

# Kinematic Modeling of Quadruped Robot

Smita A. Ganjare, V S Narwane & Ujwal Deole

<sup>1&2</sup> K. J Somaiya College of Engineering, Mumbai University,  
Product Development, Larsen & Toubro LTD, Powai

E-mail : smita.ganjare15@gmail.com, vsnarwane@yahoo.com, Ujwal.deole@larsentoubro.com

**Abstract** – This Paper presents design of a quadruped robot which tries to walk with medium speed on flat terrain. The design of the quadruped is inspired by the four legged animal, two joints of the leg enable to perform two basic motions: lifting and stepping. The kinematic modeling proposes necessary conditions for stable walk on flat terrain. Brief guidelines for the design of the leg mechanism are presented through the study of various joints, links and degrees of freedom. Also a specific semicircular trajectory is proposed which provide the movement pattern of the legs.

**Key Words** —*Legged robots, quadruped robot, forward and inverse kinematics, trajectory planning.*

## I. INTRODUCTION

This paper presents an introduction to mobile robotics. The development of mobile robotic platforms is an important and active area of research. Within this domain, the major focus has been to develop on wheeled or tracked systems that cope very effectively with flat and well-structured solid surfaces (e.g. laboratories and roads) [1]. In recent years, there has been considerable success with robotic vehicles even for off-road conditions. However, wheeled robots still have major limitations and difficulties in navigating uneven and rough terrain. These limitations and the capabilities of legged animals encouraged researchers for the past decades to focus on the construction of biologically inspired legged machines. These robots have the potential to outperform the more traditional designs with wheels and tracks in terms of mobility and versatility [2].

Over the past 40 years, a variety of engineers and scientists have embraced the opportunity of legged locomotion, building a diverse set of ingenious and inspiring legged robots. For example, see Bern's, (2006), Kar (2003) and for many examples. The Leg

Laboratory robots did a good job of demonstrated the feasibility of dynamically balanced legged systems; they had two primary limitations that would need to be addressed to build practical legged vehicles. One is the need for on-board power so the robot could operate in the field without hoses and wires. Another is the need for control algorithms that provide locomotion and stability on rough terrain [5].

Here main concentrate is on the design of quadruped involving two degrees of freedom where the robot is able for lifting and stepping only.

The assumptions for deriving the kinematic model are as follows:

- a) Surface with alternating pair of legs.
- b) The body is held at a constant height and parallel to the ground plane during locomotion.
- c) The center of gravity of the body is assumed to be at the geometric center of the body.

This paper initially introduces the formation of co-ordinate system linkage of quadruped robot and then the forward and inverse kinematic equations of the quadruped robot are formulated. Finally, 3D model of the robot is prepared using UG-NX.

## II. FORWARD KINEMATIC MODEL

The mechanical structure of the robot is mainly divided into two parts: body and legs. The body is the rigid box; the four legs are distributed symmetrically along the body and each leg has the same structure. The links are made up of a series of rigid linkages connected by rotating joints. There are two joints for each leg which are hip joint and knee joint [4].

The kinematic modeling of the robot is carried out to realize robot automatic control. Generally, there are two main problems in robot kinematics research. The

first one is to assign joint angles to robot and then calculate the position and orientation, which is called forward kinematic modeling. The other is to calculate all the corresponding joint angles of robot given position and orientation of the robot, is called inverse kinematic modeling. Forward kinematics is done initially by assigning the co-ordinate frame to each link. D-H algorithm is used to assign frames to the links. The single line diagram is as follows,

