Profit-Function of Two Similar Warm Standby System subject to failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence and controls damaged by structural failure with different repair facilities.

Ashok Kumar Saini

BLJS COLLEGE, TOSHAM (BHIWANI) HARYANA, INDIA
Email : drashokksaini2009@gmail.com

Abstract - In this paper we have taken failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence and controls damaged by structural failure with different repair facilities. When the main unit fails then warm standby system becomes operative. Failure due to controls damaged by structural failure cannot occur simultaneously in both the units and after failure the unit undergoes Type-I or Type-II or Type-III or Type IV repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

Keywords: Warm Standby, failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence, and controls damaged by structural failure, first come first serve, MTSF, Availability, Busy period, Benefit -Function.

INTRODUCTION

List of Indian Naval accidents

An article in India Today reports that since 1990, the Indian Navy has lost one warship in peacetime every five years. Since 2004, it has lost one naval combatant every two years. While peacetime losses of warships are not uncommon (since the World War II, the US Navy has lost 16 warships in accidents; Russia's nuclear submarine Kursk sank in August 2000 after a faulty torpedo exploded during a training exercise), the magazine mentioned that few global navies have such a dubious record. According to the Times of India, while some of accidents reported since August 2013 were serious, many of them were trivial incidents exaggerated in public.

These accidents have been attributed to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence, and controls damaged by structural failure. However naval commentators also argue that as India's large navy of 160 ships clocks around 12,000 ship-days at sea every year, in varied waters and weather, some incidents are inevitable. Captains of erring ships are dismissed from their command following an enquiry. The accident on board INS Sindhuratna (S59) led to the resignation of the then Chief of Naval Staff (CNS) Admiral D K Joshi on 26 February 2014, who owned moral responsibility.

2000 - 2010

- December 2005: INS Trishul (F43), a Talwar-class frigate, collided with a commercial vessel, Ambuja Laxmi, outside the Mumbai harbor, while returning from a training mission. These class of ships use stealth technologies and a special hull design to ensure a reduced radar cross section. Radar systems installed by the port authorities and those on board the Ambuja Laxmi were unable to detect INS Trishul and prevent the side on collision. No casualties were reported.
- April 2006: INS Prahar (K98), Veer class corvette, sank after colliding with the MV Rajiv Gandhi vessel about 20 nautical miles away from the Goa coast. No casualties were reported. The commanding officer of the ship, Lieutenant Commander Yogesh Tripathi was found guilty of negligence by an Indian Navy court-martial and dismissed from service.
- September 2006: INS Dunagiri (F36), Nilgiri class frigate, collided with a Shipping Corporation of India merchant vessel, the MV Kiti, off the coast of Mumbai. There were no casualties, but the Dunagiri suffered damage and required extensive repairs.
- January 2008: INS Sindhughosh (S55), a Kilo-class submarine, collided with a foreign merchant vessel MV Leeds Castle while trying to surface in waters north of Mumbai. The submarine was taking part in fleet-level war games, when the accident occurred. The Navy termed it a minor incident with no casualties reported.
- August 2009: A collision of the missile corvette INS Kuthar (P46) with destroyer INS Ranvir

(D54) in the Bay of Bengal was traced to a rudder failure, compounded by a flawed maneuver.

2010 - present

- In 2010, three crew men on destroyer INS Mumbai (D62) were instantly killed when an AK-630 Close-in weapon system went off as safety drills were not followed.
- January 2011: INS Vindhyagiri (F42), a Nilgiri-class frigate, capsized after a collision with a Cyprus-flagged merchant vessel MV Nordlake near the Sunk Rock light house, following which a major fire broke out in the ship's engine and boiler room. Everyone on board was evacuated as soon as the fire broke out and hence there were no casualties. INS Vindhyagiri was later decommissioned.
- August 2013: Blasts ripped through the torpedo compartment of the INS Sindhurakshak (S63) while it was berthed at the naval dockyard off the Mumbai coast. Fifteen Sailors and three officers were killed. Other sources state that a small explosion occurred around midnight which then triggered the two larger explosions. The disaster was thought to be the Indian navy's worst since the sinking of the frigate INS Khukri by a Pakistani submarine during the 1971 war.
- December 2013: INS Konkan (M72), a Pondicherry-class minesweeper under the Eastern Naval Command, caught fire at the naval dockyard at Visakhapatnamwhile undergoing repairs. The fire engulfed much of the ship's interior before it was put off. No casualties were reported.
- December 2013: In the second incident in the same month, INS Talwar (F40), the lead ship of the Talwar class frigate of the Indian Navy, collided with a fishing trawler injuring four of the 27 people on board the trawler and sinking it. The fishing trawler was operating without lights. The captain of the ship was subsequently stripped of command.
- December 2013: In the third incident in the same month, INS Tarkash (F50), again a Talwar class frigate, suffered damage to its hull when it hit the jetty while docking at the Mumbai naval base. The navy ordered a board of inquiry.
- January 2014: INS Betwa (F39), an indigenously built Brahmaputra class guided missile frigate, ran aground and collided with an unidentified object while approaching the Mumbai naval base. The sonar system of the frigate was cracked, leading to faulty readings and an ingress of saltwater into sensitive equipment.
- January 2014: In the second incident in the same month, INS Vipul (K46), a veer class corvette of the elite 22nd Killer Missile Vessel Squadron, was detected with a hole in its pillar compartment which forced the ship back into the harbor while it was on an operational deployment.
- February 2014: On 3 February, INS Airavat (L24), a Shardul class amphibious warfare vessel, ran aground while returning to its home base at Visakhapatnam, causing slight damage to its propellers. Following the incident, its commanding officer, Captain JPS Virk, was relieved of command pending the findings of a Board of Inquiry.
- February 2014: On 26 February, INS Sindhuratna (S59), a Kilo-class submarine, had a fire detected on board when trials were being conducted which resulted in smoke leading to suffocation and death of two officers. Seven sailors were reported injured and were airlifted to the naval base hospital in Mumbai. According to the naval board of inquiry, the fire was caused due to problems in the cables of the vessel. This particular incident led to the resignation of Chief of Naval Staff (CNS) Admiral D K Joshi on 26 February 2014, who owned moral responsibility for the incidents in the past few months.
- March 2014: INS Kolkata, had a malfunction on board which led to a toxic gas leak killing Commander Kuntal Wadhwa instantly. It seems that the ship suffered malfunction in its carbon dioxide unit while undergoing machinery trials, leading to gas leakage. Since the ship was not commissioned at the time of the incident, the enquiry into the mishap will be done by Mazagon Dock Limited, where the ship was constructed.
- May 2014: INS Ganga (F22) suffered a minor explosion in the boiler room while undergoing a refit at the Mumbai dockyard. Four people suffered minor injuries. There was no fire and no equipment was damaged.
- November 2014: A Torpedo Recovery Vessel of the Astravahini class (A-73) sank 30 NM off the Vizag coast during a routine mission to recover torpedoes fired by fleet ships during a routine exercise. The accident resulted in the tragic death of one sailor while four others were reported as missing however 23 other personnel were rescued by SAR teams deployed right after the incident.

