A Novel Procedure for Solving Multi-Support Obstacle Problems

P. V. Ramana

Department of Civil Engineering, MNIT, Jaipur, India
E-mail : pvramanaphd@gmail.com

***Abstract* - An innovative method has been applied for cable stayed and continuous bridge multi-support linear obstacle problems to improve the accuracy and convergence of above mentioned problems. Compare to conventional and numerical technique, the innovative method gives better and closed form solution. The continuous support bridge problems can easily solve with help of innovative method, which is decomposing technique and semi-analytical method. The decomposition innovative method results are found to converge very quickly and are more close to exact solution. There are many obstacles in multi-support problems like cable stayed and continuous support bridges; with ease one can solve the problem effectively using innovative method. Demonstrated results deficits that the innovative method can use any type of typical problems for which the closed form solutions are cumbersome.**

Keyword - Obstacle problem, Multi-support, Analytical Solution, Innovative decomposition technique.

I. INTRODUCTION

A novel technique applied for solving continuous support bridge problems. In recent years the developments of the high-speed digital computer and increased interest in continuous and linear phenomena have led to an intensive study of the numerical solution of ordinary and partial differential equations. The innovative decomposition method is a non-numerical method for solving linear and nonlinear differential equations, both ordinary and partial. The general direction of the paper is towards obtaining solution for ordinary and partial differential equations (PDEs). In the 1985s, Adomian [1, 2, 12,13] proposed a new and ingenious method to obtain exact solution of linear and nonlinear equations of various kinds like algebraic, differential for both ordinary and partial, integral, etc. problems. The technique uses a decomposition of the nonlinear operator as a series of Adomian functions. Each term of this series is a generalized polynomial called the Adomian polynomial. Some techniques which assume essentially that the linear and nonlinear system is almost linear after equivalent linearization will not be able to retain the originality of the problem. Present technique consists of splitting the given equation into linear, remainder and nonlinear parts, inverting the highest order differential operator contained in the linear operator on both sides, identifying the initial or boundary conditions and the terms involving the independent variable alone as initial approximation, decomposing the unknown function into a series whose components are to be determined, decomposing the linear and nonlinear function in terms of special polynomials, and finding the successive terms of the series solution. The present innovative method provides the solution in a rapidly convergent series with easily computable components. The main advantage of the method is that it can be used directly to solve, all types of differential equations with homogeneous and inhomogeneous boundary conditions. Another advantage of the method is that it reduces the computational work in a tangible manner, while maintaining higher accuracy of the numerical solution. The conventional have systematic procedure and follows some assumed rules, but using the innovative method, one can solve the problem straight way.

II. CONTINUOUS SUPPORTED CABLE STAYED BRIDGE SYSTEMS

The variational inequalities provide natural, novel and innovative settings for formulation of the multi-support, unilateral, contact and equilibrium problems arising in elasticity, transportation science, structural analysis, economics and optimization. In a variational formulation of such problems, the location of the contact area i.e. free boundary, becomes an intrinsic part of the solution and no special techniques are needed to obtain it. If the obstacle is known, then the variational inequalities can be characterized by a system of differential equations by using the penalty function method described by Lewy and Stampacchia [3]. The main computational advantage of this technique is its simple applicability for solving the system of differential equations. In recent years, Al-Said and Noor [4], Khalifa and Noor [5], Noor and Al-Said [6] and Al-Said et al. [7] have used Collocation, finite difference and spline methods for solving such a type of second order system of differential equations associated with multi-support and unilateral problems. In the current paper, the present technique is used for obtaining solutions of the nth-order boundary value problem of the type

$$\frac{d^n u}{dx^n} = u^{(n)} = \begin{cases} f(x), & a \le x \le c, \\ g(x)u(x) + f(x) + r, & c \le x \le d, \\ f(x), & d \le x \le b, \end{cases}$$

$$\frac{d^n u}{dx^n} = u^{(n)} = \begin{cases} f(x), & a \le x \le c, \\ g(x)u(x) + f(x) + r, & c \le x \le d, \\ f(x), & d \le x \le b, \end{cases}$$

(1)

with the boundary and continuity conditions

$$\begin{aligned} u(a) = u(b) &= A_1, u''(a) = u''(b) = A_2, \\ u(c) = u(d) &= B_1, u''(c) = u''(d) = B_2, \end{aligned}$$

$$\begin{aligned} u(a) = u(b) &= A_1, u''(a) = u''(b) = A_2, \\ u(c) = u(d) &= B_1, u''(c) = u''(d) = B_2, \end{aligned}$$

(2)

where f and g are continuous functions on [a, c] and [c,d] respectively. The parameters r, A_i A_i and B_i B_i, i = 1, 2, , n are real constants. Such type of systems arise in the study of multi support problems and has important applications in other branches of pure and applied sciences, referred by Adomian, Noor, Cherruault et al. [4, 8, 7, 6, 9, 10, 11,12,13].

