Cost-Benefit Analysis of a Two Similar Cold Standby System with Radar failure and All Telephone Lines Failure

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Abstract- Radar is an object-detection system that uses radio waves to determine the range, altitude, direction, or speed of objects. It can be used to detect aircraft, ships, spacecraft, guided missiles, motor vehicles, weather formations, and terrain. The radar dish or antenna transmits pulses of radio waves or micro waves that bounce off any object in their path. The object returns a tiny part of the wave's energy to a dish or antenna that is usually located at the same site as the transmitter.

Radar was secretly developed by several nations before and during World War II. The term RADAR was coined in 1940 by the United States Navy as an acronym for Radio Detection And Ranging. The term radar has since entered English and other languages as a common noun, losing all capitalization.

The modern uses of radar are highly diverse, including air traffic control, radar astronomy, air-defense systems, antimissile systems; marine radars to locate landmarks and other ships; aircraft anti collision systems; ocean surveillance systems, outer space surveillance and rendezvous systems; meteorological precipitation monitoring; altimetry and flight control systems; guided missile target locating systems; and ground-penetrating radar for geological observations. High tech radar systems are associated with digital signal processing and are capable of extracting useful information from very high noise levels.

Other systems similar to radar make use of other parts of the electromagnetic spectrum. One example is "lidar", which uses visible light from lasers rather than radio waves.

We have taken units failure due to radar failure and due to all telephone lines failure with failure time distribution as exponential and repair time distribution as General. We have found out MTSF, Availability analysis, the expected busy period of the server for repair when the failure caused due to radar failure in (0,t], expected busy period of the server for repair in (0,t], the expected busy period of the server for repair when all telephone lines fails in (0,t], the expected number of visits by the repairman for failure of units due to radar failure in (0,t], the expected number of visits by the repairman for all telephone lines fails in (0,t] and Cost-Benefit analysis using regenerative point technique. A special case using failure and repair distributions as exponential is derived and graphs have been drawn.

Keyword: Cold Standby, radar failure, all telephone lines failure, MTSF, Availability, Busy period, Cost-Benefit Analysis

INTRODUCTION

ATLANTA, Georgia (CNN) -- Air traffic controllers were forced to use their personal cell phones to reroute hundreds of flights after the Federal Aviation Administration's Memphis Center lost radar and telephone service for more than two hours, snarling air traffic in the middle of the nation.

A spokesman for FedEx, which has its hub in Memphis, Tennessee, said the package delivery company had diverted 11 aircraft to other cities. But most of its flights take off and land after 10 p.m., so FedEx expected the impact to be minimal, the spokesman said.

Air traffic was halted at 12:35 p.m. ET when a major communication line that feeds all the telephones at the FAA's Memphis Center failed, said FAA spokeswoman Kathleen Bergen.

Service was restored at 3 p.m.

The malfunction, which occurred inside a telephone company's switching office, made it impossible for air controllers at FAA's Memphis Center to communicate normally with adjoining centers to hand off control of flights, Bergen said.

In addition, three of nine long-range radar systems were lost, causing the FAA to temporarily ground traffic within a 250-mile radius of the center, affecting flights in seven states, Bergen said.

Adjacent centers in Atlanta, Georgia; Indianapolis, Indiana; Kansas City, Missouri; and Fort Worth, Texas; were pitching in to try to reroute planes, she said.

There was no indication the failure was deliberate, she said.

Doug Church, a spokesman for the National Air Traffic Controllers Association, called the failure "a major safety problem."
At the time of the outage, controllers "were thrust into an immensely chaotic situation in which they had to use personal cell phones to talk to other air traffic control facilities about specific flights that they could not communicate with themselves," he said.

"Significant delays" resulted at airports in the middle of the country, including Dallas-Fort Worth, Atlanta and Charlotte, North Carolina, he said.

Memphis Center's airspace includes 100,000 square miles of airspace, covering Tennessee, Arkansas, Mississippi and parts of Alabama and Kentucky.

Church predicted that flight operations in the affected area "are not going to be 'normal' for quite some time."

A spokesman for Northwest Airlines said the impact on the airline was "pretty minor," with 13 flights canceled and 19 others diverted out of 740 scheduled flights for the day.

