Precise Modelling of a Gantry Crane System Including Friction, 3D Angular Swing and Hoisting Cable Flexibility

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Abstract **- A crane system offers a typical control problem being an under actuated MIMO system. In this paper the precise modelling of a 2D gantry crane system with 3 DOF is considered. First a simple dynamic model of the system is obtained using Lagrange's equations of motion. Then, friction non-linearities were added to the model, which were found to decrease the output magnitudes from reference values. The model was further improved by considering the possibility of 3D angular swing which showed more accurate transient responses. Finally, the dynamics of hoisting cable flexibility was added to the system resulting in a complex model requiring time consuming simulation. But, significant change was seen in the angular swing output which will significantly affect controller performance. The models considering either flexibility or 3D load swing are comparatively less complex than the combined model. The precise model to be considered is a trade-off between safety (minimum swing angle) and precise load handling.**

Keywords— Gantry crane system, degree of freedom, Euler-Bernoulli beam equation, Lagrangian equation, friction model, cable flexibility

I. INTRODUCTION

For modelling purposes, the gantry crane system is considered as a trolley-pendulum system. Two types of crane modelling can be identified viz. Distributed mass and lumped mass models [1]. Distributed mass model is valid only when payload mass is of the same order of magnitude as the cable and trolley displacement and cable angle are small.

Lumped mass modelling is the most widely used approach in which the hoisting line is modelled as a mass less cable. The payload is lumped with the hook and modelled as a point mass. The cable-hook-payload assembly is modelled as a spherical pendulum. The resulting mathematical representation is simple and compact while capturing the complex dynamics of the payload motion. Lumped mass models are of two types 'reduced models' (all external excitations are lumped into the motion of the suspension point) and extended models (crane support mechanism and platform are added to the dynamic crane model) [1]. Gantry cranes are usually used in fixed sites inside a factory and therefore a reduced model is the most suitable.

A 2D trolley crane system transport loads in 2D, i.e. it can lift the load and move it in one horizontal direction. The torques required to move the trolley and to lift the load are generated using two motors, namely trolley and hoist motors. The friction occurring in the linear actuator and in the trolley motor has a very significant effect on the system behaviour. Empirical approach can be taken to model friction in a system by reproducing effects observed in experiments. There are different interpretations of friction and accordingly different models are available in literature.The LuGre friction model can capture most of the friction behaviour that has been, in general, observed experimentally in control systems [2].

To include the effect of flexibility of the hoisting cable additional displacement of the load due to the flexibility of the cable need to be found out. Solution of Euler-Bernoulli beam equation for a loaded beam, using assumed mode method [3] is one method to find deflection due to flexibility.

Payloads are very heavy and payload pendulations concerns the safety in the workspace and structural integrity of the crane. Thus payload pendulations need to be suppressed throughout the travel path which is a major challenge as no direct control is possible. These unwanted motions can arise as a result of inertia forces (due to motion of payload), base excitations (due to motion of the supporting structure) and/or disturbances on payload (such as wind) [1]. Closed loop techniques are particularly designed to counter the inertia excitations. The control input here is the force or torque applied to the trolley and girder motor (where available) in order to suppress pendulations due to acceleration and deceleration of the trolley. Linear feedback controllers are closed loop control techniques tuned to counter the effects of the natural frequency of the cable-payload assembly, and are thus sensitive to cable length variations. Even though dealing with a linear model is far less complex, neglecting the non linearities may significantly impact the performance of a linear controller. Thus, non-linear controllers are found to be more robust.

For the successful sway suppression and hoist control of a suspended load, it is important to know what part of the crane dynamics should be included in the control law design process and what part can be neglected [4].

II. Dynamic Modelling Of A Gantry Crane System

A 3 degree of freedom 2D gantry crane is modelled here. The system is a two input three output system.

A. *2d Non-Linear Model With Load Hoisting*

Fig 1 shows the swing motion of the load caused by trolley movement of a 3 degree of freedom (displacement of trolley, angular swing of payload and hoist cable length) gantry crane system. The trolley and the load are considered as point masses and are assumed to move in a 2D plane. Here, X is trolley moving direction Z is vertical direction, θ (t) is the sway angle of the load (rad), x(t) is displacement of the trolley (m), l(t) is hoist cable length in meters, F_x is control force applied to the trolley in the X-direction (N), F_l is control force applied to the payload in the l direction (N), m_p is payload mass(kg), and I is mass moment of inertia of the payload (kgm^2).