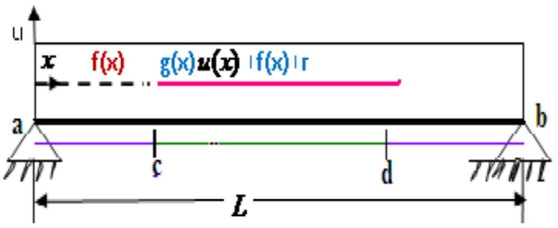


Fig. 1: Beam Orientation with different loading conditions

The figure (1) shows, beam orientation with different loading conditions and corresponding boundary values. Considering an n^{th} n^{th}-order boundary value problem,

$$\frac{d^n u}{dx^n} = u^{(n)} = g(x)u(x) + f(x) + r,$$

$$\frac{d^n u}{dx^n} = u^{(n)} = g(x)u(x) + f(x) + r,$$

(3)

with the boundary conditions

$$u(c) = u(d) = B_1, u''(c) = u''(d) = B_2,$$

$$u(c) = u(d) = B_1, u''(c) = u''(d) = B_2,$$

to solve equation (1). To begin, it is convenient to rewrite equation (3) in innovative method standard operator form $L_x u = g(x)u(x) + f(x) + r,$ where the differential operator L_x L_x is given by $L_x = \frac{d^n}{dx^n}$, $L_x = \frac{d^n}{dx^n}$. Where g(x) and f(x) are continuous functions meant to define multi supports by introducing appropriate discontinuities. The inverse operator L_x^{-1} L_x^{-1} is therefore considered as an n-fold integral operator defined by

$$L_x^{-1}(\cdot) = \int_c^x \int_c^x \int_c^x \int_c^x (\cdot)\, dxdx\, dxdx.$$

$$L_x^{-1}(\cdot) = \int_c^x \int_c^x \int_c^x \int_c^x (\cdot)\, dxdx\, dxdx.$$

(4)

Operating with L_x^{-1} L_x^{-1} on equation (3) and using the boundary conditions at x = c,

$$\begin{aligned} u(x) &= B_1 + A(x-c) + \frac{1}{2!}B_2(x-c)^2 + \frac{1}{3!}B(x-c)^3 + \frac{1}{4!}B_3(x-c)^4 \\ &+ + L_x^{-1}(g(x)u) + L_x^{-1}(f(x)), \end{aligned}$$

$$\begin{aligned} u(x) &= B_1 + A(x-c) + \frac{1}{2!}B_2(x-c)^2 + \frac{1}{3!}B(x-c)^3 + \frac{1}{4!}B_3(x-c)^4 \\ &+ + L_x^{-1}(g(x)u) + L_x^{-1}(f(x)), \end{aligned}$$

(5)

Here c and d are defined as the obstacle points, where the constants $A = u'(c), B_i = u_i'''(c)$ $A = u'(c), B_i = u_i'''(c)$ will be determined later by using the boundary conditions at x = c and d. Now, decompose the unknown function u(x) as a sum of components defined by the series

$$u(x) = \sum_{i=0}^{\infty} u_i(x).$$

$$u(x) = \sum_{i=0}^{\infty} u_i(x).$$

(6)

The zero th th or u_0 component is usually taken to be all terms arising from the boundary conditions and the integration of the independent term f(x) and r

$$\begin{aligned} u_0 &= B_1 + A(x-c) + \frac{1}{2!}B_2(x-c)^2 + \frac{1}{3!}B(x-c)^3 \\ &+ \frac{1}{4!}B_3(x-c)^4 + + L_x^{-1}(f(x) + r). \end{aligned}$$

$$\begin{aligned} u_0 &= B_1 + A(x-c) + \frac{1}{2!}B_2(x-c)^2 + \frac{1}{3!}B(x-c)^3 \\ &+ \frac{1}{4!}B_3(x-c)^4 + + L_x^{-1}(f(x) + r). \end{aligned}$$

(7)