Controls damaged by structural failure

American Airlines Flight 96, a McDonnell Douglas DC-10, on 12 June 1972. The failure of the rear cargo door caused an explosive decompression, which in turn caused the rear main cabin floor to collapse and severed flight controls. The pilots had only limited ailerons and elevators; the rudder was jammed. The number two engine also ran down to idle at the time of decompression. The aircraft landed safely at Detroit-Metropolitan Airport**.**

- Japan Airlines Flight 123, a Boeing 747, on 12 August 1985. A faulty repair years earlier had weakened the aircraft's rear pressure bulkhead, which failed in flight. The vertical stabilizerand much of the aircraft's empennage was blown off during the decompression. The decompression also ruptured all four hydraulic lines which controlled the aircraft's mechanical flight controls. The pilots were able to continue flying the aircraft with very limited control, but after 32 minutes the aircraft crashed into a mountain, killing 520 of the 524 people aboard in the deadliest single aircraft disaster in history.

- Turkish Airlines Flight 981, a McDonnell Douglas DC-10, on 3 March 1974. Similar to American Airlines Flight 96, the flight experienced a explosive decompression, when flying over the town of Meaux, France, caused by a rear cargo door failure. The rear main cabin floor collapsed and severed all flight controls. While the plane went into a vertical dive, the captain called for "Speed!" meaning increasing engine thrust to pull the plane's nose up. The plane began to level out, but had lost too much altitude and slammed into the Ermenonville Forest. All 346 people on board were killed upon impact, and it became the worst single aircraft disaster without survivors, and the fourth deadliest aviation death count ever.
- Reeve Aleutian Airways Flight 8, a Lockheed L-188 Electra, on 8 June 1983. Flying over Cold Bay, Alaska, the plane's number 4 engine propeller separated and cut a hole in the plane, causing an explosive decompression, jammed flight controls, snapped throttle cables, and left the flight deck crew of three with only autopilot that had no lateral control. After managing to wrench the ailerons and elevators into minimal working condition, the crew tried to land at Anchorage at high speed. They had to make a go-around, but landed on the second attempt, saving all 10 passengers on board. Air Midwest Flight 5481, a Beechcraft 1900D, on 8 January 2003. On takeoff from Charlotte/Douglas International Airport, it pitched up into a vertical ascent and stalled, only 37 seconds later smashing into a US Airways hangar, despite the captain applying full elevator down. There were 21 fatalities. The NTSB found out that the plane had been overweight and that during maintenance, the tension turnbuckles that governed elevator movement had been set incorrectly by an inexperienced mechanic. This caused the elevators to lose control upon takeoff.
- Air Transat Flight 961, an Airbus A310, on 6 March 2005, catastrophic structural failure: the rudder detached from the aircraft with a loud bang. The pilots regained enough control to land the aircraft safely.

Stochastic behavior of systems operating under changing environments has widely been studied. Dhillon , B.S. and Natesan, J. (1983) studied an outdoor power systems in fluctuating environment . Kan Cheng (1985) has studied reliability analysis of a system in a randomly changing environment. Jinhua Cao (1989) has studied a man machine system operating under changing environment subject to a Markov process with two states. The change in operating conditions viz. fluctuations of voltage, corrosive atmosphere, very low gravity etc. may make a system completely inoperative. Severe environmental conditions can make the actual mission duration longer than the ideal mission duration. In this paper we have taken failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence, and controls damaged by structural failure. When the main operative unit fails then warm standby system becomes operative. Failure due to controls damaged by structural failure cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of Type- II by ordinary repairman or Type III, Type IV by multispecialty repairman immediately when failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence.The repair is done on the basis of first fail first repaired.

