Predictions of BOD-DO Concentration in River/Stream: A Mathematical Approach

Ritu Malik
Asst. Prof. (Mathematics), Deptt. of Applied Sciences Gurgaon Institute of Tech. and Management, Rohtak

Abstract: Surface water, is used by human beings for many purposes like irrigation, domestic and recreational activities, an essential source for life to sustain on the earth. Various natural and man-made activities play an important role in polluting surface water. Pollutant enters into surface water naturally through rainwater and melting of ice. Uncontrolled discharge of industrial waste into river bodies is a main cause of polluting surface water. These natural and man-made activities further result in decreasing Dissolved Oxygen (DO) level in river which is the main concern for water quality managers and environment engineers. The biodegradable contaminant in water is measured by Biochemical Oxygen Demand (BOD), a parameter most related to DO. Various mathematical models have been developed to predict BOD-DO concentration in surface water. In this paper a mathematical approach on various one dimensional and two dimensional models is presented.

Key Words: Biochemical-Chemical Oxygen Demand (BOD), Dissolved Oxygen (DO), Mathematical Modeling, Contaminant.

I. INTRODUCTION

The pollutant transport in river water is of great concern in the field of environmental engineering and the processes controlling mixing and movement of waste discharged in water bodies have been extensively studied in relation to different sinks and sources so that pollutant transport can be modeled effectively and accurately. Mathematical models are often used as a tool to predict BOD-DO condition in river after an organic waste is discharged into it. Most of these mathematical models are in the form of a differential equation which represents the physical system in which various physical/chemical and/or biological processes are taking place. These models have a wide range of application including evaluation of risks from accidental spillage and consequently in formulation of pollution control strategies (Thomann R.V., 1998).

Many complex models including various processes like advection, dispersion, reaeration, photosynthesis etc. have been developed so far but simpler approaches are often used to estimate DO concentration in rivers when a pollutant is disposed into it. The first published mathematical model represented (Thommon & Muller, 1987) model which is a steady state model representing the BOD-DO balance in a river.

\[-u \frac{dB}{dx} - k_d B = 0 \]  \hspace{1cm} (1)

\[-u \frac{dC}{dx} - k_d B + k_s (C_s - C) = 0 \]  \hspace{1cm} (2)

The coupled one dimensional differential equations are associated with the initial conditions

\[B = B_0, \text{ and } C = C_0 \text{ at } x = 0 \]  \hspace{1cm} (3)

In this model advection is considered to be the only relevant transport mechanism and all BOD is considered to be in soluble form. The interesting characteristic of S-P model is the idea that the river may be represented by a single one – dimensional system. The model was well suited with the computational capability of that time but the assumption restricts the model validity in the situation where source strength varies with time and when the oxygen consuming waste also contains settleable part along with the dissolved part.

The concentration of DO as obtained from the above stated model is given by the following equation:

\[C = C_s - \frac{k_d B_0}{k_s - k_d} \left[ \exp \left( -k_d \frac{x}{u} \right) - \exp \left( -k_a \frac{x}{u} \right) \right] + \left( C_s - C_0 \right) \exp \left( -k_a \frac{x}{u} \right) \]  \hspace{1cm} (4)

The solution of Eq.(4), in terms of DO deficit, D(=C_s-\text{C}) with condition \[D = D_0 \text{ at } x = 0 \]  is given by

\[D = \frac{k_d B_0}{k_a - k_d} \left[ \exp \left( -k_d \frac{x}{u} \right) - \exp \left( -k_a \frac{x}{u} \right) \right] + D_0 \exp \left( -k_d \frac{x}{u} \right) \]  \hspace{1cm} (5)

Figs. 1(a, b) show the distribution of DO and DO deficit with distance downstream as given by Equations (4) and (5) respectively.
Though the classical Streeter-Phelps model is considered land mark in mathematical modeling, but the assumption restricts the model validity in the situation where source strength varies with time and where the oxygen consuming wastes contain settleable part. In this situation the effect of dispersion should also be included. Li, C.W. (1991, 1994) found the distribution of dissolve oxygen due to unsteady initial oxygen content and BOD load.

Various dispersion model developed are one dimensional models Beltaos, S.,(1980), Beer, T and Young, P.C.(1983), Chatwin, P.C.(1980), Day,T.J.(1975) and Fischer, H.B. (1967a). Various models have been proposed by different authors (Hays et.al. 1966, Bencala and Walters 1983, Runkel and Chapra 1993) to describe the physical phenomena in rivers. Taylor (1954) presented in his one dimensional model that how a substance transported following turbulent flow, in a certain time. Aris (1956) confirmed Taylor’s result through independently using method of moments.