The remaining components $u_n(x), n \ge 1$ $u_n(x), n \ge 1$, can be completely determined such that each term is computed by using the previous term, since $u_0 u_0$ is known. Therefore,

$$u_n = L_x^{-1}(g(x)u_{n-1}), n \geq 1.$$

$$u_n = L_x^{-1}(g(x)u_{n-1}), n \geq 1. \quad (8)$$

The innovative decomposition method procedure described is followed, where in the initial term containing the known boundary conditions(defined before) is kept aside as $u_0 u_0$ and the terms one by one viz A(x-c), B $(x-c)^2$ $(x-c)^2$ are taken along with the inverse operations. They are u_1, u_2, u_3, u_1, u_2, u_3, etc, as shown below. Since the computation in equation (8) depends heavily on $u_0 u_0$, the whole computations to find the solution will be simplified considerably. For example an alternative to equation (7) might be

$u_0 = B_1$, $u_1 = A(x-c) + L_x^{-1}f(x) + L_x^{-1}(g(x)u_0)$, $u_2 = \frac{1}{2!}B_2(x-c)^2 + L_x^{-1}(g(x)u_1)$, $u_3 = \frac{1}{3!}B(x-c)^3 + L_x^{-1}(g(x)u_2)$, $u_4 = \frac{1}{4!}B_3(x-c)^4 + L_x^{-1}(g(x)u_3)$,

$u_0 = B_1$, $u_1 = A(x-c) + L_x^{-1}f(x) + L_x^{-1}(g(x)u_0)$, $u_2 = \frac{1}{2!}B_2(x-c)^2 + L_x^{-1}(g(x)u_1)$, $u_3 = \frac{1}{3!}B(x-c)^3 + L_x^{-1}(g(x)u_2)$, $u_4 = \frac{1}{4!}B_3(x-c)^4 + L_x^{-1}(g(x)u_3)$,

$\ldots u_n = L_x^{-1}(g(x)u_{n-1}), n \geq 2$

$u_n = L_x^{-1}(g(x)u_{n-1}), n \geq 2$. Finally an N-term approximate solution is given by

$$u(x) = \sum_{i=0}^{n} u_i(x), i \geq 1.$$

$$u(x) = \sum_{i=0}^{n} u_i(x), i \geq 1. \quad (9)$$

To show the effectiveness of the proposed innovative decomposition method and to give a clear overview of the methodology. Now using the idea of Lewy et al.[3], then equation (10) can be written for fourth order form as

$$u^{(4)} + \mathrm{v}[u-\psi](u-\psi) = f, \quad (10)$$

$$u^{(4)} + \mathrm{v}[u-\psi](u-\psi) = f, \quad (11)$$

where $\psi \psi$ is the obstacle function and v(t) is a penalty function defined by

$$v(t) = \begin{cases} 4, t \geq 0, \\ 0, t < 0. \end{cases} v(t) = \begin{cases} 4, t \geq 0, \\ 0, t < 0. \end{cases} \quad (12)$$

The singularity function $\psi(x)\psi(x)$ is chosen as

$$\psi(x) = \begin{cases} -\frac{1}{4} \text{ for } -1 \leq x \leq -\frac{1}{2} \text{ and } \frac{1}{2} \leq x \leq 1, \\ \frac{1}{4} \text{ for } -\frac{1}{2} \leq x \leq \frac{1}{2}, \end{cases}$$

$$\psi(x) = \begin{cases} -\frac{1}{4} \text{ for } -1 \leq x \leq -\frac{1}{2} \text{ and } \frac{1}{2} \leq x \leq 1, \\ \frac{1}{4} \text{ for } -\frac{1}{2} \leq x \leq \frac{1}{2}, \end{cases} \quad (13)$$

III. AN INNOVATIVE DECOMPOSITION METHOD: MULTI-SUPPORT OBSTACLE APPLICATIONS

An application of continuous support equation (10) can be written for fourth order form as

$$u^{(4)} + v[u-\psi](u-\psi) = f,$$

$$u^{(4)} + v[u-\psi](u-\psi) = f, \quad (14)$$

where $\psi \psi$ is the obstacle function and v(t) is a penalty function defined by

$$v(t) = \begin{cases} 4, t \geq 0, \\ 0, t < 0. \end{cases} v(t) = \begin{cases} 4, t \geq 0, \\ 0, t < 0. \end{cases} \quad (15)$$

Assume that the obstacle function $\psi(x)$ is defined by

$$\psi(x) = \begin{cases} -\frac{1}{4} \text{ for } -1 \leq x \leq -\frac{1}{2} \text{ and } \frac{1}{2} \leq x \leq 1, \\ \frac{1}{4} \text{ for } -\frac{1}{2} \leq x \leq \frac{1}{2}, \end{cases}$$

$$\psi(x) = \begin{cases} -\frac{1}{4} \text{ for } -1 \leq x \leq -\frac{1}{2} \text{ and } \frac{1}{2} \leq x \leq 1, \\ \frac{1}{4} \text{ for } -\frac{1}{2} \leq x \leq \frac{1}{2}, \end{cases} \quad (16)$$

