Profit-Function of Two- Similar Cold Standby Nuclear Missiles System subject to failure due to engineering or failure shuts down Squadron of Nuclear Missiles

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Abstract- Failure Shuts Down Squadron of Nuclear Missiles

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President Obama was briefed this morning on an engineering failure at F.E. Warren Air Force Base in Wyoming that took 50 nuclear intercontinental ballistic missiles (ICBMs), one-ninth of the U.S. missile stockpile, temporarily offline on Saturday in their bunkers could no longer communicate with the missiles themselves. LF Down status also means that various security protocols built into the missile delivery system, like intrusion alarms and warhead separation alarms, were offline. In LF Down status, the missiles are still technically launch-able, but they can only be controlled by an airborne command and control platform like the E-4B NAOC aircraft or perhaps the TACAMO fleet, which is primarily used to communicate with nuclear submarines. Had the country been placed on a higher state of nuclear alert, those platforms would be operating automatically because the frequencies used to transmit nuclear codes would be interfacing with separate systems, according to officials.

According to the official, engineers believe that a launch control center computer (LCC), responsible for a package of at least five missiles, usually ten of them, began to "ping" out of sequence, resulting in a surge of "noise" through the system. The LCCs interrogate each missile in sequence, so if they begin to send signals out when they're not supposed to, receivers on the missiles themselves will notice this and send out error codes.

In this paper we have taken failure due to engineering or failure causing shuts down Squadron of Nuclear Missiles. When the main unit fails due to engineering then cold standby system becomes operative. Failure due to engineering cannot occur simultaneously in both the units and after failure the unit undergoes very costly repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

Keywords: Cold Standby, failure due to engineering, failure causing shuts down Squadron of Nuclear Missiles, first come first serve, MTSF, Availability, Busy period, Benefit -Function.

INTRODUCTION

Since LCCs ping out of sequence on occasion, missileers tried quick fixes. But as more and more missiles began to display error settings, they decided to take off-line all five LCCs that the malfunctioning center was connected to. That left 50 missiles in the dark. The missileers then restarted one of the LCCs, which began to normally interrogate the missile transceiver. Three other LCCs were successfully restarted. The suspect LCC remains off-line.

Commanders at the Air Force Base sent warning notices to colleagues at the country's two other nuclear missile command centers, as well as the to the National Military Command Center in Washington. At that point, they did not know what was causing the failure, and they did not know whether other missile systems were experiencing similar symptoms.

According to the official, engineers discovered that similar hardware failures had triggered a similar cascading failure 12 years ago at Minot AFB in North Dakota and Malmstrom AFB in Montana. That piece of hardware is the prime suspect.

The defense official said that there had not been a power failure, though the official acknowledged that that explanation had made its way through public affairs channels. Engineers working on the system presented a draft of their initial findings late this afternoon, the official said.

An administration official, speaking about the president's ability to control nuclear forces, said: "At no time did the president's ability decrease," an administration official said. "

Still, the Chairman of the Joint Chiefs of Staff, Adm. Michael Mullen, was immediately notified on Saturday, and he, in turn, briefed Secretary of Defense Robert Gates.
"We've never had something as big as this happen," a military officer who was briefed on the incident said. Occasionally, one or two might blink out, the officer

said, and several warheads are routinely out of service for maintenance. At an extreme, "[w]e can deal with maybe 5, 6, or 7 at a time, but we've never lost complete command and control and functionality of 50 ICBMs."There was a temporary interruption and the missiles themselves were always protected by multiple, redundant, safety, security and command and control features. At no time was there any danger to the public,"she said.

Another military official said the failure triggered an emergency inspection protocol, and sentries were dispatched to verify in person that all of the missiles were safe and properly protected.

When on alert, the missiles are the property of the U.S. Strategic Command, which controls all nuclear forces. When not on alert status, the missiles are under the control of a subordinate organization, the Global Strike Command,

A White House spokesperson referred questions about the incident to the Office of the Secretary of Defense and to the Air Force. A spokesperson for the Global Strike Command did not immediately respond to questions.

The cause of the failure remains unknown, although it is suspected to be a breach of underground cables deep beneath the base, according to a senior military official.

