(s) \]
where
\[ N_4(s) = (1 - \hat{Q}_{22}^{(3)}(s)) \{ \hat{Q}_{10}(s) \hat{Q}_{14}(s) \]
The expected number of visits by the repairman for repairing due to high blood pressure causing heart failure in (0,t]

\[ V_d(t) = Q_{01}(t)[cV_1(t) + Q_{02}(t)[cV_2(t) + Q_{00}(t)[cV_0(t)]] \]

\[ V_1(t) = Q_{12}(t)[cV_2(t) + Q_{14}(t)[cV_4(t)] \]

\[ V_2(t) = Q_{20}(t)[cV_2(t) + Q_{22}(s)[t][cV_2(t)] \]

\[ V_d(t) = Q_{0d}^{(3)}(t)[cV_d(t) \]

\[ V_d(t) = Q_{0d}(t)[cV_0(t) \]

\[ V_l(t) = (Q_{2l}(t)[1+V_2(t)] + Q_{22}(s)[t)] [cV_2(t) + Q_{24}(t)[cV_4(t)] \]

\[ V_d(t) = Q_{0d}(t)[cV_0(t) \]

\[ V_1(t) = (Q_{10}(t) + Q_{02}(s))[t][cV_2(t)] \]

\[ \text{Taking Laplace-Stieltjes transform of eq. (61-68) and solving for } \]

\[ V_0^*(s) = N_*(s) / D_*(s) \]

\[ \text{where } N_*(s) = Q_{01}^*(s) Q_{22}^*(s) [1 - Q_{22}^{(3)}(s)] \text{ and } D_*(s) \text{ is the same as } D_3(s) \]

\[ \text{In the long run, } V_0 = \frac{N_d(0)}{D_d(0)} \]

\[ \text{where } N_d(0) = p_{20} p_{10} p_{12}, \text{ and } D_3(0) \text{ is already defined.} \]

**Gain-Function Analysis**

The Gain- function of the system considering mean up-time, expected busy period of the system due to high blood pressure causing heart failure when the units stops automatically, expected busy period of the server for repair of unit and due to high blood pressure causing brain damage, expected number of visits by the repairman for non-identical units failure, expected number of visits by the repairman due to high blood pressure causing heart failure.

The expected total Gain-function incurred in (0,t]

\[ C(t) = \text{Expected total revenue in (0,t]} \]

- expected number of visits by the repairman for repairing due to high blood pressure causing heart failure in (0,t]

\[ C_2 = \text{cost per unit time for which the system is due to high blood pressure causing heart failure} \]

\[ C_3 = \text{cost per unit time for which the system is due to high blood pressure causing kidney failure} \]

\[ C_4 = \text{cost per unit time for which the system is due to high blood pressure causing brain damage} \]

\[ C_5 = \text{cost per visit by the repairman for non-identical units repair.} \]

\[ C_6 = \text{cost per visit by the repairman due to high blood pressure causing heart failure} \]

**CONCLUSION**

After studying the system, we have analyzed graphically that when the failure rate due to high blood pressure causing heart failure, kidney failure and brain damage increases, the MTSF and steady state availability decreases and the Gain-Function also decreased as the failure increases.

**REFERENCES**


Fig. 1 The State Transition Diagram

- **Up state**: 
- **Down state**: 

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