From equations (14) - (16), one can obtain the following system of equations

$$u^{(4)} = \begin{cases} f, & \text{for } -1 \leq x \leq -\frac{1}{2} \text{ and } \frac{1}{2} \leq x \leq 1, \\ 1 - 4u + f, & \text{for } -\frac{1}{2} \leq x \leq \frac{1}{2}. \end{cases}$$

$$u^{(4)} = \begin{cases} f, & \text{for } -1 \leq x \leq -\frac{1}{2} \text{ and } \frac{1}{2} \leq x \leq 1, \\ 1 - 4u + f, & \text{for } -\frac{1}{2} \leq x \leq \frac{1}{2}. \end{cases} \quad (17)$$

with the boundary conditions

$$u(-1) = u(-\tfrac{1}{2}) = u(\tfrac{1}{2}) = u(1) = 0,$$
$$u''(-1) = u''(-\tfrac{1}{2}) = u''(\tfrac{1}{2}) = u''(1) = 0,$$
$$u(-1) = u(-\tfrac{1}{2}) = u(\tfrac{1}{2}) = u(1) = 0,$$
$$u''(-1) = u''(-\tfrac{1}{2}) = u''(\tfrac{1}{2}) = u''(1) = 0, \quad (18)$$

and the conditions of continuity of u and $u'' u''$ at x = $-\frac{1}{2} -\frac{1}{2}$ and $\frac{1}{2}\frac{1}{2}$.

3.1. 4th Order Homogeneous Obstacle Continuous Bridge Problem

Considering the present method to solve the system of differential equation (17) when f = 0, viz.,

$$u^{(4)} = \begin{cases} 0, & \text{for } -1 \le x \le -\frac{1}{2} \text{ and } \frac{1}{2} \le x \le 1, \\ 1-4u, & \text{for } -\frac{1}{2} \le x \le \frac{1}{2}. \end{cases}$$

$$u^{(4)} = \begin{cases} 0, & \text{for } -1 \le x \le -\frac{1}{2} \text{ and } \frac{1}{2} \le x \le 1, \\ 1-4u, & \text{for } -\frac{1}{2} \le x \le \frac{1}{2}. \end{cases}$$

(19)

with the boundary conditions

$$\begin{aligned} u(-1) &= u(-\tfrac{1}{2}) = u(\tfrac{1}{2}) = u(1) = 0, \\ u''(-1) &= -u''(-\tfrac{1}{2}) = u''(\tfrac{1}{2}) = -u''(1) = \varepsilon, \end{aligned}$$

$$\begin{aligned} u(-1) &= u(-\tfrac{1}{2}) = u(\tfrac{1}{2}) = u(1) = 0, \\ u''(-1) &= -u''(-\tfrac{1}{2}) = u''(\tfrac{1}{2}) = -u''(1) = \varepsilon, \end{aligned}$$ (20)

where $\varepsilon \to 0$.

Innovative decomposition Solution: The above equation (19) can be written as $L_x u = 0$ and $1-4u$ $L_x u = 0$ and $1-4u$, where the linear operator is $L_x = \frac{d^4}{dx^4}$ $L_x = \frac{d^4}{dx^4}$. Pre-multiplying both sides of the equation (19) by L_x^{-1} L_x^{-1} for each case. Apply the boundary conditions to determine the unknown variables viz. α_1 α_1 to α_4 α_4.

Like case 2, the remaining cases 1 and 3 also can solve easily and apply the boundary conditions to determine the unknown variables viz. α_1 α_1 to α_4 α_4. The innovative decomposition method solution is

$$u(x) = \begin{cases} -\frac{x}{4000000} - \frac{\left(x+\frac{1}{2}\right)^2}{2000000} - \frac{1}{8000000}, & \text{for } -1 \le x \le -\frac{1}{2}, \\ 0.04x^4 - 0.06x + 67\exp^{-5} x^6 - 1\exp^{-4} x^8 - \\ 5.3\exp^{-7} x^{10} + 3\exp^{-8} x^{12} + 9\exp^{-11} x^{13} - \\ 3\exp^{-13} x^{16} - 4.8\exp^{-15} x^{18} + 1\exp^{-17} x^{20} + \\ 1.1\exp^{-18} x^{22} - 1.7\exp^{-21} x^{24} - \\ 0.06\,(x-0.5)^2 + 0.0275, & \text{for } -1 \le x \le \\ \frac{x}{4000000} + \frac{(x-1)^2}{2000000} - \frac{1}{4000000}, & \text{for } \frac{1}{2} \le x \le 1. \end{cases}$$