It is next to impossible for these systems to be hacked, so the military does not believe the incident was caused by malicious actors. Half dozen individual silos were affected by Saturday's failure.

There are about 450 ICBMs in America's nuclear arsenal, some of them bearing multiple warheads. 150 are based at Minot and about 150 are housed at Malmstrom AFB in Montana. The chessboard of nuclear deterrence, a game-theory-like intellectual contraption that dates from the Cold War, is predicated upon those missiles being able to target specific threat locations across the world. If a squadron goes down, that means other missiles have to pick up the slack. The new START treaty would reduce the number of these missiles by 30 percent, but the cuts are predicated upon the health of the current nuclear stockpile, from warhead to delivery system to command and control.

An administration official said that "to make too much out of this would be to sensationalize it. It's not that big of a deal. Everything worked as planned."

Senate Republicans have been pressing Senate Democrats to spend more money ensuring the current strategic nuclear arsenal, which dates to the early 1980s, is ready to go. The treaty requires the vote of two-thirds of the Senate to be ratified.

In 2008, Gates fired the Secretary of the Air Force and its chief of staff after a series of incidents suggested to Gates that the service wasn't taking its nuclear duties seriously enough. At one point, a B-52 bomber flew across the continental U.S. without realizing that its nuclear weapons were "hot."

National Journal's Megan Scully contacted a spokesperson for Sen. Jon Kyl, a top GOP critic of START, who said that "We don't know what happened and why." The spokesperson refused to comment on "media reports."

The base is a main locus of the United States' strategic nuclear forces. The 90th Missile Wing headquartered there, controls 150 Minuteman III intercontinental ballistic nuclear missiles. They're on full-time alert and are housed in a variety of bunkers across several states.

On Saturday morning, according to people briefed on what happened, a squadron of ICBMs suddenly dropped down into what's known as "LF Down" status, meaning that the missileers

The military contends that command and control -- "C2" in their parlance -- was not lost.

An Air Force spokesperson, Christy Nolta, said the failure lasted less than an hour. "

Stochastic behavior of systems operating under changing environments has widely been studied. . Dhillon , B.S. and Natesan, J. (1983) studied an outdoor power systems in fluctuating environment . Kan Cheng (1985) has studied reliability analysis of a system in a randomly changing environment. Jinhua Cao (1989) has studied a man machine system operating under changing environment subject to a Markov process with two states. The change in operating conditions viz. fluctuations of voltage, corrosive atmosphere, very low gravity etc. may make a system completely inoperative. Severe environmental conditions can make the actual mission duration longer than the ideal mission duration. In this paper we have taken **failure due to engineering or failure causing shuts down Squadron of Nuclear Missiles.** When the main operative unit fails then cold standby system becomes operative. **Failure due to engineering** cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of very high cost in case of **failure due to engineering** immediately. The repair is done on the basis of first fail first repaired.

Assumptions

1. λ_1 , λ_2, are constant failure rates for **failure due to engineering, failure causing shuts down Squadron of Nuclear Missiles** respectively. The CDF of repair time distribution of Type I and Type II are $G_1(t)$ and $G_2(t)$.
2. The failure due to **failure due to engineering** is non-instantaneous and it cannot come simultaneously in both the units.
3. The repair starts immediately after the **failure due to engineering or failure causing shuts down Squadron of Nuclear Missiles** works on the principle of first fail first repaired basis.

4. The repair facility does no damage to the units and after repair units are as good as new.
5. The switches are perfect and instantaneous.
6. All random variables are mutually independent.
7. When both the units fail, we give priority to operative unit for repair.
8. Repairs are perfect and failure of a unit is detected immediately and perfectly.
9. The system is down when both the units are non-operative.

Notations

λ_1, λ_2 are the **failure due to engineering, failure causing shuts down Squadron of Nuclear Missiles** respectively. $G_1(t)$, $G_2(t)$ – repair time distribution Type -I, Type-II due to **engineering failure, failure causing shuts down Squadron of Nuclear Missiles** respectively.

p, q - probability of **failure due to engineering, failure causing shuts down Squadron of Nuclear Missiles** respectively such that p+ q=1

$M_i(t)$ System having started from state i is up at time t without visiting any other regenerative state

$A_i(t)$ state is up state as instant t

R_i (t) System having started from state i is busy for repair at time t without visiting any other regenerative state.