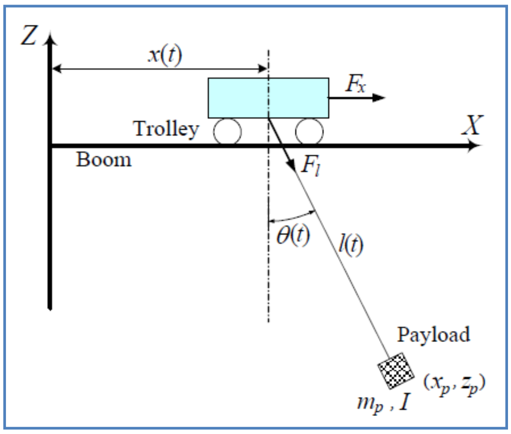

Fig 1. Trolley-Cart 2D model of gantry crane system with load hoisting[4]

For the under actuated system, forces F_x and F_l are the two inputs and trolley displacement x, angular swing θ and hoist cable length are the three outputs.

The following assumptions are made for deriving the dynamic equations of the model: i) the payload and trolley are connected by a mass less rigid cable. ii) the trolley and payload mass and the position of the trolley are known. iii) all frictional elements in the trolley and hoist motions can be neglected. iv) the rod elongation is negligible. v) trolley and load moves in a 2D plane. vii) viscous damping is neglected. vii) external disturbances such as wind are neglected. The coordinates of the payload is (x_p, z_p) and is given by,

$$x_p = x + l sin\theta \text{ , } z_p = -l cos\theta \qquad (1)$$

The kinetic and potential energies of the system are given by

$$K.E = \frac{1}{2}m_x\dot{x}^2 + \frac{1}{2}m_y\dot{y}^2 + \frac{1}{2}m_l\dot{l}^2 + \frac{1}{2}m_p{v_m}^2 + \frac{1}{2}I\dot{\theta}^2 \qquad (2)$$

$$m_p l cos\theta\ddot{x} + (m_p l^2 + I)\ddot{\theta} + 2m_p l\dot{l}\theta + m_p g l sin\theta = 0$$

where

$$v_m{}^2 = \dot{y_m}^2 + \dot{z_m}^2 \qquad (3)$$

Substituting,

$$K.E = \frac{1}{2}(m_t + m_p)\dot{x}^2 + \frac{1}{2}(m_p + m_l)\dot{l}^2 + \frac{1}{2}m_p(l\dot{\theta})^2 + m_p\dot{x}(l cos\theta\dot{\theta} + sin\theta\dot{l}) + \frac{1}{2}I\dot{\theta}^2$$

$$P.E = -m_p g l cos\theta \qquad (4)$$

For the given system, the generalised coordinate vector *q* and force vector *F* are given by,

$$q(t) = [x(t)\ l(t)\ \theta(t)]^T$$
$$F = [F_x\ F_l\ 0]^T \qquad (5)$$

Using the above equations Lagrange equations for the system is obtained as,

$$(m_t + m_p)\ddot{x} + m_p sin\theta l + m_p cos\theta l\ddot{\theta} + 2m_p cos\theta\dot{l}\dot{\theta} - m_p sin\theta l\dot{\theta}^2 = F_x$$

$$m_p sin\theta \ddot{x} + \left(m_p + m_l\right)\ddot{l} - m_p l\dot{\theta}^2 - m_p g cos\theta = F_l$$
$$m_p l cos\theta \ddot{x} + \left(m_p l^2 + I\right)\ddot{\theta} + 2m_p l\dot{l}\theta + m_p g l sin\theta = 0$$

B. *Friction Model*

To derive appropriate friction model from physical laws alone is impossible. Empirical approach can be taken to model friction in a system by reproducing effects observed in experiments. Among different interpretations of friction the Lu-Gre friction model is found to capture most of the friction behaviour that has been, in general, observed experimentally in control systems [2].