$$u(x) = \begin{cases} -\frac{x}{4000000} - \frac{\left(x+\frac{1}{2}\right)^2}{2000000} - \frac{1}{8000000}, & \text{for } -1 \le x \le -\frac{1}{2}, \\ 0.04x^4 - 0.06x + 67\exp^{-5} x^6 - 1\exp^{-4} x^8 - \\ 5.3\exp^{-7} x^{10} + 3\exp^{-8} x^{12} + 9\exp^{-11} x^{13} - \\ 3\exp^{-13} x^{16} - 4.8\exp^{-15} x^{18} + 1\exp^{-17} x^{20} + \\ 1.1\exp^{-18} x^{22} - 1.7\exp^{-21} x^{24} - \\ 0.06\,(x-0.5)^2 + 0.0275, & \text{for } -1 \le x \le \\ \frac{x}{4000000} + \frac{(x-1)^2}{2000000} - \frac{1}{4000000}, & \text{for } \frac{1}{2} \le x \le 1. \end{cases}$$

(23)

Exact Solution:

The exact solution [14] is given below. The result is shown in figure (2).

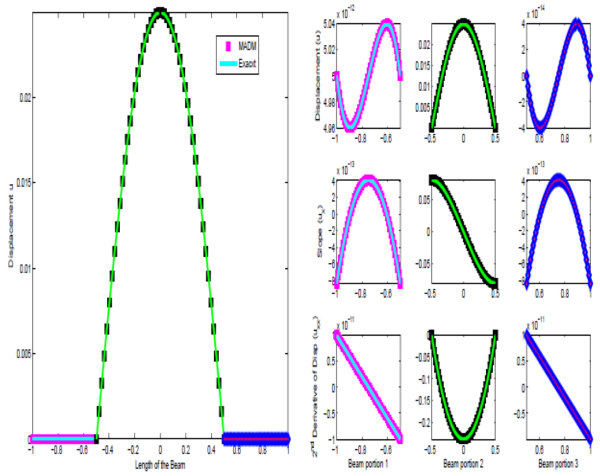
$$u(x) = \begin{cases} \left(-\frac{2}{3}x^3 - \frac{3}{2}x^2 - \frac{13}{12}x + \frac{1}{4}\right)\varepsilon & \text{for } -1 \le x \le -\frac{1}{2}, \\ 0.5 - \frac{1}{\phi_3}\left(\phi(\sin(x)\sinh(x)) + \phi_2(\cos(x)\cosh(x))\right), & \text{for } -\frac{1}{2} \le x \le \frac{1}{2}, \\ \left(-\frac{2}{3}x^3 + \frac{3}{2}x^2 - \frac{13}{12}x + \frac{1}{4}\right)\varepsilon, & \text{for } \frac{1}{2} \le x \le 1, \end{cases}$$

$$u(x) = \begin{cases} \left(-\frac{2}{3}x^3 - \frac{3}{2}x^2 - \frac{13}{12}x + \frac{1}{4}\right)\varepsilon & \text{for } -1 \le x \le -\frac{1}{2}, \\ 0.5 - \frac{1}{\phi_3}\left(\phi(\sin(x)\sinh(x)) + \phi_2(\cos(x)\cosh(x))\right), & \text{for } -\frac{1}{2} \le x \le \frac{1}{2}, \\ \left(-\frac{2}{3}x^3 + \frac{3}{2}x^2 - \frac{13}{12}x + \frac{1}{4}\right)\varepsilon, & \text{for } \frac{1}{2} \le x \le 1, \end{cases}$$

(24)

Where $\phi_1 = \sin(\frac{1}{2})\sinh(\frac{1}{2}), \phi_2 = \cos(\frac{1}{2})\cosh(\frac{1}{2})$ $\phi_1 = \sin(\frac{1}{2})\sinh(\frac{1}{2}), \phi_2 = \cos(\frac{1}{2})\cosh(\frac{1}{2})$ and $\phi_3 = \cos(1) + \cosh(1)$ $\phi_3 = \cos(1) + \cosh(1)$.

The boundary value problem from example above was solved by the present method over the intervals $[-1, -\frac{1}{2}]$ $[-1, -\frac{1}{2}]$, $[-\frac{1}{2}, \frac{1}{2}]$ and $[\frac{1}{2}, 1]$ $[\frac{1}{2}, 1]$, when $\varepsilon = 10^{-6}$ $\varepsilon = 10^{-6}$.



[Displacement u (Present technique & Exact).]
[Obtained results with three different portions]

Figure 2: Variation of Displacement, Slope and 2^{nd} 2^{nd} Derivative of Displacement (u_{xx}) (u_{xx}) diagrams with x (Present technique & Exact).