$B_i(t)$ the server is busy for repair at time t.

$H_i(t)$ Expected number of visits by the server for repairing given that the system initially starts from regenerative state i

Symbols for states of the System

Superscripts O, CS, EF, SDNMF

Operative, Cold Standby, failure due to engineering, failure causing shuts down Squadron of Nuclear Missiles respectively

Subscripts nsdnm, sdnm, ef, ur, wr, uR

No failure due to failure due to failure causing shuts down Squadron of Nuclear Missiles, failure causing shuts down Squadron of Nuclear Missiles, engineering failure, under repair, waiting for repair, under repair continued from previous state respectively

Up states – 0, 1, 2, 7, and 8;

Down states – 3, 4, 5, 6

regeneration point – 0,1,2, 7, 8

States of the System

0(O_{nsdnm}, CS_{nsdnm})

One unit is operative and the other unit is cold standby and there is no failure causing shuts down Squadron of Nuclear Missiles in both the units.

1($SDNMF_{sdnm,\ ur}$, O_{nsdnm})

The operating unit fails due to shuts down Squadron of Nuclear Missiles and is under repair immediately of Type- II and standby unit starts operating with no failure causing shuts down Squadron of Nuclear Missiles

2($EF_{ef,\ ur}$, O_{nsdnm})

The operative unit fails due to EF resulting from failure due to engineering and undergoes repair of very costly Type I and the standby unit becomes operative with no failure causing shuts down Squadron of Nuclear Missiles .

3($SDNMF_{sdnm,uR}$, $EF_{ef,wr}$)

The first unit fails due to shuts down Squadron of Nuclear Missiles and under Type-II repair is continued from state 1 and the other unit fails due to EF resulting from failure due to engineering and is waiting for repair of very costly Type -I.

4($SDNMF_{sdnm,uR}$, $SDNMF_{sdnm,wr}$)

The repair of the unit is failed due to SDNMF resulting from failure due to failure causing shuts down Squadron of Nuclear Missiles is continued from state 1and the other unit failed due to SDNMF resulting from failure due to failure causing shuts down Squadron of Nuclear Missiles is waiting for repair of Type-II.

5($EF_{ef,\ uR}$, $EF_{ef,\ wr}$)

The operating unit fails due to failure due to engineering (EF mode) and under repair of very costly Type - I continue from the state 2 and the other unit fails also due to failure due to engineering is waiting for repair of very costly Type- I.

6($EF_{ef,uR}$, $SDNMF_{sdnm,wr}$)

The operative unit fails due to EF resulting from failure due to engineering and under repair continues from state 2 of Type –I and the other unit is failed due to SDNMF resulting from failure due to failure causing shuts down Squadron of Nuclear Missiles and under Type-II

7(O_{nsdnm} , $SDNMF_{sdnm,ur}$)

The repair of the unit failed due to SDNMF resulting from failure due to shuts down Squadron of Nuclear Missiles failure is completed and there is no failure due to engineering and the other unit is failed due to SDNMF resulting from failure due to failure causing shuts down Squadron of Nuclear Missiles is under repair of Type-II

8(O_{nsdnm} , $EF_{ef,ur}$)

The repair of the unit failed due to SDNMF resulting from failure causing shuts down Squadron of Nuclear Missiles failure is completed and there is no failure due to engineering and the other unit is failed due to EF

resulting from failure due to engineering is under repair of Type-I.

Transition Probabilities

Simple probabilistic considerations yield the following expressions:

$p_{01} = p,\quad p_{02} = q,$

$p_{10} = pG_1^*(\lambda_1)+q\,G_1^*(\lambda_2)= p_{70},$

$p_{20} = pG_2^*(\lambda_1)+q\,G_2^*(\lambda_2)= p_{80},$

$p_{11}^{(3)}= p(1- G_1^*(\lambda_1))= p_{14} = p_{71}^{(4)}\ p_{28}^{(5)}= q(1- G_2^*(\lambda_2))= p_{25} = p_{82}^{(5)}$ (1)