Assumptions

1. $\lambda_{1,}\lambda_2\ \lambda_3$ are constant failure rates when failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence, and controls damaged by structural failure respectively. The CDF of repair time distribution of Type I, Type II and multispecialty repairmen Type-III, IV are $G_1(t)$, $G_2(t)$ and $G_3(t)$ $G_4(t)$.
2. The failure due to controls damaged by structural failure is non-instantaneous and it cannot come simultaneously in both the units.
3. The repair starts immediately after failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence, and controls damaged by structural failure and works on the principle of first fail first repaired basis. The repair facility does no damage to the units and after repair units are as good as new.
4. The switches are perfect and instantaneous.
5. All random variables are mutually independent.
6. When both the units fail, we give priority to operative unit for repair.
7. Repairs are perfect and failure of a unit is detected immediately and perfectly.
8. The system is down when both the units are non-operative.

Symbols for states of the System

Superscripts O, CS, ASDAF, CDSF,

Operative, Warm Standby, failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence, and controls damaged by structural failure respectively.

Subscripts nasdaf, asdaf, cdsf, ur, wr, uR

No failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence, failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence, failure due to controls damaged by structural failure, under repair, waiting for repair, under repair continued from previous state respectively

Up states – 0, 1, 2, 3, 10 ; Down states – 4, 5, 6, 7,8,9,11

regeneration point – 0,1,2, 3, 8, 9,10

States of the System

0(O_{nasdaf}, CS_{nasdaf}) One unit is operative and the other unit is warm standby and there is no failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence of both the units.

1($ASDAF_{asdaf, urI}$, O_{nasdaf}) The operating unit failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence is under repair immediately of Type- I and standby unit starts operating with no failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence

2($CDSF_{cdsf, urII}$, O_{nasdaf}) The operative unit failure due to controls damaged by structural failure and undergoes repair of type II and the standby unit becomes operative with no failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence

3($CDSF_{cdsf, urIII}$, O_{nasdaf}) The first unit failure due to controls damaged by structural failure and under Type-III multispecialty repairman and the other unit is operative with no failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence

4($ASDAF_{asdaf,uR1}$, $ASDAF_{asdaf,wrI}$) The unit failed due to ASDAF resulting from failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence under repair of Type- I continued from state 1and the other unit failed due to ASDAF resulting from failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence is waiting for repair of Type-I.

5($ASDAF_{asdaf,uR1}$, $CDSF_{cdsf, wrII}$) The unit failed due to ASDAF resulting from failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence is under repair of Type- I continued from state 1and the other unit fails due to controls damaged by structural failure is waiting for repair of Type- II.

6($CDSF_{cdsf, uRII}$, $ASDAF_{asdaf ,wrI}$) The operative unit failed due to controls damaged by structural failure is under repair continues from state 2 of Type –II and the other unit failed due to ASDAF resulting from failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence is waiting under repair of Type-I.

7($CDSF_{cdsf ,uRII}$, $ASDAF_{asdaf,wrII}$) The one unit failed due to controls damaged by structural failure is continued to be under repair of Type II and the other unit failed due to ASDAF resulting from failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence is waiting for repair of Type-II.

8($ASDAF_{asdaf,urIII}$, $CDSF_{cdsf, wrII}$) The one unit failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence is under multispecialty repair of Type-III and the other unit failed due to controls damaged by structural failure is waiting for repair of Type-II.

9($ASDAF_{asdaf,urIII}$, $CDSF_{cdsf, wrI}$) The one unit failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence is under multispecialty repair of Type-III and the other unit failed due to controls damaged by structural failure is waiting for repair of Type-I

10(O_{nasdaf} $CDSF_{cdsf, urIV}$)

The one unit is operative with no failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence and warm standby unit fails due to controls damaged by structural failure and undergoes repair of type IV.

11(O_{nasdaf} $CDSF_{cdsf, uRIV}$)

The one unit is operative with no failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence and warm standby unit fails due to controls damaged by structural failure and repair of type IV continues from state 10.

Transition Probabilities

Simple probabilistic considerations yield the following expressions:

$p_{01} = \lambda_1 / \lambda_1 + \lambda_2 + \lambda_3$,

$p_{02} = \lambda_2 / \lambda_1 + \lambda_2 + \lambda_3$,

$p_{0,10} = \lambda_3 / \lambda_1 + \lambda_2 + \lambda_3$

$p_{10} = pG_1^*(\lambda_1)+q\,G_2^*(\lambda_2)$,

$p_{14} = p- pG_1^*(\lambda_1) = p_{11}^{(4)}$,

$p_{15} = q- q\,G_1^*(\lambda_2) = p_{12}^{(5)}$,

$p_{23} = pG_2^*(\lambda_1)+q\,G_2^*(\lambda_2)$,

$p_{26} = p- pG_2^*(\lambda_1) = p_{29}^{(6)}$,

$p_{27} = q- qG_2^*(\lambda_2) = p_{28}^{(7)}$,

$p_{30} = p_{82} = p_{91} = 1$

$p_{0,10} = pG_4^*(\lambda_1)+q\,G_4^*(\lambda_2)$

$p_{10,1} = p- pG_4^*(\lambda_1) = p_{10,1}^{(11)}$

$p_{10,2} = q- q\,G_4^*(\lambda_2) = p_{10,2}^{(11)}$ (1)