In this paper, we have failure due to radar failure and failure due to all telephone lines failure which are non-instantaneous in nature.. Here, we investigate a two identical cold standby -a system in which offline unit cannot fail. The failure is due to All telephone lines failure and due to radar failure. When there is radar failure within specified limit, it operates as normal as before but if these are beyond the specified limit the operation of the unit is stopped to avoid excessive damage of the unit and as the radar failure continues going on some characteristics of the unit change which we call failure of the unit. After failure due to radar failure the failed unit undergoes repair immediately according to first come first served discipline.

ASSUMPTIONS
1. The system consists of two similar cold standby units. The failure time distributions of the operation of the unit stopped automatically, the radar failure and all telephone lines failure are exponential with rates λ₁, λ₂ and λ₃ whereas the repairing rates for repairing the failed system due to radar failure and due to All telephone lines failure are arbitrary with CDF G₁(t) & G₂(t) respectively.

2. When there is radar failure within specified limit, it operates as normal as before but if these are beyond the specified limit the operation of the unit is avoided and as the radar failure continues goes on some characteristics of the unit change which we call failure of the unit.

3. The radar failure actually failed the units. The radar failure are non-instantaneous and it cannot occur simultaneously in both the units.

4. The repair facility works on the first fail first repaired (FCFS) basis.

5. The switches are perfect and instantaneous.

6. All random variables are mutually independent.

Symbols for states of the System

Superscripts
- O, CS, SO, FR, FATL
Operative, cold Standby, Stops the operation, Failed due to radar failure, failed due to all telephone lines failure respectively

Subscripts
- nrf, urf, atl, ur, wr, uR
No radar failure, under radar failure, all telephone lines failure, under repair, waiting for repair, under repair continued respectively

States of the System

0(O_{nrf}, CS_{urf})
One unit is operative and the other unit is cold standby and there is no radar failure in both the units.

1(SO_{urf}, O_{urf})
The operation of the first unit stops automatically due to radar failure and cold standby unit starts operating with no radar failure.

2(SO_{urf}, FATL_{urf, fail, ur})
The operation of the first unit stops automatically due to radar failure and the other unit fails due to all telephone lines failure and undergoes repair..

3(FR_{ur}, O_{urf})
The first unit fails due to radar failure and undergoes repair and the other unit continues to be operative with no radar failure.

4(FR_{ur}, SO_{urf})
The one unit fails due to radar failure and continues to be undergone repair and

![Fig.1 The State Transition Diagram](image-url)
the other unit also stops automatically due to radar failure.

5(FR$_{ad}$, FR$_{ar}$)

The repair of the first unit is continued from state 4 and the other unit failed due to radar failure is waiting for repair.

6(FR$_{ad}$, SO$_{rt}$)

The repair of the first unit is continued from state 3 and unit fails due to radar failure and operation of other unit stops automatically due to radar failure.

7(FR$_{rt}$, FATI$_{att}$. att)

The repair of failed unit due to all telephone lines failure is continued from state 2 and the first unit is failed due to radar failure is waiting for repair.

Transition Probabilities

Simple probabilistic considerations yield the following expressions:

\[ P_{01} = \frac{\lambda_1}{\lambda_1 + \lambda_3}, \quad P_{02} = \frac{\lambda_3}{\lambda_1 + \lambda_3} \]

\[ P_{13} = \frac{\lambda_2}{\lambda_1 + \lambda_2}, \quad P_{14} = \frac{\lambda_1}{\lambda_1 + \lambda_2} \]

\[ p_{23} = \lambda_3 G_2^*(\lambda_2), \quad p_{23}^{(7)} = \lambda_2 G_2^*(\lambda_2), \quad p_{24} = G_2^*(\lambda_2), \]

\[ p_{30} = G_1^*(\lambda_1), \quad p_{33}^{(6)} = G_1^*(\lambda_1) \]

\[ p_{43} = G_1^*(\lambda_2), \quad p_{43}^{(5)} = G_1^*(\lambda_2) \]

(1)

we can easily verify that

\[ P_{01} + P_{02} = 1, \quad P_{13} + P_{14} = 1, \]

\[ P_{23} + P_{23}^{(7)} + P_{24} = 1, \quad P_{30} + P_{33}^{(6)} = 1, \]

\[ p_{43} + p_{43}^{(5)} = 1 \]