We can easily verify that

$p_{01} + p_{02} = 1,$

$p_{10} + p_{17}^{(4)} (= p_{14}) + p_{18}^{(3)} (= p_{13}) = 1,$

$p_{80} + p_{82}^{(5)} + p_{87}^{(6)} = 1$ (2)

And mean sojourn time is

$\mu_0 = E(T) = \int_0^\infty P[T > t]dt$

Mean Time To System Failure

$\emptyset_0(t) = Q_{01}(t)[s]\ \emptyset_1(t) + Q_{02}(t)[s]\ \emptyset_2(t)$

$\emptyset_1(t) = Q_{10}(t)[s]\ \emptyset_0(t) + Q_{13}(t) + Q_{14}(t)$

$\emptyset_2(t) = Q_{20}(t)[s]\ \emptyset_0(t) + Q_{25}(t) + Q_{26}(t)$ (3-5)

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-5) and solving for

$\emptyset_0^*(s) = N_1(s) / D_1(s)$ (6)

where

$N_1(s) = Q_{01}^*[\,Q_{13}^*(s) + Q_{14}^*(s)\,] + Q_{02}^*[\,Q_{25}^*(s) + Q_{26}^*(s)\,]$

$D_1(s) = 1 - Q_{01}^*\ Q_{10}^* - Q_{02}^*\ Q_{20}^*$

Making use of relations (1) & (2) it can be shown that $\emptyset_0^*(0) = 1$, which implies that $\emptyset_0(t)$ is a proper distribution.

$$MTSF = E[T] = \left.\frac{d}{ds}\,\emptyset_0^*(s)\right|_{s=0}$$

$$= (D_1'(0) - N_1'(0)) / D_1(0)$$

$$= (\mu_0 + p_{01}\ \mu_1 + p_{02}\ \mu_2) / (1 - p_{01}\,p_{10} - p_{02}\,p_{20})$$

where

$\mu_0 = \mu_{01} + \mu_{02},$

$\mu_1 = \mu_{01} + \mu_{17}^{(4)} + \mu_{18}^{(3)},$

$\mu_2 = \mu_{02} + \mu_{27}^{(6)} + \mu_{28}^{(5)}$

Availability analysis

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

$M_0(t) = e^{-\lambda_1 t}e^{-\lambda_2 t},$

$M_1(t) = p\,G_1(t)\,e^{-(\lambda_1+\lambda_2)} = M_7(t)$

$M_2(t) = q\,G_2(t)\,e^{-(\lambda_1+\lambda_2)} = M_8(t)$

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$

$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{18}^{(3)}(t)[c]A_8(t) + q_{17}^{(4)}(t)[c]A_7(t)$

$A_2(t) = M_2(t) + q_{20}(t)[c]A_0(t) + [q_{28}^{(5)}(t)[c]\,A_8(t) + q_{27}^{(6)}(t)]\,[c]A_7(t)$

$A_7(t) = M_7(t) + q_{70}(t)[c]A_0(t) + [q_{71}^{(4)}(t)[c]\,A_1(t) + q_{78}^{(3)}(t)]\,[c]A_8(t)$

$A_8(t) = M_8(t) + q_{80}(t)[c]A_0(t) + [q_{82}^{(5)}(t)[c]\,A_2(t) + q_{87}^{(6)}(t)]\,[c]A_7(t)$ (7-11)

Taking Laplace Transform of eq. (7-11) and solving for $\hat{A}_0(s)$

$$\hat{A}_0(s) = N_2(s) / D_2(s)$$ (12)

where

$N_2(s) = \hat{M}_0(1 - \hat{q}_{78}^{(3)} - \hat{q}_{87}^{(6)}) - \hat{q}_{82}^{(5)}$

$(\hat{q}_{27}^{(6)}\,\hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)} - \hat{q}_{71}^{(4)}$

$(\hat{q}_{17}^{(4)} + \hat{q}_{87}^{(6)}\,\hat{q}_{18}^{(3)}) + \hat{q}_{71}^{(4)}\,\hat{q}_{82}^{(5)}$

$(\hat{q}_{17}^{(4)} - \hat{q}_{27}^{(6)}\,\hat{q}_{18}^{(3)})] + \hat{q}_{01}[\hat{M}_1(1 -$