We can easily verify that

$p_{01} + p_{02} + p_{03} = 1$,

$p_{10} + p_{14} (=p_{11}^{(4)}) + p_{15} (=p_{12}^{(5)}) = 1$,

$p_{23} + p_{26} (=p_{29}^{(6)}) + p_{27} (=p_{28}^{(7)}) = 1 \quad p_{30} = p_{82} = p_{91} = 1$

$p_{10,0} + p_{10,1}^{(11)} (=p_{10,1}) + p_{10,2}^{(12)} (=p_{10,2}) = 1 \qquad (2)$

And mean sojourn time is

$\mu_0 = E(T) = \int_0^\infty P[T > t]dt$

Mean Time To System Failure

$Ø_0(t) = Q_{01}(t)[s]\, Ø_1(t) + Q_{02}(t)[s]$

$\quad Ø_2(t) + Q_{0,10}(t)[s]\, Ø_{10}(t)$

$Ø_1(t) = Q_{10}(t)[s]\, Ø_0(t) + Q_{14}(t) +$

$\quad Q_{15}(t)$

$Ø_2(t) = Q_{23}(t)[s]\, Ø_3(t) + Q_{26}(t) +$

$\quad Q_{27}(t)$

$Ø_3(t) = Q_{30}(t)[s]\, Ø_0(t)$

$Ø_{10}(t) = Q_{10,0}(t)[s]\, Ø_{10}(t) + Q_{10,2}(t)[s]$

$\quad Ø_1(t) + Q_{10,2}(t)[s]\, Ø_2(t) \qquad (3\text{-}6)$

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-7) and solving for

$ø_0^*(s) = N_1(s) / D_1(s) \qquad (7)$

where

$N_1(s) = \{Q_{01}^* + Q_{0,10}^* Q_{10,1}^*\} [Q_{14}^*(s) + Q_{15}^*(s)] + \{Q_{02}^* + Q_{0,10}^* Q_{10,2}^*\} [Q_{26}^*(s) + Q_{27}^*(s)]$

$D_1(s) = 1 - \{Q_{01}^* + Q_{0,10}^* Q_{10,1}^*\}\ Q_{10}^* - \{Q_{02}^* + Q_{0,10}^* Q_{10,2}^*\}\ Q_{23}^* Q_{30}^* - Q_{0,10}^* Q_{10,0}^*$

Making use of relations (1) & (2) it can be shown that $ø_0^*(0) = 1$, which implies that $ø_0(t)$ is a proper distribution.

$$\text{MTSF} = E[T] = \left.\frac{d}{ds}\, ø_0^{*}(s)\right|_{s=0}$$

$= (D_1'(0) - N_1'(0)) / D_1(0)$

$= (\mu_0 + \mu_1 (p_{01} + p_{0,10}\, p_{10,1}) + (p_{02} + p_{0,10}\, p_{10,2})(\mu_2 + \mu_3) + \mu_{10}\, p_{0,10} / (1 - (p_{01} + p_{0,10}\, p_{10,1})\, p_{10} - (p_{02} + p_{0,10}\, p_{10,2})\, p_{23}) - p_{0,10}\, p_{10,0}$

where

$\mu_0 = \mu_{01} + \mu_{02} + \mu_{0,10}$,

$\mu_1 = \mu_{10} + \mu_{11}^{(4)} + \mu_{12}^{(5)}$,

$\mu_2 = \mu_{23} + \mu_{28}^{(7)} + \mu_{29}^{(6)}$,

$\mu_{10} = \mu_{10,0} + \mu_{10,1} + \mu_{10,2}$

Availability analysis

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

$M_0(t) = e^{-\lambda_1 t}\, e^{-\lambda_2 t}\, e^{-\lambda_3 t}$

, $M_1(t) = p\, \overline{G_1(t)}\, e^{-\lambda_1 t}$

$M_2(t) = q\, \overline{G_2(t)}\, e^{-\lambda_2 t}$,

$M_3(t) = \overline{G_3(t)}$, $M_{10}(t) = G_4(t)\, e^{-\lambda_3 t}\,\overline{\ }$

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) +$

$\quad q_{02}(t)[c]A_2(t) + q_{0,10}(t)[c]A_{10}(t)$

$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) +$

$\quad q_{12}^{(5)}(t)[c]A_2(t) + q_{11}^{(4)}(t)[c]A_1(t)$,

$A_2(t) = M_2(t) + q_{23}(t)[c]A_3(t) +$

$\quad q_{28}^{(7)}(t)[c]\, A_8(t) + q_{29}^{(6)}(t)]\, [c]A_9(t) \quad A_3(t) = M_3(t) + q_{30}(t)[c]A_0(t)$