The friction occurring in the linear actuator and in the trolley motor has a very significant effect on the system behaviour. The hoist motor friction is mainly due to the gear. The efficiency of the hoist motor with the attached gear box is 50 % for the laboratory scale gantry crane model in [2]. Thus it is assumed that 50 % of the torque is lost because of friction, therefore T_{fric} = -0.5T_{load} for the hoist motor. The friction in the trolley motor and in the linear actuator is modelled using the Lu-Gre friction model. The standard parameterization of the Lu-Gre model is given by

$$\frac{dz}{dt} = v - \sigma_0 \frac{|v|}{g(v)} z$$
$$g(v) = \alpha_0 + \alpha_1 e^{\left(v/v_0\right)^2}$$
$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \alpha_2 v \tag{7}$$

Where, α_i and v_0 are the static parameters and σ_i are the dynamic ones. The state variable z is related to the bristle interpretation of friction and is the average bristle deflection. z is not measurable. The friction torque is given by F, which is a function of the trolley speed v. The friction torques calculated are subtracted from the corresponding inputs.

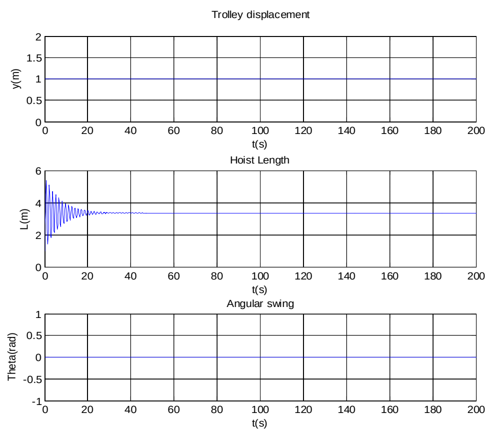

Fig 2. 2D model output without friction

The PID controlled output without friction is obtained, as in Fig 2. The trolley displacement reached a magnitude 1 instantaneously due to lightly damped nature of crane and absence of friction. The cable length magnitude initially oscillates due to the combined effect of gravity and the controller. It finally settles to the reference value, in about 20 seconds, due to the control action. It is observed that angular swing is zero for zero friction when trolley velocity is zero. Also it is noticed that angular swing is not affected by cable length variation in the absence of friction. Fig 3 shows the effect of friction on the output. Due to friction the trolley displacement and cable length are found to settle at values below their corresponding reference values. The swing angle is reaching up to about 5 radians initially and also all three outputs are taking much time to settle. Therefore a more suitable controller is required.

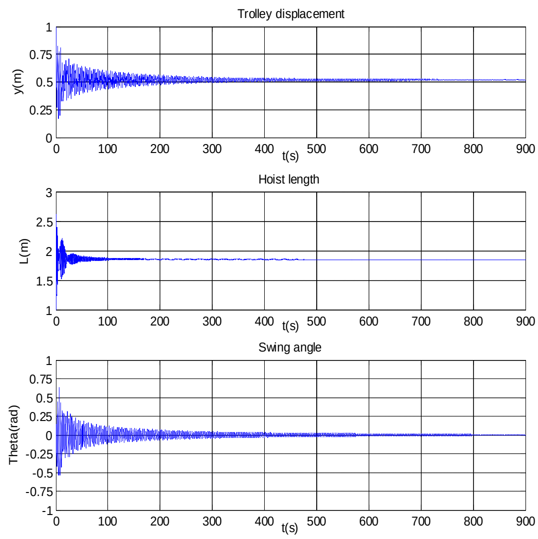

Fig 3. 2D model output with friction

C. *2d Non-Linear Model With 3d Angular Swing*

Fig 4 shows the swing motion of the load caused by trolley movement of a 4DOF (x displacement of trolley, y displacement of trolley, hoisting cable length and angular swing) gantry crane system. The trolley and the load are considered as point masses and are assumed to move in a 3D plane. The trolley can move along the girder in Y direction and the girder moves along the X direction. Here, θ_x is angular swing component in x direction (rad), θ_y is angular swing component in y direction (rad), x is displacement of the girder (m), y is displacement of trolley along girder(m), l is hoist cable length in meters, f_x is control force applied to the trolley in the X-direction (N) , f_y is control force applied to the trolley in the Y-direction (N), f_l is control force applied to the payload in the l -direction (N), m_p is payload mass(kg) and I is mass moment of inertia of the payload (kgm^2).