$\hat{q}_{78}^{(3)}\,\hat{q}_{87}^{(6)}) + \hat{q}_{71}^{(4)}(\hat{M}_7 + \hat{q}_{78}^{(3)}$

$\hat{M}_8) + \hat{q}_{18}^{(3)}(\hat{M}_7\,\hat{q}_{87}^{(6)} - \hat{M}_8) -$

$\hat{q}_{82}^{(5)}(\hat{M}_1(\hat{q}_{27}^{(6)}\,\hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) +$

$\hat{q}_{17}^{(4)}(-\hat{M}_2(\hat{q}_{78}^{(3)} + \hat{M}_7\hat{q}_{28}^{(5)}) -$

$\hat{q}_{18}^{(3)}(\hat{M}_2 + \hat{M}_7\hat{q}_{27}^{(6)})\}]\ \hat{q}_{02}[\hat{M}_2(1 - \hat{q}_{78}^{(3)}\ \hat{q}_{87}^{(6)}) + \hat{q}_{27}^{(6)}($

$\hat{M}_7 + \hat{q}_{78}^{(3)}\,\hat{M}_8) + \hat{q}_{28}^{(5)}(\hat{M}_7\,\hat{q}_{87}^{(6)} + \hat{M}_8) - \hat{q}_{71}^{(4)}($

$\hat{M}_1(- \hat{q}_{27}^{(6)} - \hat{q}_{28}^{(5)} +$

$\hat{q}_{87}^{(6)}) + \hat{q}_{17}^{(4)}(\hat{M}_2 + \hat{q}_{28}^{(5)}\,\hat{M}_8) - \hat{q}_{18}^{(3)}(-\hat{M}_2\,\hat{q}_{87}^{(6)} +$

$\hat{M}_8\,\hat{q}_{27}^{(6)})\}]$

$\hat{q}_{18}^{(3)}(\hat{M}_2 + \hat{M}_7\hat{q}_{27}^{(6)})\}]$

$D_2(s) = (1 - \hat{q}_{78}^{(3)} - \hat{q}_{87}^{(6)}) - \hat{q}_{82}^{(5)}($

$\hat{q}_{27}^{(6)}\ \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) - \hat{q}_{71}^{(4)}$

$(\hat{q}_{17}^{(4)} + \hat{q}_{87}^{(6)}\ \hat{q}_{18}^{(3)}) + \hat{q}_{71}^{(4)}\ \hat{q}_{82}^{(5)}\ (\hat{q}_{17}^{(4)}\ \hat{q}_{28}^{(5)} - \hat{q}_{18}^{(3)})] + \hat{q}_{01}[-\hat{q}_{10}\ (1 -$

$\hat{q}_{78}^{(3)}\ \hat{q}_{87}^{(6)}) - \hat{q}_{71}^{(4)}(\ \hat{q}_{70} + \hat{q}_{78}^{(3)}$

$\hat{q}_{80}) - \hat{q}_{18}^{(3)}(\ \hat{q}_{70}\ \hat{q}_{87}^{(6)} - \hat{q}_{80}) -$

$\hat{q}_{82}^{(5)}(-\hat{q}_{10}(\ \hat{q}_{27}^{(6)}\ \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) +$

$\hat{q}_{17}^{(4)}(\hat{q}_{20}\ (\ \hat{q}_{78}^{(3)} - \hat{q}_{70}\hat{q}_{28}^{(5)}) +$

$\hat{q}_{18}^{(3)}(\ \hat{q}_{20} + \hat{q}_{70}\hat{q}_{27}^{(6)})\}]\ \hat{q}_{02}[-\hat{q}_{20}(1 - \hat{q}_{78}^{(3)}\ \hat{q}_{87}^{(6)})$

$-\hat{q}_{27}^{(6)}(\ \hat{q}_{70} + \hat{q}_{78}^{(3)}\ \hat{q}_{80}) - \hat{q}_{28}^{(5)}(\ \hat{q}_{70}\hat{q}_{87}^{(6)} + \hat{q}_{80}) -$

$\hat{q}_{71}^{(4)}($

$\hat{q}_{10}\ (\ \hat{q}_{27}^{(6)} + \hat{q}_{28}^{(5)}\ \hat{q}_{87}^{(6)}) - \hat{q}_{17}^{(4)}(\ \hat{q}_{20} -$