$A_8(t) = q_{82}(t)[c]A_2(t)$

$A_9(t) = q_{91}(t)[c]A_1(t)$

$A_{10}(t) = M_{10}(t) + q_{10,0}(t)[c]A_0(t) + q_{10,1}^{(11)}(t)[c]A_1(t) + q_{10,2}^{(11)}(t)[c]A_2(t) \qquad (8\text{-}15)$

Taking Laplace Transform of eq. (7-15) and solving for $\hat{A}_0(s)$

$\hat{A}_0(s) = N_2(s) / D_2(s) \qquad (16)$

where

$N_2(s) = \{ \hat{q}_{0,10} \widehat{M}_{10} + \widehat{M}_0 \} [\{1 -$

$\hat{q}_{11}^{(4)}\}\{1 - \hat{q}_{28}^{(7}\, \hat{q}_{82}\} - \hat{q}_{12}^{(5)}\, \hat{q}_{29}^{(6)}$

$\hat{q}_{91}] + \{ \hat{q}_{01} + \hat{q}_{0,10}\, \hat{q}_{10,1}^{(11)}\}[\widehat{M}_1$

$\{1 - \hat{q}_{28}^{(7)}\, \hat{q}_{82}\} + \hat{q}_{12}^{(5)}\, \hat{q}_{23} \widehat{M}_3 +$

$\widehat{M}_2] + \{ \hat{q}_{02} + \hat{q}_{0,10}\, \hat{q}_{10,2}^{(11)}\} [\{$

$\hat{q}_{23} \widehat{M}_3\}\{1 - \hat{q}_{11}^{(4)}\} + \hat{q}_{29}^{(6)}\, \hat{q}_{91}$

$\widehat{M}_1]$

$D_2(s) = \{1 - \hat{q}_{11}^{(4)}\}\{1 - \hat{q}_{28}^{(7}\, \hat{q}_{82}\} -$

$\hat{q}_{12}^{(5)}\, \hat{q}_{29}^{(6)}\, \hat{q}_{91} - \{ \hat{q}_{01} + \hat{q}_{0,10}$

$\hat{q}_{10,1}^{(11)} \}[\hat{q}_{10} \{1 - \hat{q}_{28}^{(7)}\, \hat{q}_{82}\} +$

$\hat{q}_{12}^{(5)}\, \hat{q}_{23}\, \hat{q}_{30}] - \{ \hat{q}_{02} + \hat{q}_{0,10}$

$\hat{q}_{10,2}^{(11)}\}\{[\hat{q}_{23}\, \hat{q}_{30} \{1 - \hat{q}_{11}^{(4)}\} +$

$\hat{q}_{29}^{(6)}\, \hat{q}_{91}\, \hat{q}_{10}]$

(Omitting the arguments s for brevity)

The steady state availability

$A_0 = \lim_{t\to\infty}[A_0(t)]$

$= \lim_{s\to 0}[s\,\hat{A}_0(s)] \quad = \lim_{s\to 0}\frac{s\,N_2(s)}{D_2(s)}$

Using L' Hospitals rule, we get

$$A_0 = \lim_{s\to 0}\frac{N_2(s)+s\;N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)} \qquad (17)$$

Where

$N_2(0) = \{p_{0,10}\,\hat{M}_{10}(0) + \hat{M}_0(0)\}\,[\{1 - p_{11}^{(4)}\}\{1 - p_{28}^{(7)}\} - p_{12}^{(5)}\,p_{29}^{(6)}] + \{p_{01} + p_{0,10}\,p_{10,1}^{(11)}\}[\hat{M}_1(0)\{1 - p_{28}^{(7)}\} + p_{12}^{(5)}\,p_{23}\,\hat{M}_3(0) + \hat{M}_2(0)] + \{p_{02} + p_{0,10}\,p_{10,2}^{(11)}\}\,[\{p_{23}\,\hat{M}_3(0) + \hat{M}_2(0)\}\{1 - p_{11}^{(4)}\} + p_{29}^{(6)}\,\hat{M}_1(0)]$

$D_2'(0) = \mu_0[p_{10}(1 - p_{28}^{(7)}\} + p_{12}^{(5)}\,p_{23}] + \mu_1[p_{29}^{(6)} + p_{01}\,p_{23} - p_{0,10}\{p_{10,0}\{1 - p_{28}^{(7)}\} + p_{23}\,p_{10,2}^{(11)}\,p_{23}\}] + \mu_2[(1 - p_{11}^{(4)}) - p_{01}\,p_{10} - p_{0,10}\,(p_{10} - p_{10}\,p_{10,2}^{(11)} + p_{12}^{(5)}\,p_{10,0})]\} + \mu_3\;[p_{23}[p_{12}^{(5)}\{p_{01} + p_{0,10}\,p_{10,1}^{(11)}\} + (1 - p_{11}^{(4)})\{p_{02} + p_{0,10}\,p_{10,2}^{(11)}\}] + \mu_8\;[p_{28}^{(7)}(1 - p_{0,10}\,p_{10,0} - p_{10}\{p_{01} + p_{0,10}\,p_{10,1}^{(11)}\})] + \mu_9\;[p_{29}^{(6)}\{p_{12}^{(5)}\;(1 - p_{0,10}\,p_{10,0} + (p_{02} + p_{0,10}\,p_{10,2}^{(11)}\})] + \mu_{10}\;[p_{29}^{(6)}\{p_{12}^{(5)}\;(1 - p_{0,10}\,p_{10,0} + (p_{02} + p_{0,10}\,p_{10,2}^{(11)}\})]$

and

$\mu_3 = \mu_{30}\;,\; \mu_9 = \mu_{91}\;,\; \mu_8 = \mu_{81}$

The expected up time of the system in (0,t] is

$\lambda_u(t) = \int_0^\infty A_0(z)\,dz$

So that $\overline{\lambda_u}(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)} \qquad (18)$