Fischer (1966a) quantified it for natural rivers.Furthermore Taylors theory was applied to pipe, has been verified by several laboratory studies (Sayre, 1968; Sayre and Chang, 1968).

Hays et. al. (1966) described the transient storage model in the form of two partial differential equations that showing the natural behavior of longitudinal concentration distribution more accurately than the solution of classical advection-dispersion equation, one for main zone of river and the other for storage zone.  

Bencal and Walter (1983) included the term due to lateral inflow in the dead zone model proposed by Hays (1966) and found that effect of lateral inflow was negligible. A commonly referenced model known as Transient storage model (TSM) or Dead Zone Model (DZM) is based on the classical advection-dispersion phenomenon with exchange of pollutants between the main zone and stagnant water zones such as pools, gravel beds, side arms of the river (Bencala and Walters 1983, Runkel and Chapra 1993).

Subsequently, several other authors (Harvey et.al. 1996) have been made experimental studies to characterize the physical transport phenomena and hydrological processes in rivers. Singh, S.K. (2003) presented treatment of storage zone in riverine advective-dispersion.

None of the above stated model included the effect of dispersion on stream’s DO. Various Authors(ValentineandWood,1977); Rutherford and Bride(1984) Holley, et.al 1982 etc.) developed models that also included dispersion to predict the DO condition in river but all these dispersion model are one-dimensional models and these models do not account the effect of settleable part of BOD. Bhargava (1986, 1986a, 1986b) Models incorporates the settleable component of BOD along with the dissolved part, but not the dispersion effect. Tyagi et. al (1999) incorporates the dispersion effect along with advection, and settleable and soluble component of BOD. The model is presented by set of equations as follows:

$$B_{i}(x,t) = B_{0-i}(0,t - \tau) \left[1 - \frac{V}{d} \tau \right] \tau \leq T_{s} \quad (6)$$

$$= 0 \quad \tau > T_{s}$$

$$\frac{\partial B}{\partial t} + u(x) \frac{\partial B}{\partial x} = \frac{1}{A(x)} \frac{\partial}{\partial x} \left[ D_{l}(x)A(x) \frac{\partial B}{\partial x} \right] - k_{B} B_{s} \quad (7)$$

$$\frac{\partial C}{\partial t} + u(x) \frac{\partial C}{\partial x} = \frac{1}{A(x)} \frac{\partial}{\partial x} \left[ D_{l}(x)A(x) \frac{\partial C}{\partial x} \right] - k_{B} B_{s} - mB_{i} + k_{C} (C_{s} - C) \quad (8)$$

Where $B_{i}$ and $B_{s}$ are the settleable and dissolved part of BOD while $C$ represents the concentration of DO in the stream.

The system is solved with the following initial and boundary conditions:

$$B(0,t) = (B_{0-s} + B_{0-i}) \exp(-k_{t}t) = B_{0} \exp(-k_{t}t) ; t \geq 0 \quad (9)$$
\[ C = C_r - \frac{k_r B_x}{k_t - k_t} \{ \exp(-k_t \tau) - \exp(-k_t \tau) \} ; I \geq 0 \quad \ldots (10) \]

\[ B(x,0) = 0; C(x,0) = C_r; x \geq 0 \quad \ldots (11) \]

And

\[ B(x,t) \to 0. C(x,0) \to C_r \text{ as } x \to \infty \quad \ldots (12) \]

If one dimensional model is not applicable, transport is often described by two dimensional models (Gowda, T.P.H., (1984); Harden, T.O. and Shen, H.T. (1979); Holley, F.M. and Nerat, J., (1982); Leonard, B.P. (1979); Luk, G.K.Y., Lau, Y.L. and Watt, W.E., (1990); Yotsukura, N. and Sayre, W.W., (1976 1977)). But two-dimensional Modelling of transport requires a considerable amount of hydraulic data, which has to be estimated. Rough estimates of parameters lead to the partial loss of accuracy gained.

\section*{II CONCLUSION}

Variety of models is developed to date. Some of them are one dimensional and other are two and three dimensional but for practical purposes one dimensional model are preferred. As two dimensional models requires considerable amount of estimated hydraulic data which leads to loss of accuracy. All developed one dimensional models are not universally applicable. These models are applicable according to particular situation.

\section*{REFERENCES}


[22] Li, C.W.,”Simulation advection dispersion by minimization characteristics and alternate
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