$\hat{q}_{28}^{(5)}\ \hat{q}_{80}) - \hat{q}_{18}^{(3)}(\ \hat{q}_{20}\ \hat{q}_{87}^{(6)} + \hat{q}_{80}$

$\hat{q}_{27}^{(6)})\}]$

(Omitting the arguments s for brevity)

The steady state availability

$A_0 = \lim_{t\to\infty}[A_0(t)]$

$= \lim_{s\to 0}[s\,\hat{A}_0(s)] = \lim_{s\to 0}\frac{s\,N_2(s)}{D_2(s)}$

Using L' Hospitals rule, we get

$A_0 = \lim_{s\to 0}\frac{N_2(s)+s\,N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)}$ (13)

The expected up time of the system in (0,t] is

$\lambda_u(t) = \int_0^\infty A_0(z)dz$

So that $\bar{\lambda}_u(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)}$ (14)

The expected down time of the system in (0,t] is

$\lambda_d(t) = t - \lambda_u(t)$

So that $\bar{\lambda}_d(s) = \frac{1}{s^2} - \bar{\lambda}_u(s)$ (15)

The expected busy period of the server when there is EF - Failure due to engineering or SDNMF- failure causing shuts down Squadron of Nuclear Missiles in (0,t]

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t)$

$R_1(t) = S_1(t) + q_{10}(t)[c]R_0(t) +$

$q_{18}^{(3)}(t)[c]\,R_8(t) + q_{17}^{(4)}(t)[c]R_7(t)$

$R_2(t) = S_2(t) + q_{20}(t)[c]R_0(t) + q_{28}^{(5)}(t)$

$R_8(t)\ +q_{27}^{(6)}(t)][c]R_7(t)$

$R_7(t) = S_7(t) + q_{70}(t)[c]R_0(t) + Q_{71}^{(4)}(t)$

$R_1(t)\ +q_{78}^{(3)}(t)][c]R_8(t)$

$R_8(t) = S_8(t) + q_{80}(t)[c]R_0(t) + Q_{82}^{(5)}(t)$

$R_2(t)\ +q_{87}^{(6)}(t)][c]R_7(t)$ (16-20)

Taking Laplace Transform of eq. (16-20) and solving for $\bar{R}_0(s)$

$\bar{R}_0(s) = N_3(s)\ /\ D_2(s)$ (21)

where

$N_3(s) = \hat{q}_{01}[\,\hat{S}_1(1 - \hat{q}_{78}^{(3)}\ \hat{q}_{87}^{(6)}) +$

$\hat{q}_{71}^{(4)}(\,\hat{S}_7 + \hat{q}_{78}^{(3)}\,\hat{S}_8) + \hat{q}_{18}^{(3)}(\,\hat{S}_7$

$\hat{q}_{87}^{(6)} - \hat{S}_8)] - \hat{q}_{01}\hat{q}_{82}^{(5)}(\,\hat{S}_1\ \hat{q}_{27}^{(6)}\ \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) + \hat{q}_{17}^{(4)}(\hat{S}_2\ \hat{q}_{78}^{(3)} + \hat{S}_7\hat{q}_{28}^{(5)}) -$

$\hat{q}_{18}^{(3)}(\,\hat{S}_2 + \hat{S}_7\hat{q}_{27}^{(6)})] + \hat{q}_{02}[\hat{S}_2(1 -$

$\hat{q}_{78}^{(3)}\ \hat{q}_{87}^{(6)}) + \hat{q}_{27}^{(6)}(\,\hat{S}_7 + \hat{q}_{78}^{(3)}\,\hat{S}_8) + \hat{q}_{28}^{(5)}(\,\hat{S}_7\,\hat{q}_{87}^{(6)} + \hat{S}_8) - \hat{q}_{02}\hat{q}_{71}^{(4)}(\,\hat{S}_1(-$

$\hat{q}_{27}^{(6)} - \hat{q}_{28}^{(5)}\,\hat{q}_{87}^{(6)}\,\hat{q}_{17}^{(4)}(\hat{S}_2 + \hat{q}_{28}^{(5)}\,\hat{S}_8) - \hat{q}_{18}^{(3)}(-\hat{S}_2\,\hat{q}_{87}^{(6)} + \hat{S}_8\,\hat{q}_{27}^{(6)})]$

and

$D_2(s)$ is already defined.