The expected down time of the system in (0,t] is

$\lambda_d(t) = t - \lambda_u(t)$

So that $\overline{\lambda_d}(s) = \frac{1}{s^2} - \overline{\lambda_u}(s) \qquad (19)$

The expected busy period of the server when there is failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence, and controls damaged by structural failure in (0,t]-R_0

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t) + q_{0,10}(t)[c]R_{10}(t)$

$R_1(t) = S_1(t) + q_{10}(t)[c]R_0(t) + q_{12}^{(5)}(t)[c]\,R_2(t) + q_{11}^{(4)}(t)[c]R_1(t)$

$R_2(t) = S_2(t) + q_{23}(t)[c]R_3(t) + q_{28}^{(7)}(t)\,R_8(t) + q_{29}^{(6)}(t)][c]R_9(t)$

$R_3(t) = S_3(t) + q_{30}(t)[c]R_0(t)$

$R_8(t) = S_8(t) + q_{82}(t)[c]R_2(t)$

$R_9(t) = S_9(t) + q_{91}(t)[c]R_1(t)$

$R_{10}(t) = S_{10}(t) + q_{10,0}(t)[c]R_0(t) + q_{10,1}^{(11)}(t)[c]R_1(t) + q_{10,2}^{(11)}(t)[c]R_2(t) \qquad (20\text{-}26)$

where

$S_1(t) = p\,G_1\overline{(t)}\;e^{-\lambda_1 t}$,

$S_2(t) = q\,G_2\overline{(t)}\;e^{-\lambda_2 t}$

$S_3(t) = S_8(t) = S_9(t) = G_3(t)$ —

$S_{10}(t) = G_4\overline{(t)} \qquad (27)$

Taking Laplacė Transform of eq. (16-26) and solving for $\overline{R_0}(s)$

$\overline{R_0}(s) = N_3(s)\,/\,D_2(s) \qquad (23)$

where

$N_3(s) = \{\hat{q}_{01} + \hat{q}_{0,10}\,\hat{q}_{10,1}^{(11)}\}[\hat{S}_1(1 - \hat{q}_{28}^{(7)}\,\hat{q}_{82}\} + \hat{q}_{12}^{(5)}[\hat{S}_2 + \hat{q}_{23}\hat{S}3 + \hat{q}_{28}^{(7)}\,\hat{S}8 + \hat{q}_{29}^{(6)}\,\hat{S}9)]] + \{\hat{q}_{02} + \hat{q}_{0,10}\,\hat{q}_{10,2}^{(11)}\}[\{\hat{S}_2 + \hat{q}_{23}\hat{S}_3 + \hat{q}_{28}^{(7)}\,\hat{S}_8 + \hat{S}_9\,\hat{q}_{29}^{(6)})(1 - \hat{q}_{11}^{(4)}) + \hat{S}_1\,\hat{q}_{29}^{(6)}\,\hat{q}_{91}] + \hat{q}_{0,10}\,\hat{S}_{10}[\{1 - \hat{q}_{28}^{(7)}\,\hat{q}_{82}\}\{1 - \hat{q}_{11}^{(4)}\} - \hat{q}_{29}^{(6)}\,\hat{q}_{91}\,\hat{q}_{12}^{(5)}]$

and $D_2(s)$ is already defined.

(Omitting the arguments s for brevity)

In the long run, $R_0 = \frac{N_3(0)}{D_2'(0)} \qquad (24)$

Where

$N_3(0) = \{p_{01} + p_{0,10}\,p_{10,1}^{(11)}\}[\hat{S}_1(1 - p_{28}^{(7)}\} + p_{12}^{(5)}[\hat{S}_2 + p_{23}\,\hat{S}3 + p_{28}^{(7)}\,\hat{S}8 + p_{29}^{(6)}\,\hat{S}9)]] + \{p_{02} + p_{0,10}\,p_{10,2}^{(11)}\}[\{\hat{S}_2 + p_{23}\hat{S}_3 + p_{28}^{(7)}\,\hat{S}_8 + \hat{S}_9\;p_{29}^{(6)})(1 - p_{11}^{(4)}) + \hat{S}_1 p_{29}^{(6)}] + p_{0,10}\;\hat{S}_{10}[\{1 - p_{28}^{(7)}\}\{1 - p_{11}^{(4)}\} - p_{29}^{(6)}\,p_{12}^{(5)}]$

and $D_2'(0)$ is already defined.