(2)

and mean sojourn time is

\[ \mu_0 = E(T) = \int_0^\infty P[T > t]dt = -1/\lambda_1 \]

Similarly

\[ \mu_1 = 1/\lambda_2 \]

\[ \mu_2 = \int_0^\infty e^{-\lambda_1 G_1(t)}dt \]

\[ \mu_4 = \int_0^\infty e^{-\lambda_2 G_1(t)}dt \]

(3)

Mean Time To System Failure

We can regard the failed state as absorbing

\[ \theta_0(t) = Q_{01}(t)[s]\theta_1(t) + Q_{02}(t) \]

\[ \theta_1(t) = Q_{13}(t)[s]\theta_3(t) + Q_{14}(t), \quad \theta_3(t) = Q_{30}(t)[s]\theta_0(t) + Q_{33}^{(6)}(t) \]

(4-6)

Taking Laplace-Stieltjes transforms of eq. (4-6) and solving, we get

\[ Q_0'(s) = N_1(s) / D_1(s) \]

(7)

where

\[ N_1(s) = Q_{01}(s) \{ Q_{13}(s) Q_{23}^{(6)}(s) + Q_{14}(s) \} + Q_{02}(s) \]

\[ D_1(s) = 1 - Q_{01}(s) Q_{13}(s) Q_{20}(s) \]

Making use of relations (1) & (2) it can be shown that \( Q_0'(0) = 1 \), which implies that \( \theta_1(t) \) is a proper distribution.

\[ MTSF = E[T] = \frac{d}{ds}\theta_0(s) \]

\[ s=0 \]

\[ = (D_1(0) - N_1(0)) / D_1(0) \]

\[ = (\mu_0 + \mu_1 \mu_1 + \mu_0 \mu_1 \mu_3) / (1 - \mu_0 \mu_1 \mu_3) \]

(8)

where

\[ \mu_0 = \mu_{01} + \mu_{02} \]

\[ \mu_1 = \mu_{13} + \mu_{14} \]

\[ \mu_2 = \mu_{23} + \mu_{23}^{(1)} + \mu_{24} \]

\[ \mu_3 = \mu_{30} + \mu_{33}^{(6)} \]

\[ \mu_4 = \mu_{43} + \mu_{43}^{(5)} \]

Availability analysis
Let $M_i(t)$ be the probability of the system having started from state $i$ is up at time $t$ without making any other regenerative state. By probabilistic arguments, we have

The value of $M_0(t) = e^{-\lambda_1 t} e^{-\lambda_3 t}$, $M_i(t) = e^{-\lambda_i t}$.

The point wise availability $A_i(t)$ have the following recursive relations

$$A_0(t) = M_0(t) + q_{01}(t) [c] A_1(t) + q_{02}(t) [c] A_2(t)$$

$$A_1(t) = M_1(t) + q_{13}(t) [c] A_0(t) + q_{14}(t) [c] A_1(t).$$

$$A_2(t) = \{ q_{23}(t) + q_{23}^{(7)}(t) \} [c] A_1(t) + q_{23}^{(6)}(t) [c] A_3(t)$$

$$A_3(t) = M_3(t) + [ q_{30}(t) + q_{33}^{(6)}(t) ] [c] A_0(t)$$

$$A_4(t) = [ q_{40}(t) + q_{43}^{(5)}(t) ] [c] A_3(t).$$

Taking Laplace Transform of eq. (10-14) and solving for $\tilde{A}_0(s)$

$$\tilde{A}_0(s) = \frac{N_2(s)}{D_2(s)} \tag{15}$$

where

$$N_2(s) = (1 - \tilde{q}_{33}^{(6)}(s)) \tilde{M}_d(s) + [ \tilde{q}_{01}(s) \{ \tilde{M}_1(s) + (\tilde{q}_{13}(s) + \tilde{q}_{14}(s) (\tilde{q}_{43}(s) + \tilde{q}_{43}^{(5)}(s))) + \tilde{q}_{20}(s) ]$$

$$\tilde{q}_{23}(s) + \tilde{q}_{23}^{(7)}(s) + \tilde{q}_{24}(s) (\tilde{q}_{43}(s) + \tilde{q}_{43}^{(5)}(s)) \} \tilde{M}_d(s)$$