(Omitting the arguments s for brevity)

In the long run, $R_0 = \frac{N_3(0)}{D_2'(0)}$ (22)

The expected period of the system under EF - failure due to engineering or SDNMF- failure causing shuts down Squadron of Nuclear Missiles in (0,t] is

$\lambda_{rv}(t) = \int_0^\infty R_0(z)dz$ So that $\bar{\lambda}_{rv}(s) = \frac{\bar{R}_0(s)}{s}$

The expected number of visits by the repairman for repairing the identical units in (0,t]

$H_0(t) = Q_{01}(t)[s][1 + H_1(t)] +$

$Q_{02}(t)[s][1 + H_2(t)]$

$H_1(t) = Q_{10}(t)[s]H_0(t)] + Q_{18}^{(3)}(t)[s]$

$H_8(t) + Q_{17}^{(4)}(t)][s]H_7(t)$,

$H_2(t) = Q_{20}(t)[s]H_0(t) + Q_{28}^{(5)}(t)[s]$

$H_8(t) + Q_{27}^{(6)}(t)][c]H_7(t)$

$H_7(t) = Q_{70}(t)[s]H_0(t) + Q_{71}^{(4)}(t)[s]$

$H_1(t) + Q_{78}^{(3)}(t)][c]H_8(t)$

$H_8(t) = Q_{80}(t)[s]H_0(t) + Q_{82}^{(5)}(t)[s]$

$H_2(t) + Q_{87}^{(6)}(t)][c]H_7(t)$ (23-27)

Taking Laplace Transform of eq. (23-27) and solving for $H_0^*(s)$

$H_0^*(s) = N_4(s)\ /\ D_3(s)$ (28)

In the long run, $H_0 = N_4(0)\ /\ D_3'(0)$ (29)

Benefit- Function Analysis

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to engineering or failure causing shuts down Squadron of Nuclear Missiles, expected number of visits by the repairman for unit failure.

The expected total Benefit-Function incurred in (0,t] is

C(t) = Expected total revenue in (0,t]

- expected busy period of the system under failure due to engineering or failure causing shuts down Squadron of Nuclear Missiles for repairing the units in (0,t]

- expected number of visits by the repairman for repairing of identical the units in (0,t]

The expected total cost per unit time in steady state is

$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2 C(s))$

$= K_1A_0 - K_2R_0 - K_3H_0$

where

K_1 - revenue per unit up-time,

K_2 - cost per unit time for which the system is under repair of type- I or type- II

K_3 - cost per visit by the repairman for units repair.

CONCLUSION

After studying the system , we have analyzed graphically that when the failure rate due to failure causing shuts down Squadron of Nuclear Missiles or failure due to engineering increases, the MTSF and steady state availability decreases and the Benefit-function decreased as the failure increases.

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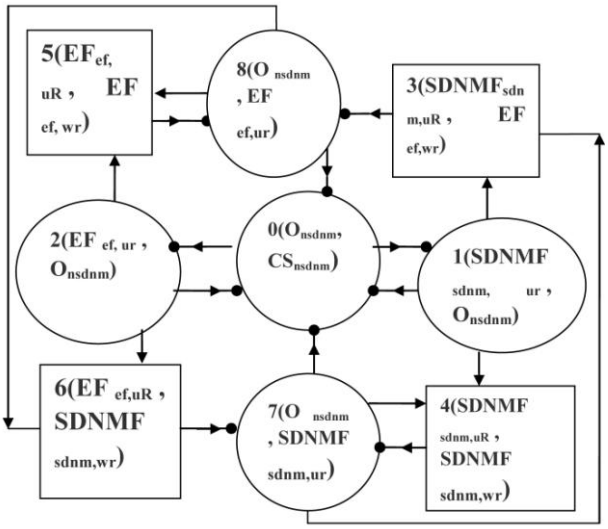


Fig. The State Space Diagram

◯ Up state ▭ down state

• Regeration point

◈◈◈