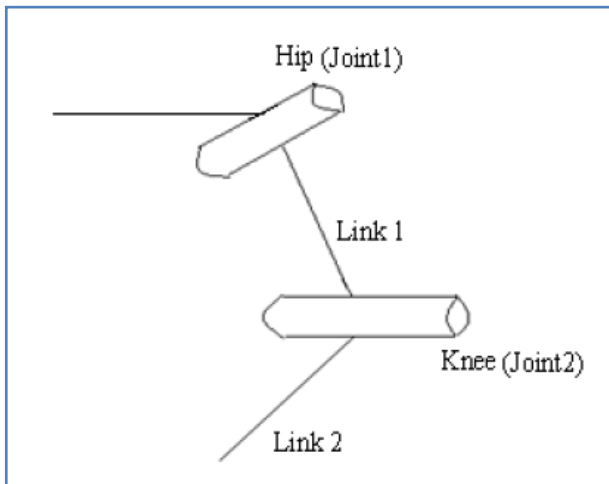


Fig 1: Single line diagram of each link

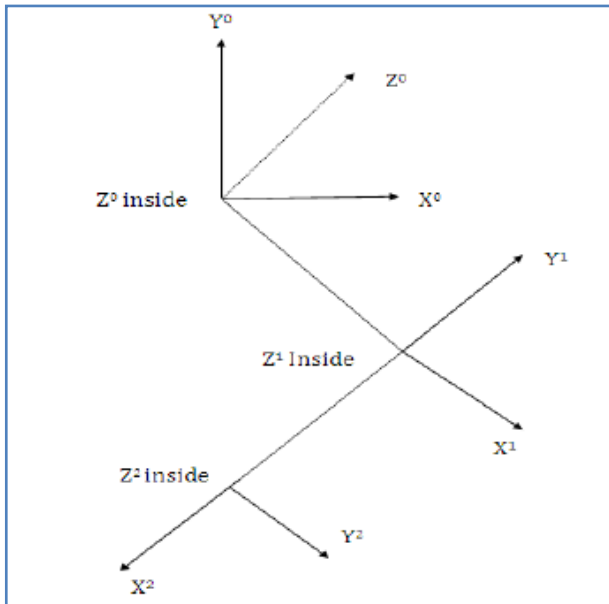


Fig 2: Co-ordinate system of each linkage

The final homogeneous transformation matrix of co-ordinate system is given by,

$${}^0_1T = \begin{bmatrix} C_1 & -S_1 & 0 & a_1C_1 \\ S_1 & C_1 & 0 & a_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1_2T = \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transformation matrix  ${}^0_2T$  of foot co-ordinate system {2} with respect to base co-ordinate system {0} is given by,

$${}^0_2T = \begin{bmatrix} C_1C_2 - S_1S_2 & -C_1S_2 - C_2S_2 & 0 & a_1C_1 + a_2C_1C_2 - a_2S_1S_2 \\ C_1S_2 + C_2S_1 & C_1C_2 - S_1S_2 & 0 & a_1S_1 + a_1C_1S_2 + a_2C_2S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

As the four legs of the robot have same symmetry, we can calculate the forward and inverse kinematic model for one leg and then apply it to remaining 3 legs. Each leg has 2 degrees of freedom, hence total there are total 8 degrees of freedom for four legs.

### III. INVERSE KINEMATIC MODEL

The analysis of forward kinematic problem can be used to determine whether the motion of the robot meets with the requirements or not, it can also be used as preparation for the analysis of inverse kinematic problem. The inverse kinematic model verifies whether the robot can achieve the required position and orientation or not. The inverse kinematic model finds out the rotation angle with which the robot can move [10]. The solution to the inverse kinematic problem is found out with the help of Tool Configuration Vector (TCV) [10]. TCV (Tool configuration vector) is the compact representation of the foot tip position and foot orientation. The direction in which the foot is oriented or pointed is given by the approach vector or the last column of rotation matrix specifies the roll, but does not give any information about yaw and pitch.

The tool configuration vector is given by,

$$W(q) = \begin{bmatrix} w^1 \\ w^2 \end{bmatrix} = \begin{bmatrix} p \\ (exp_{\pi}^{qn}) * r^3 \end{bmatrix}$$

Where,

$w$  = Tool Configuration Vector

$w^1$  = First three components of  $w$ , represents the foot position  $p$ .

$w^2$  = Next three components of  $w$ , which represents foot orientation.

$$T_n = \begin{bmatrix} R_{11} & R_{12} & R_{13} & P_1 \\ R_{21} & R_{22} & R_{23} & P_2 \\ R_{31} & R_{32} & R_{33} & P_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Equating equation of T and  $T_n$ ,

$$p_1 = w_1 = a_1 C_1 + a_2 C_1 C_2 - a_2 S_1 S_2 \quad \text{----- (1)}$$

$$p_2 = w_2 = a_1 S_1 + a_1 C_1 S_2 + a_2 C_2 S_1 \quad \text{----- (2)}$$

$$p_3 = w_3 = 0$$

Squaring and adding  $w_1$  and  $w_2$ .

$$w_1^2 + w_2^2 = a_1^2 C_1^2 + a_2^2 C_1^2 C_2^2 - a_2^2 S_1^2 S_2^2 + a_1^2 S_1^2 + a_2^2 C_1^2 S_2^2 + a_2^2 C_2^2 S_1^2$$

After solving the equation for knee angle,

$$\theta_2 = \cos^{-1} \left( \frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2a_1 a_2} \right)$$