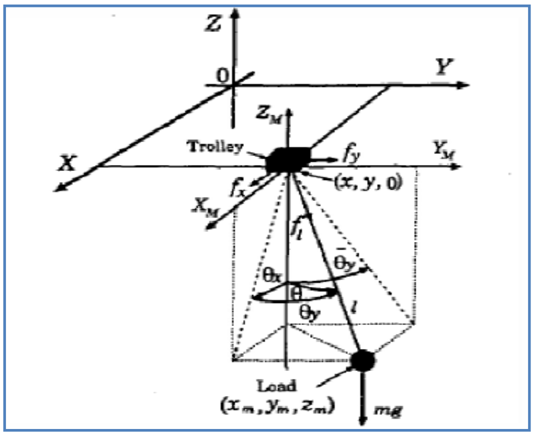

Fig 4. Trolley-cart model of 3D overhead crane [5]

The gantry crane system considered in this paper is of 3 degree of freedom having no x displacement. When the girder is in rest and trolley is in motion, the three dimensional crane model resembles the two-dimensional crane model [6].Thus the girder displacement x and its derivatives are zero. But forces in X direction can be present in the form of disturbances acting directly on payload and is taken as F_x. Therefore we get,

$$K.E = \frac{1}{2}\left(m_y\dot{y}^2 + m_l\dot{l}^2\right) + \frac{1}{2}m_pV_m^2$$
$$P.E = mgl(1 - \cos\theta_x \cos\theta_y)$$

(8)

For the given system, the generalised coordinate vector q and force vector F are given by,

$$q = [\,y\; l\; \theta_x\; \theta_y]^T$$
$$F = [f_y\; f_l\; F_x\; 0]^T$$

(9)

Deriving the Lagrangian equations using the above data, the dynamic equations of the system is obtained.

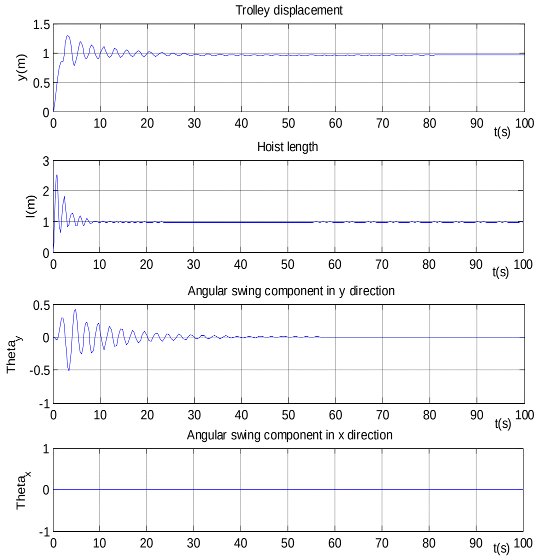

Fig 5. Model output without disturbances in X direction

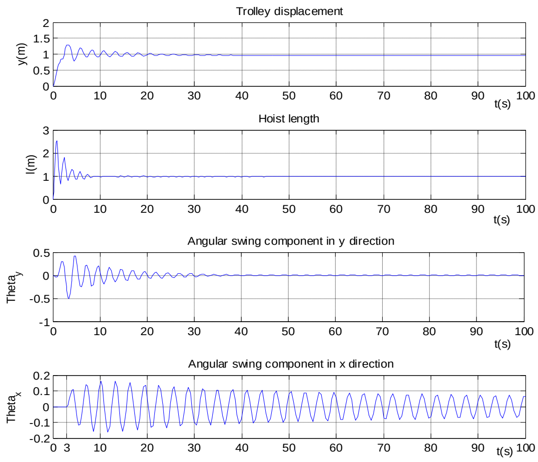

Fig 6. Model output with disturbances in X direction

The PID controlled output without payload disturbances (Fig 5) are obtained by giving F_x = 0. Comparing with Fig 2, the new model output without friction (Fig 5) shows initial oscillations for trolley displacement and also presence of angular swing of load. From Fig 5 and Fig 6 it can be seen that angular swing component in X direction, θ_x , is present only when disturbance forces F_x is acting directly on the payload. But angular swing is present in the Y direction due to trolley motion in Y direction. Here, F_x has been given as a single pulse at time 3 seconds. Angular swing component in Y direction is seen to settle faster than that in X direction since it is influenced by the y displacement which is settling whereas x displacement is absent. Thus swing output can be controlled using trolley displacement.