3.2. Homogeneous 2^nd Order Multi-Support Cable Stayed Bridge

The continuous support 2^{nd} 2^{nd} order multi-support cable stayed bridge equation and boundary conditions shown are

$$u''(x) = \begin{cases} f & \text{for } 0 \le x \le \frac{\pi}{4} \text{ and } \frac{3\pi}{4} \le x \le \pi, \\ u + f - 1 & \text{for } \frac{\pi}{4} \le x \le \frac{3\pi}{4}, \end{cases}$$

$$u''(x) = \begin{cases} f & \text{for } 0 \le x \le \frac{\pi}{4} \text{ and } \frac{3\pi}{4} \le x \le \pi, \\ u + f - 1 & \text{for } \frac{\pi}{4} \le x \le \frac{3\pi}{4}, \end{cases} \tag{25}$$

with the boundary conditions $u(0) = u(\pi) = 0,$ $u(0) = u(\pi) = 0,$ (26)

Considering the system of differential equations (25) when f = 0

$$u'' = \begin{cases} 0, & \text{for } 0 \le x \le \frac{\pi}{4} \text{ and } \frac{3\pi}{4} \le x \le \pi, \\ u - 1, & \text{for } \frac{\pi}{4} \le x \le \frac{3\pi}{4}, \end{cases}$$

$$u'' = \begin{cases} 0, & \text{for } 0 \le x \le \frac{\pi}{4} \text{ and } \frac{3\pi}{4} \le x \le \pi, \\ u - 1, & \text{for } \frac{\pi}{4} \le x \le \frac{3\pi}{4}, \end{cases} \tag{27}$$

with the boundary conditions in equation (26).

Innovative decomposition Solution:

The above equation (27) can be written as $L_x u = 0$ and $u - 1$ $L_x u = 0$ and $u - 1$, where the linear operator is $L_x = \frac{d^2}{dx^2}$ $L_x = \frac{d^2}{dx^2}$. Pre-multiplying both sides of the equation (27) by L_x^{-1} L_x^{-1} for each case. To calculate the components of the decomposition series (27) for u(x) by using the innovative decomposition approach outlined in the previous problem, considering the following three cases:

Case I: In the innovative decomposition approach, split the terms (28) into two parts, one is assigned to the $zero^{th}$ $zero^{th}$ component u_0 u_0(x) and the remaining part is assigned to $u_1(x)$ $u_1(x)$ among other terms, for 0 $\le$ x $\le$ $\frac{\pi}{4}$ $\frac{\pi}{4}$. In this case, implement the innovative decomposition approach and obtain the recursive relation

$$\begin{aligned} u_0 &= u(0) = \alpha_1 \\ u_1 &= u'(0)x + L_x^{-1}(u_0) \\ &\vdots \\ u_n &= L_x^{-1}(u_{n-1}), n \ge 1. \\ u_0 &= u(0) = \alpha_1 \\ u_1 &= u'(0)x + L_x^{-1}(u_0) \\ &\vdots \\ u_n &= L_x^{-1}(u_{n-1}), n \ge 1. \end{aligned} \tag{28}$$

The boundary condition u'(0) is directly taken from the equation (26). Nonetheless, such approach is needed to evaluate the accuracy of the numerical scheme.

Case II: For $\frac{\pi}{4} \le x \le \frac{3\pi}{4}$, to determine the components u_n u_n, $n > 0$, the innovative decomposition approach will be implemented in this case. On these identifications, the obtained recursive relation

$$\begin{aligned} u_0 &= u(\tfrac{\pi}{4}) \\ u_1 &= u'(\tfrac{\pi}{4})(x - \tfrac{\pi}{4}) + L_x^{-1}(-1) + L_x^{-1}(u_0) \\ &\vdots \\ u_n &= L_x^{-1}(u_{n-1}), n \ge 2. \\ u_0 &= u(\tfrac{\pi}{4}) \\ u_1 &= u'(\tfrac{\pi}{4})(x - \tfrac{\pi}{4}) + L_x^{-1}(-1) + L_x^{-1}(u_0) \\ &\vdots \\ u_n &= L_x^{-1}(u_{n-1}), n \ge 2. \end{aligned}$$

The boundary conditions u $(\frac{\pi}{4})\&u'(\frac{\pi}{4})$ $\frac{\pi}{4})\&u'(\frac{\pi}{4})$ are taken directly from solution obtained in case I.