The expected busy period of the server when there is failure due to failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence, and controls damaged by structural failure in (0,t] is

$\lambda_{rv}(t) = \int_0^\infty R_0(z)\,dz$

So that $Q_{01}{}^{*}\,\overline{\lambda_{rv}}(s) = \frac{\hat{R}_0(s)}{s}$

The expected number of visits by the repairman Type-I or Type-II for repairing the identical units in (0,t]-H_0

$H_0(t) = Q_{01}(t)[s][1+ H_1(t)] + Q_{02}(t)[s][1+H_2(t)]+Q_{0,10}(t)[s]\ H_{10}(t)]$

$H_1(t) = Q_{10}(t)[s]H_0(t)] + Q_{12}^{(5)}(t)[s]$

$H_8(t) + Q_{11}^{(4)}(t)]\ [s]H_1(t)$,

$H_2(t) = Q_{23}(t)[s]H_3(t) + Q_{28}^{(7)}(t)\ [s]$

$H_8(t) +Q_{29}^{(6)}(t)]\ [c]H_9(t)$

$H_3(t) = Q_{30}(t)[s]H_0(t)$

$H_8(t) = Q_{82}(t)[s]H_2(t)$

$H_9(t) = Q_{91}(t)[s]H_1(t)$

$H_{10}(t) = Q_{10,0}(t)[s]H_{10}(t)] + Q_{10,1}^{(11)}(t)[s]H_1(t)]+Q_{10,2}^{(11)}(t)[s]\ H_2(t)]$ (25-30)

Taking Laplace Transform of eq. (25-30) and solving for $H_0^*(s)$

$H_0^*(s) = N_4(s) / D_3(s)$ (31)

$N_4(s) = \{ Q_{01}^* + Q_{02}^*\}[\{ 1 – Q_{11}^{(4)*}\}\{1-Q_{28}^{(7)*} Q_{82}^* \} – Q_{12}^{(5)*} Q_{29}^{(6)*} Q_{91}^*]$

And

$D_3(s) = \{1 – Q_{11}^{(4)*}\} \{ 1- Q_{28}^{(7)*} Q_{82}^*\} – Q_{12}^{(5)*} Q_{29}^{(6)*} Q_{91}^*](1- Q_{0,10}^* Q_{10,0}^*)-\{ Q_{01}^*+ Q_{0,10}^* Q_{10,1}^{(11)*}\}[Q_{10}^*\{ 1 – Q_{28}^{(7)*} Q_{82}^* \}+ Q_{12}^{(5)*} Q_{23}^* Q_{30}^*] – \{Q_{02}^* + Q_{0,10}^* Q_{10,2}^{(11)*}\}[Q_{23}^* Q_{30}^*\{1 – Q_{11}^{(4)*}\}+ Q_{29}^{(6)*} Q_{91}^* Q_{10}^*]$

(Omitting the arguments s for brevity)

In the long run,

$H_0 = N_4(0) / D_3'(0)$ (32)

where

$N_4(0) =\{1 – p_{0,10}\}[\{1 – p_{11}^{(4)}\} \{ 1-$

$p_{28}^{(7)} \} – p_{12}^{(5)} p_{29}^{(6)}]$

The expected number of visits by the multispecialty repairman Type-III for repairing the identical units in (0,t]-W_0

$W_0(t)=Q_{01}(t)[s]W_1(t)+Q_{02}(t)[s]\ W_2(t) + Q_{10,0}(t)[s]\ W_{10}(t)$

$W_1(t) = Q_{10}(t)[s]W_0(t)] + Q_{12}^{(5)}(t)[s]$

$W_2(t) + Q_{11}^{(4)}(t)]\ [s]W_1(t)$,

$W_2(t) = Q_{23}(t)[s]W_3(t) + Q_{28}^{(7)}(t)\ [s]$

$W_8(t) +Q_{29}^{(6)}(t)]\ [c]W_9(t)$

$W_3(t) = Q_{30}(t)[s][1+W_0(t)]$

$W_8(t) = Q_{82}(t)[s][1+W_2(t)]$

$W_9(t) = Q_{91}(t)[s][1+W_1(t)]$

$W_{10}(t)=Q_{10,0}(t)[s]W_0(t)+ Q_{10,1}^{(11)}(t)[s]\ W_1(t) + Q_{10,2}^{(12)}(t)[s]\ W_2(t)$ (33-39)

Taking Laplace Transform of eq. (33-39) and solving for $H_0^*(s)$

$H_0^*(s) = N_5(s) / D_3(s)$ (40)

$N_5(s) = \{Q_{01}^*+ Q_{0,10}^*Q_{0,10}^{(11)*} \}[Q_{12}^{(5)*} [Q_{23}^* Q_{30}^* + Q_{28}^{(5)*} Q_{82}^* + Q_{29}^{(6)*} Q_{91}^*] + \{Q_{02}^* + Q_{0,10}^*Q_{10,2}^{(11)*}\}[[Q_{23}^* Q_{30}^* + Q_{28}^{(5)*} Q_{82}^* + Q_{29}^{(6)*} Q_{91}^* \{1 – Q_{11}^{(4)*}\}]$

(Omitting the arguments s for brevity)

In the long run,

$W_0 = N_5(0) / D_3'(0)$ (41)

where $N_5(0) = \{p_{01}+ p_{0,10}\ p_{10,1}^{(11)} \}$

$p_{12}^{(5)} + \{ p_{02}+ p_{0,10}\ p_{10,2}^{(11}\} \{1 –$

$p_{11}^{(4)}\}]$

The expected number of visits by the multispecialty repairman Type-III for repairing the identical units in (0,t]-Y_0