Adding friction to the new 2D model, the trolley displacement and hoist cable length does not reach the reference values as observed previously. But, compared to new model output without friction (Fig 5), output with friction (Fig 7) shows that oscillations are taking longer time to settle due to the combined effect of friction and control action

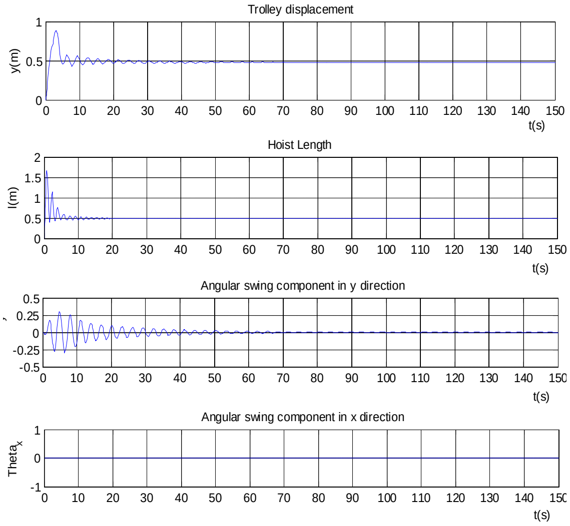

Fig 7. Model output with 3D angular swing and friction

A. 2d Non-Linear Model With Load Hoisting And Flexible Cable

Fig 8 shows the load swing in a trolley-pendulum model of a 2D 3DOF gantry crane system with a flexible cable. *v(x,t)* is the deflection of a point on the cable at a distance *x* from the trolley end of cable in meters. All the rest of the notations are the same as that in Fig 1.Here, *v* is a function of *x* as well as time *t*. Then, deflection of the cable tip is *v(l,t)* where *l* is the length of the cable.

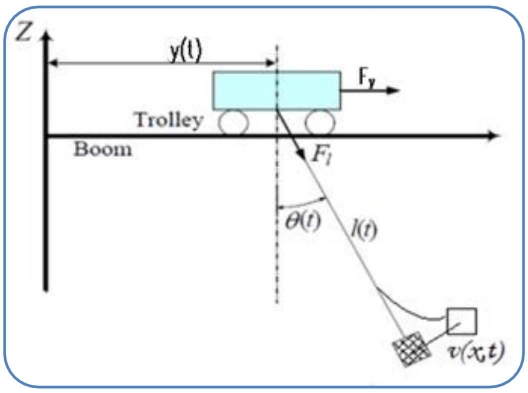

Fig 8. 2D gantry crane with load hoisting and flexible rod

v(l,t) can be found using the solution of Euler-Bernoulli beam equation using assumed modes method. In this method, deflection is expressed as a sum of two functions: a function of displacement along the length of the cable (Shape function$\emptyset_i(x)$) and a function of time (generalized co-ordinates $\delta_i(t)$) [7].

From the solution of Euler-Bernoulli beam equation [8], deflection *v* at a distance *x* along the cable is,

$$v(x,t) = \sum_{i=1}^{n} \emptyset_i(x)\delta(t) \tag{10}$$

$$\emptyset_i(x) = A_i\left(cosh\frac{\beta_i x}{l} - cos\frac{\beta_i x}{l} - \gamma_i\left(sinh\frac{\beta_i x}{l} - sin\frac{\beta_i x}{l}\right)\right) = A_i\varphi_i(x) \tag{11}$$

Where,

$$\gamma_i = \frac{cos\,\beta_i + cosh\,\beta_i}{sin\,\beta_i + sinh\,\beta_i}$$

$$A_i = \left[\int_0^l \varphi_i^2(x)dx + \frac{m_p}{\rho}\varphi_i^2(l)\right]^{-1/2} \tag{12}$$

$$1 + \cosh\beta_i \cos\beta_i + \frac{m_p\beta_i}{\rho l}(\sinh\beta_i\cos\beta_i - \cosh\beta_i\sin\beta_i) = 0$$

for a loaded beam [3].