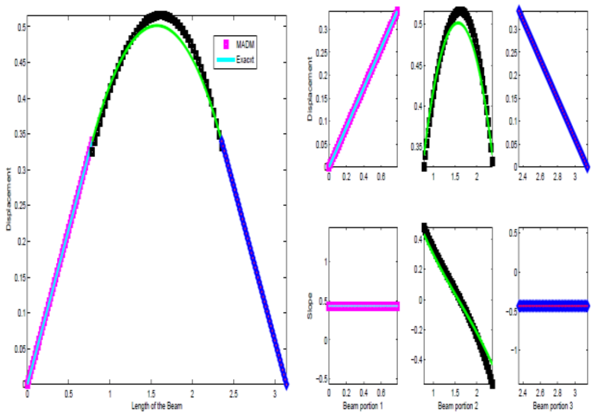
Case III: For $\frac{3\pi}{4} \le x \le \pi \frac{3\pi}{4} \le x \le \pi$, as in case I, obtain the recursive relation

$$\begin{aligned} u_0 &= u(\tfrac{3\pi}{4}) \\ u_1 &= u'(\tfrac{3\pi}{4})(x - \tfrac{3\pi}{4}) + L_x^{-1}(u_0) \\ &\vdots \\ u_n &= L_x^{-1}(u_{n-1}), n \ge 1. \\ u_0 &= u(\tfrac{3\pi}{4}) \\ u_1 &= u'(\tfrac{3\pi}{4})(x - \tfrac{3\pi}{4}) + L_x^{-1}(u_0) \\ &\vdots \\ u_n &= L_x^{-1}(u_{n-1}), n \ge 1. \end{aligned}$$
(29)

The innovative decomposition solution is

$$u(x) = \begin{cases} \frac{6124173888997887}{4503599627370496}\frac{x}{\pi}, & \text{for } 0 \le x \le \frac{\pi}{4}, \\ -0.05922\,x + 9.37635\exp^{-17} x^{20} - \\ +1.187\exp^{-18} x^{21}, & \text{for } \frac{\pi}{4} \le x \le \frac{3\pi}{4}, \\ 1.35984 - 0.43285\,x & \text{for } \frac{3\pi}{4} \le x \le \pi, \end{cases}$$

$$u(x) = \begin{cases} \frac{6124173888997887}{4503599627370496}\frac{x}{\pi}, & \text{for } 0 \le x \le \frac{\pi}{4}, \\ -0.05922\,x + 9.37635\exp^{-17} x^{20} - \\ +1.187\exp^{-18} x^{21}, & \text{for } \frac{\pi}{4} \le x \le \frac{3\pi}{4}, \\ 1.35984 - 0.43285\,x & \text{for } \frac{3\pi}{4} \le x \le \pi, \end{cases}$$
(30)



[Displacement u (Present technique & Exact)] [Displacement & Slope versus x with three different portions] Figure 3: Variation of Displacement and Slope diagrams with x (Present technique & Exact).

Exact Solution: The exact solution for this problem is referred by Khalifa et al. [5] for contact problems and result is shown in figure (3).

$$u = \begin{cases} \frac{4}{\gamma_1}x, & \text{for } 0 \le x \le \frac{\pi}{4} \text{ and } \frac{3\pi}{4} \le x \le \pi, \\ 1 - \frac{4}{\gamma_2}\cosh(\frac{\pi}{2} - x), & \text{for } \frac{\pi}{4} \le x \le \frac{3\pi}{4}, \end{cases}$$

$$u = \begin{cases} \frac{4}{\gamma_1}x, & \text{for } 0 \le x \le \frac{\pi}{4} \text{ and } \frac{3\pi}{4} \le x \le \pi, \\ 1 - \frac{4}{\gamma_2}\cosh(\frac{\pi}{2} - x), & \text{for } \frac{\pi}{4} \le x \le \frac{3\pi}{4}, \end{cases}$$
(31)

where $\gamma_1 = \pi + 4\coth(\frac{\pi}{4}) \gamma_1 = \pi + 4\coth(\frac{\pi}{4})$ and $\gamma_2 = \pi\sinh(\frac{\pi}{4}) + 4\cosh(\frac{\pi}{4}) \gamma_2 = \pi\sinh(\frac{\pi}{4}) + 4\cosh(\frac{\pi}{4})$.