$$D_2(s) = (1 - \tilde{q}_{33}^{(6)}(s)) - \tilde{q}_{30}(s) \tilde{q}_{01}(s)$$

$$[ \tilde{q}_{13}(s) + \tilde{q}_{14}(s) (\tilde{q}_{43}(s) + \tilde{q}_{43}^{(5)}(s)) ]$$

$$\tilde{q}_{20}(s) [ \tilde{q}_{23}(s) + \tilde{q}_{23}^{(7)}(s) + \tilde{q}_{24}(s) (\tilde{q}_{43}(s) + \tilde{q}_{43}^{(5)}(s)] \} \tilde{M}_d(s)$$

The steady state availability

$$A_0 = \lim_{t\to\infty} [A_0(t)]$$

$$= \lim_{s\to 0} \frac{N_2(s)}{D_2(s)} \tag{16}$$

where

$$N_2(0) = p_{30} \tilde{M}_d(0) + p_{01} \tilde{M}_1(0) \tilde{M}_d(0)$$

$$D_2(0) = \mu_3 + [ \mu_0 + p_{01} (\mu_1 + p_{14} \mu_4 + p_{02}( \mu_2 + p_{24} \mu_4 ) ] \tilde{p}_{30}$$

The expected up time of the system in $(0,t]$ is

$$\lambda_u(t) = \int_0^t A_0(\tau) d\tau \quad \text{So that} \quad \tilde{\lambda}_u(s) = \frac{\tilde{A}_0(s)}{s} = \frac{N_2(s)}{s D_2(s)} \tag{17}$$

The expected down time of the system in $(0,t]$ is

$$\lambda_d(t) = t - \lambda_u(t) \quad \text{So that} \quad \tilde{\lambda}_d(s) = \frac{1}{s^2} - \tilde{\lambda}_u(s) \tag{18}$$

The expected busy period of the server when the operation of the unit stops automatically when there is radar failure in $(0,t]$ is

$$R_0(t) = q_{01}(t) [c] R_1(t) + q_{02}(t) [c] R_2(t)$$

$$R_1(t) = S_1(t) + q_{13}(t) [c] R_3(t) + q_{14}(t) [c] R_4(t)$$

$$R_2(t) = S_2(t) + q_{23}(t) [c] R_3(t) + q_{23}^{(7)}(t) [c] R_3(t) + q_{24}(t) [c] R_4(t)$$

$$R_3(t) = q_{30}(t) [c] R_0(t) + q_{33}^{(6)}(t) [c] R_0(t)$$

$$R_4(t) = S_4(t) + (q_{43}(t) + q_{43}^{(5)}(t)) [c] R_0(t) \tag{19-23}$$

where

$$S_1(t) = e^{-\lambda_1 t} e^{-\lambda_3 t} t$$

$$S_2(t) = e^{-\lambda_1 t} e^{-\lambda_4 t} \tilde{G}_1(t)$$

$$S_4(t) = e^{-\lambda_1 t} e^{-\lambda_4 t} \tilde{G}_1(t) \tag{24}$$

Taking Laplace Transform of eq. (19-23) and solving for $\tilde{R}_0(s)$

$$\tilde{R}_0(s) = \frac{N_3(s)}{D_2(s)} \tag{25}$$

where

$$N_3(s) = (1 - \tilde{q}_{33}^{(6)}(s)) \{ \tilde{q}_{01}(s) \tilde{S}_1(s) + \tilde{q}_{14}(s) \tilde{S}_4(s) + \tilde{q}_{23}(s) \tilde{S}_2(s) + \tilde{q}_{23}^{(7)}(s) \tilde{S}_3(s) + \tilde{q}_{24}(s) \tilde{S}_2(s) \}$$

$$\tilde{S}_4(s)$$

and $D_2(s)$ is already defined.

In the long run, $R_0 = \frac{N_3(0)}{D_2(0)} \tag{26}$

where

$$N_3(0) = p_{30} [ p_{01} (\tilde{S}_1(0) + p_{14} \tilde{S}_4(0) ) + p_{02} (\tilde{S}_2(0) + p_{24} \tilde{S}_3(0) )$$

and $D_2(0)$ is already defined.