The coordinates of the payload is given by,

$$y_m = y + l sin\,\theta + v(l,t)\,cos\,\theta$$
$$z_m = -l\,cos\,\theta_x + v(l,t)\,sin\,\theta$$

Kinetic and potential energies of the system is given by,

$$K.E = \frac{1}{2}(M_y\dot{y}^2 + M_l\dot{l}^2) + \frac{1}{2}mV_m^2 + K_{cable}$$

$$P.E = mg(v(l,t)\,sin\,\theta - l\,cos\,\theta) + \frac{1}{2}EI\int_0^l \emptyset''^2(x)\delta^2(t)dx \tag{14}$$

If the cable is assumed to be mass-less the kinetic energy of the cable K_{cable} can be neglected. For the given system, the generalised coordinate vector *q* and force vector *F* are given by,

$$q = [y\ l\ \theta\ \delta]^T$$
$$F = [F_y\ F_l\ 0\ 0]^T \tag{15}$$

Using the above equations, the dynamic equations of the system are obtained using Lagrange's equation of motion

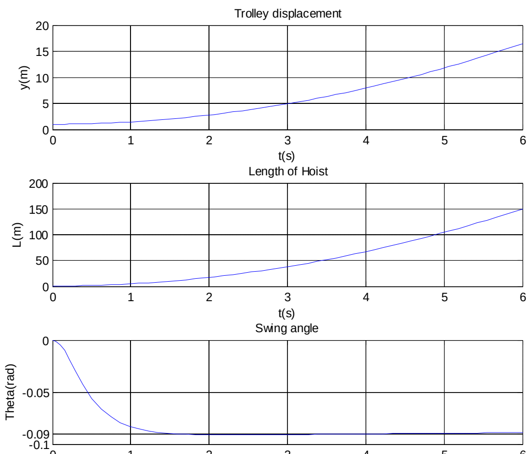

Fig 9. 2D model open loop response without friction and cable flexibility for 6s

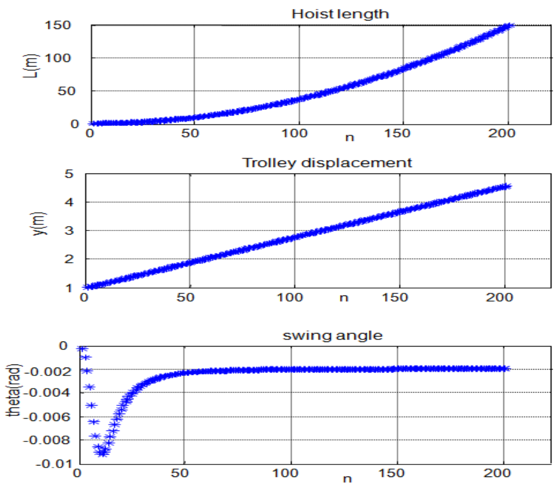

Fig 10. 2D model open loop response with friction and cable flexibility for 6s (t = 0.03n)

Comparing Fig 9 and Fig 10, the effect of friction is observed to decrease the rate of increase of trolley displacement and hoist cable length. The effect of flexibility is seen in angular swing of payload. The swing angle is seen to decrease significantly compared to the model output without considering cable flexibility. A possible explanation to this phenomenon can be explained with Fig 11.

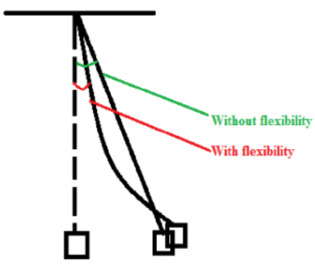

Fig 11. Deflection of a flexible rope

This difference in the actual angle will be having important implications on controller performance.