IV. DISCUSSION OF RESULTS:

It is a problem of continuous support cable stayed bridge with obstacles at the cable points. The equation (27) is solved over the intervals $(0,\frac{\pi}{4}) (0,\frac{\pi}{4})$, $(\frac{\pi}{4},\frac{3\pi}{4}) (\frac{\pi}{4},\frac{3\pi}{4})$ and $(\frac{3\pi}{4},\pi) (\frac{3\pi}{4},\pi)$ using the innovative decomposition approach developed. An innovative decomposition approach and exact solutions agree almost perfectly with errors smaller than $10^{-5} 10^{-5}$. It is possible to remedy the error further by considering more terms of the new polynomials. Figure (3) shows displacements and slopes within regions $(0,\frac{\pi}{4}) (0,\frac{\pi}{4})$, and $(\frac{\pi}{4},\frac{3\pi}{4}) (\frac{\pi}{4},\frac{3\pi}{4})$. While figure (3b) shows all of the above in one figure on a larger scale. Figure (3b) shows in region $(0,\frac{\pi}{4}) (0,\frac{\pi}{4})$, and $(\frac{3\pi}{4},\pi) (\frac{3\pi}{4},\pi)$ results deficits linear variation in displacement and constant in slope. The region $(\frac{\pi}{4},\frac{3\pi}{4}) (\frac{\pi}{4},\frac{3\pi}{4})$ shows parabolic nature in displacement and linear variation in slope. All the results are match well with available analytical solution.

V. CONCLUSIONS

A number of linear obstacle cable stayed and continuous support bridge problems which are generally encountered in engineering are solved using innovative decomposition approach. Therefore the results from innovative decomposition approach are compared with the exact solution. The innovative decomposition approach procedure is simple to apply for multi-support problems also. Like analytical solution, one can get convergent solution using present novel technique. An innovative decomposition approach procedure is systematic and simple. The novel technique can apply to any type ordinary or partial differential equations. The above problems deficits that it is possible to remedy the error further by considering more terms of the innovative decomposition method polynomials. Above figure (2) shows displacements, slopes and 2 $^{nd\ nd}$ derivative of the displacements positive sine nature, negative sine nature and slope is same as displacement nature and linear variation along the beam for 2 $^{nd\ nd}$ derivative region. All the results are match well with available analytical solution. In the second problem, figure (3) shows displacements and slopes deficits linear variation in displacement and constant in slope. The region $(\frac{\pi}{4}, \frac{3\pi}{4})(\frac{\pi}{4}, \frac{3\pi}{4})$ shows parabolic nature in displacement and linear variation in slope. Demonstrated results deficits that the innovative method can use any type of typical problems like obstacle linear problems like cable stayed bridges and also for which the closed form solutions are cumbersome.

VI. REFERENCES

[1] P V Ramana, 2012. An innovative technique for solving laminated beams, Springer special edition, 2, 18-35.

[2] Adomian, G. 1994. Solving frontier problems of physics: The decomposition method (2nd edn). Kluwer, Dordrecht.

[3] Lewy, H. and Stampacchia, G. 1969. On the regularity of the solutions of the variational inequalities Communication for Pure Application of Mathematics, 22, 153-188.

[4] Al-Said, E.A. and Noor, M.A. 1998. Numerical solutions of a system of fourth order boundary value problems International Journal of Computational Mathematics, 71, 347-355

[5] Khalifa, A. K. and Noor, M. A. 1990. Quintic splines solutions of a class of contact problems Mathematical and Computational Modeling, 13, 51-58.

[6] Noor, M. A. and Al-Said, E. A. 2000. Numerical solutions of fourth order variational inequalities International Journal of Computational Mathematics, 75, 107-116.

[7] Noor, and Jian-Lin, Li. 2009. Adomian's decomposition method and homotopy perturbation method in solving nonlinear equations, Journal of Computational and Applied Mathematics, 228(1), 168-173.

[8] Al-Said, E. A. and Noor, M. A. 2002. Quartic spline method for solving fourth order obstacle boundary value problem ,application for Mathematical Computation , 43, 107-116.

[9] Noor, M. A. and Tirmizi, S. I. 1986. Numerical methods for unilateral problems , Journal of Computational Application and Mathematics, 16, 387-395.

[10] Noor, M. A. and Tirmizi, S. I. 1991. Numerical methods for a class of contact problems ,International Journal of Engineering Science, 29, 513-521.

[11] Repaci, A. 1990. Nonlinear dynamical systems: on the accuracy of Adomian's decomposition method, Applied Mathematical Literature, 3, 35-39.

[12] Wazwaz, A. M. 2002. A new method for solving singular initial value problems in the second order ordinary differential equations,Applied Mathematics and Computation,128,45-57.

[13] Morawetz 1991. The decay of solution of the exterior initial boundary value problem for the wave equations. Pure Applied mathematics, 14, 561–568.

[14] Aluru, N. (2000). A point collocation method based on reproducing kernel approximations. International Journal for Numerical Methods in Engineering, 47, 1083–1121.

[15] J.N. Reddy. An Introduction to the Finite Element Method. McGraw Hill Book Company, 1995

◈◈◈