The expected period of the system under radar failure in $(0,t]$ is

$$\lambda_{r1}(t) = \int_0^t R_0(\tau) d\tau \quad \text{so that} \quad \tilde{\lambda}_{r1}(s) = \frac{\tilde{R}_0(s)}{s} \tag{27}$$

The expected busy period of the server for repair when there is radar failure in $(0,t]$ is

$$B_0(t) = q_{01}(t) [c] B_1(t) + q_{02}(t) [c] B_2(t)$$

$$B_1(t) = q_{13}(t) [c] B_3(t) + q_{14}(t) [c] B_4(t)$$

$$B_2(t) = q_{23}(t) [c] B_3(t) + q_{23}^{(7)}(t) [c] B_3(t) + q_{24}(t) [c] B_4(t)$$

$$B_3(t) = T_3(t) + q_{30}(t) [c] B_0(t) + q_{33}^{(6)}(t) [c] B_0(t)$$

$$B_4(t) = T_4(t) + [ q_{43}(t) + q_{43}^{(5)}(t) ] [c] B_0(t) \tag{28-32}$$

where

$$T_3(t) = e^{-\lambda_1 t} e^{-\lambda_3 t} \tilde{G}_1(t)$$

$$T_4(t) = e^{-\lambda_1 t} e^{-\lambda_4 t} \tilde{G}_1(t) \tag{33}$$
Taking Laplace Transform of eq. (28-32) and solving for $B_0(s)$

$$\bar{B}_0(s) = \frac{N_d(s)}{D_2(s)} \quad (34)$$

where

$$N_d(s) = \overline{\lambda r}(s) - q_0(t) \overline{\lambda r}(s) = \frac{\bar{B}_0(s)}{s}$$

And $D_2(s)$ is already defined.

In steady state, $B_0 = \frac{N_d(0)}{D_2(0)} \quad (35)$

where $N_d(0)=\overline{\lambda r}_0(0)+\overline{\lambda r}_d(0) \{ p_{30} (p_{01} p_{14} + p_{02} p_{24} ) \} \text{ and } D_2(0)$ is already defined.

The expected busy period of the server for repair in $[0,t]$ is

$$\lambda_{ru}(t) = \int_{0}^{t} \bar{B}_0(z)dz \quad \text{So that } \lambda_{ru}(s) = \frac{\bar{B}_0(s)}{s} \quad (36)$$

The expected busy period of the server for repair when there is all telephone lines failure in $[0,t]$

$$P_0(t) = \overline{q_0(t)|c|P_1(t) + \overline{q_0(t)|c|P_2(t)$$

$$P_1(t) = q_1(t)|c|P_1(t) + \overline{q_3(t)|c|P_1(t)$$

$$P_2(t) = L_2(t) + q_3(t)|c|P_2(t) + \overline{q_3(t)|c|P_2(t)$$

$$P_3(t) = q_0(t)|c|P_3(t) + q_3(t)|c|P_3(t)$$

$$P_4(t) = q_3(t)|c|P_3(t) + q_3(s)|c|P_3(t) \quad (37-41)$$

where $L_2(t) = e^{-\lambda_1 t} \overline{\lambda r}_d(t)$

Taking Laplace Transform of eq. (37-41) and solving for $\bar{P}_0(s)$

$$\bar{P}_0(s) = \frac{N_d(s)}{D_2(s)} \quad (43)$$

where $N_d(s) = \overline{q_0(t)} \overline{L}_2(s) \{ 1 - \overline{q_3(s)} \}$ and $D_2(s)$ is defined earlier.

In the long run, $P_0 = \frac{N_d(0)}{D_2(0)} \quad (44)$

where $N_d(0)=p_{30} p_{01} L_2(0)$ and $D_2(0)$ is already defined.