E. 3d Non-Linear Model With Load Hoisting And Flexible Cable

It has been observed previously that the transients are better captured by the model including 3D angular swing dynamics. Therefore, the same derivation is attempted here including the dynamics of cable flexibility.

The 3D gantry crane model used before is again considered with having a flexible cable (Fig 12). The two components of the additional deflection due to the flexibility of the cable *v(x,t)* are given by *v(l_y,t)* and *v(l_x,t)*.

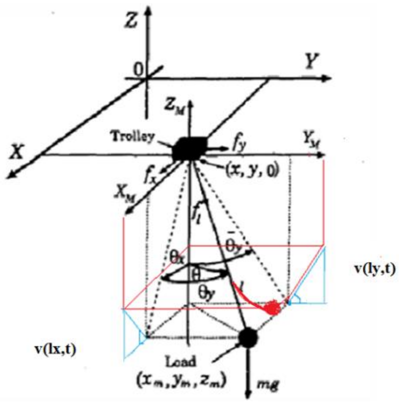

Fig 12. 3D gantry crane with load hoisting and flexible cable

Where,

$$l_x = l\cos\theta_y$$
$$l_y = l\cos\theta_x \tag{16}$$

The coordinates of the payload is given by,

$$x_m = x + l\sin\theta_x\cos\theta_y + v(l\cos\theta_y, t)\cos\theta_x$$
$$y_m = y + l\sin\theta_y + v(l\cos\theta_x, t)\cos\theta_y$$
$$z_m = -(l\cos\theta_x\cos\theta_y - v(l\cos\theta_x, t)\sin\theta_y \tag{17}$$

The kinetic and potential energies of the system will be

$$K.E = \frac{1}{2}\left(M_y\dot{y}^2 + M_x\dot{x}^2 + M_l\dot{l}^2\right) + \frac{1}{2}mV_m^2 + K_{cable}$$

$$P.E = mg(v(l\cos\theta_x, t)\sin\theta_y - l\cos\theta_x\cos\theta_y + \frac{1}{2}EI\int_0^l \phi''^2(x)\delta^2(t)dx \tag{18}$$

For the given system, the generalised coordinate vector q and force vector F are given b ,

$$\begin{aligned} q &= [x\ y\ l\ \theta_x\ \theta_y\ \delta]^T \\ F &= [F_x\ F_y\ F_l\ 0\ 0\ 0]^T \end{aligned} \quad (19)$$

Deriving the Lagrangian equations using the above data, the dynamic equations of the system is obtained.

The obtained model is 3D with 4 degrees of freedom. To get 2D 3DOF model with 3D angular swing, the girder displacement or displacement in X direction is taken as zero, i.e. x and its derivatives are taken as zero in the dynamic equations obtained above. Further adding friction as the complete model is obtained.

The simulation of the obtained model is complex and highly time consuming. It was observed that 3D swing angle is only present in the case of disturbances acting directly on the load. This complex model with 3D swing angle needs to be used only if these direct disturbances are to be considered. Still, it was also observed that the transients in the trolley displacement and angular swing were shown only by this model. For the lightly damped crane system the controller should consider both the transient and steady state response [1].

III. Conclusion

A dynamic non-linear modelling of a 2D gantry crane system with 3 DOF has been considered in this paper. First, a 2D 3DOF gantry crane model was obtained using Lagrangian equations of motion. Friction non-linearities were then added to the model which was found to decrease the output magnitudes. The model was further improved by considering the possibility of 3D angular swing which showed more accurate transient responses. Finally, the dynamics of hoisting cable flexibility was added to the system resulting in a complex model requiring time consuming simulation. But, significant change was seen in the angular swing output which will have important effect in controller performance. The model without considering flexibility is much simpler. But the significant difference in the open loop response of the models with and without considering cable flexibility demands the inclusion of the dynamics of cable flexibility. The models considering either flexibility or 3D load swing are comparatively less complex than the combined model. The solution lies in whether minimization of angular swing or precise positioning of load is given higher importance. Gantry cranes used in construction sites will demand safety i.e. minimal angular swing rather than precision load handling. Transportation industries demand precision while safety aspect depends on the work site and size of load. Manufacturing industries can compromise on load swing if work space is not cluttered and is fully automated.

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