$Y_0(t)=Q_{01}(t)[s]Y_1(t)+Q_{02}(t)[s]\ Y_2(t) + Q_{0,10}(t)[s][1+Y_{10}(t)]$

$Y_1(t) = Q_{10}(t)[s]Y_0(t) + Q_{12}^{(5)}(t)[s]$

$Y_2(t) + Q_{11}^{(4)}(t)]\ [s]Y_1(t)$,

$Y_2(t) = Q_{23}(t)[s]Y_3(t) + Q_{28}^{(7)}(t)\ [s]$

$Y_8(t) +Q_{29}^{(6)}(t)]\ [c]Y_9(t)$

$Y_3(t) = Q_{30}(t)[s][1+Y_0(t)]$

$Y_8(t) = Q_{82}(t)[s]Y_2(t)$

$Y_9(t) = Q_{91}(t)[s]Y_1(t)$

$Y_{10}(t)=Q_{10,0}(t)[s]Y_0(t)+ Q_{10,1}^{(11)}(t)[s]\ Y_1(t) + Q_{10,2}^{(12)}(t)[s]\ Y_2(t)$ (42-48)

Taking Laplace Transform of eq. (42-48) and solving for$Y_0^*(s)$,we get

$Y_0^*(s) = N_6(s) / D_3(s)$ (49)

$N_6(s) = Q_{0,10}^* [\{1 – Q_{11}^{(4)*}\}(1- Q_{28}^{(5)*} Q_{82}^* \} - Q_{12}^{(5)*}Q_{29}^{(6)*} Q_{91}^*\{1- Q_{0,10}^*Q_{,10,0}^* \}+\{Q_{02}^* + Q_{0,10}^*Q_{10,2}^{(11)*}\}[[Q_{23}^* Q_{30}^* \{1 – Q_{11}^{(4)*}\}+ Q_{10}^* Q_{29}^{(6)*} Q_{91}^*]$

(Omitting the arguments s for brevity)

In the long run,

$W_0 = N_6(0) / D_3'(0)$ (50)

where $N_6(0) = p_{0,10}[\{1-p_{11}^{(4)} \}\{1- p_{28}^{(7)}\}- p_{12}^{(5)} p_{29}^{(6)}]$

$p_{12}^{(5)} + \{ p_{02}+ p_{0,10}\ p_{10,2}^{(11}\} \{1 –$

$p_{11}^{(4)}\}]$

Benefit- Function Analysis

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence, and controls damaged by structural failure, expected number of visits by the repairman for unit failure. The expected total Benefit-Function incurred in (0,t] is

C(t) = Expected total revenue in (0,t]

- expected busy period of the server when there is failure due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence, and controls damaged by structural failure in (0,t]

- expected number of visits by the repairman Type- I or Type- II for repairing of identical the units in (0,t]

- expected number of visits by the multispecialty repairman Type- III for repairing of identical the units in (0,t]

- expected number of visits by the multispecialty repairman Type- IV for repairing of identical the units in (0,t]

$$C = \lim_{t\to\infty}(C(t)/t) \quad = \lim_{s\to 0}(s^2 C(s))$$

$$= K_1A_0 - K_2R_0 - K_3H_0 - K_4W_0 - K_5Y_0$$

where

K_1 - revenue per unit up-time,

K_2 - cost per unit time for which the system is busy under repairing,

K_3 - cost per visit by the repairman type- I or type- II for units repair,

K_4 - cost per visit by the multispecialty repairman Type- III for units repair

K_5 - cost per visit by the multispecialty repairman Type- IV for units repair

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to ageing ships in need of maintenance, delayed acquisitions by the Ministry of Defence, and controls damaged by structural failure increases, the MTSF, steady state availability decreases and the Profit-function decreased as the failure increases.

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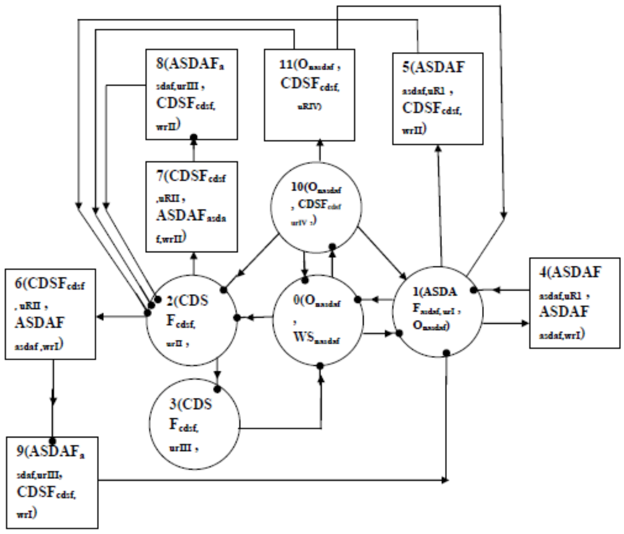

Fig. The State Transition Diagram

○ Up-State □ Down-State
regeneration point

◈ ◈ ◈