The expected busy period of the server for repair when failure caused due to all telephone lines in $[0,t]$ is

$$\lambda_{ru}(t) = \int_{0}^{t} \bar{P}_0(z)dz \quad \text{So that } \lambda_{ru}(s) = \frac{\bar{P}_0(s)}{s} \quad (45)$$

The expected number of visits by the repairman for repairing the units when there is radar failure in $[0,t]$

$$H_0(t) = Q_0(t)[s]H_1(t) + Q_0(t)[s]H_2(t)$$

$$H_1(t) = Q_0(t)[s][1+H_1(t)] + Q_0(t)[s][1+H_2(t)]$$

$$H_2(t) = \{Q_2(t) + Q_3(t)[s]\} [s][1+H_1(t)] + Q_2(t)[s][1+H_2(t)]$$

$$H_3(t) = Q_0(t)[s]H_4(t) + Q_3(t)[s]H_3(t)$$

$$H_4(t) = (Q_0(t)+Q_3(s)[s]) [s]H_4(t) \quad (46-50)$$

Taking Laplace Transform of eq. (46-50) and solving for $H_0(s)$

$$H_0(s) = \frac{N_d(s)}{D_3(s)} \quad (51)$$

where

$$N_d(s) = \{1 - Q_3(s)^{1/3}\} \{Q_0(s)[s]Q_1(s) + Q_1(t)[s]\} + Q_0(t)[s] \quad (52)$$

where $N_d(0)=p_{30}$ and $D_3(0)$ is already defined.

The expected number of visits by the repairman for repairing when there is all telephone lines failure in $[0,t]$

$$V_0(t) = Q_0(t)[s]V_1(t) + Q_0(t)[s][1+V_2(t)]$$

$$V_1(t) = Q_3(t)[s]V_2(t) + Q_3(t)[s]V_3(t)$$

$$V_2(t) = Q_0(t)[s][1+V_4(t)] + [Q_2(t) + Q_3(t)[s][1+V_4(t)]$$

$$V_3(t) = Q_0(t)[s]V_4(t) + Q_3(t)[s]V_3(t) \quad (53-57)$$

Taking Laplace-Stieltjes transform of eq. (53-57) and solving for $V_0(s)$

$$V_0(s) = \frac{N_d(s)}{D_4(s)} \quad (58)$$

where $N_d(s) = \{1 - Q_3(s)^{1/3}\} \{Q_0(s)[s]Q_1(s) + Q_1(t)[s]\} + Q_0(t)[s] \quad (52)$

and $D_4(s)$ is the same as $D_3(s)$

In the long run, $V_0 = \frac{N_d(0)}{D_4(0)} \quad (59)$

where $N_d(0)=p_{30} [p_{01} p_{14} + p_{02} p_{24}]$ and $D_4(0)$ is already defined.

Cost Benefit Analysis

The cost-benefit function of the system considering mean up-time, expected busy period of the system under radar failure when the units stops automatically, expected busy period of the server for repair when there is radar failure, expected total repair cost for repairing
the units when there is all telephone lines failure, expected number of visits by the repairman when there is radar failure, expected number of visits by the repairman for all telephone lines failure.

The expected total cost-benefit incurred in (0,t] is
\[ C(t) = \text{Expected total revenue in } (0,t] \]
- expected busy period of the system under radar failure when the units automatically stop in (0,t]
- expected total repair cost when there is radar failure in (0,t]
- expected total repair cost for repairing the units when there is all telephone lines failure in (0,t]
- expected number of visits by the repairman for repairing when there is radar failure in (0,t]
- expected number of visits by the repairman for repairing the units when there is all telephone lines failure in (0,t]

The expected total cost per unit time in steady state is
\[
C = \lim_{t \to \infty} \left( C(t) / t \right) = \lim_{s \to 0} \left( s^2 C(s) \right) = K_1 A_0 - K_2 R_0 - K_3 B_0 - K_4 P_0 - K_5 H_0 - K_6 V_0
\]

Where
- \( K_1 \) - revenue per unit up-time,
- \( K_2 \) - cost per unit time for which the system is under radar failure when units automatically stop.
- \( K_3 \) - cost per unit time for which the system is under unit repair when there is radar failure
- \( K_4 \) - cost per unit time for which the system failure due to all telephone lines failure
- \( K_5 \) - cost per visit by the repairman when there is radar failure,
- \( K_6 \) - cost per visit by the repairman when there is all telephone lines failure.

**CONCLUSION**

After studying the system, we have analyzed graphically that when the failure rate due to operation of the unit stops automatically, due to radar failure and due to all telephone lines increases, the MTSF and steady state availability decreases and the cost function decreased as the failure increases.

**REFERENCES**