Further, for calculating  $\theta_1$ .

$$w_1 = a_1 C_1 + a_2 C_1 C_2 - a_2 S_1 S_2$$

$$w_2 = a_1 S_1 + a_1 C_1 S_2 + a_2 C_2 S_1$$

Expand  $C_1 C_2$  and  $S_1 S_2$  using sum of sines and cosines; isolate  $C_1$ ,  $S_1$ . Write in matrix form, collect all  $C_1$  terms and  $S_1$  terms and then find out  $\theta_1$ .

$$w_1 = a_1 C_1 + a_2 C_1 C_2 - a_2 S_1 S_2 \\ = (a_1 + a_2 C_2) C_1 - (a_2 S_2) S_1 \quad \text{----- (3)}$$

$$w_2 = a_1 S_1 + a_1 C_1 S_2 + a_2 C_2 S_1 \\ = (a_2 S_2) C_1 + (a_1 + a_2 C_2) S_1 \quad \text{----- (4)}$$

The hip angle is given by,

$$\theta_1 = \tan^{-1} \left( \frac{(a_1 + a_2 C_2) - (a_2 S_2) w_1}{(a_1 + a_2 C_2) w_1 + (a_2 S_2) w_2} \right)$$

#### IV. TRAJECTORY PLANNING

The goal of trajectory planning is to describe the requisite motion of the manipulator as a time sequence of joints or links locations and derivatives of locations, which are generated by “interpolating” or “approximating” the desired path by the polynomial function. These time base sequence locations, also called time law or time history, obtained from the trajectory planning serve as reference input or ‘control set points’ to the manipulator’s control system. The control system, in turn, assures that the manipulator executes the planned trajectories.

Every robot leg has at least capability to move from an initial location (position and orientation) to a final desired location. The kinematics and dynamics of the manipulator govern the transition of posture with actuators exerting the desired torques. As no limits must be violated and no unmodelled behavior should be executed, it is necessary to devise planning algorithms that generate smooth trajectories.

One of the main objectives of any trajectory planning algorithm is to achieve a smooth motion of the

foot. A smooth function is one that is continuous and has a continuous first derivative [11]. The smooth motion has the important advantage of reducing the vibrations and wear of the mechanical system. Here we consider the leg encounter a semicircular path. Some assumptions are made for this kind of trajectory which is:

- Some data has been assumed from the Bigdog and HyQ robots, which are rough terrain mechanical mule. The body length of the robot should be 1m long and is at near about 0.8 m from the ground.
- The legs of the robot are placed at a distance of 0.6 m from each other and all the legs have the same symmetry.
- The alternate pair of legs moves at a time i.e leg 1 and leg 3 move simultaneously. This is called as trotting. The distance between two legs i.e front and rear one is 0.6m and the distance covers in one stance is 0.3m. The speed of the robot is 2 m/sec.

Firstly the 3D model of the robot has been developed in UG-NX, some of the design parameters are  $L1=L2=0.5m$ .

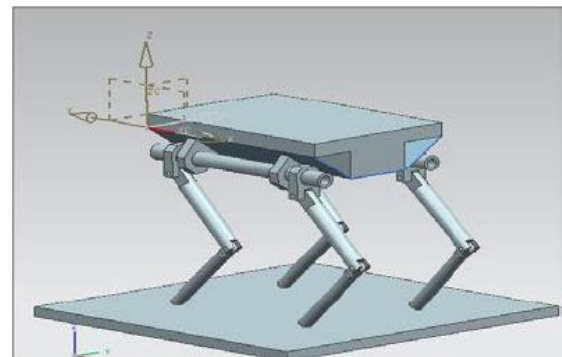


Fig 3: 3D CAD model of quadruped robot

Secondly, with the help of semicircular trajectory, parametric equation for this kind of quadruped robot is deduced.

$$x = r - r * \cos(\pi * t)$$

$$y = r * \sin(\pi * t)$$

From these equations, the x and y co-ordinates of the robot feet is derived. By using these co-ordinates and with the help of inverse kinematic model, the hip and knee angles of each leg can be found out. Angles for alternate pair of legs are calculated, considering leg1-leg3 simultaneously moves first and leg2-leg4 will follow [4].

The trajectory of the robot’s leg is set with the help of parametric equations and the values of angles are found out from the inverse kinematics. One step should

take 6.67 sec with uniform speed. The graphics developed using Matlab2011a, is shown in fig 3 to fig 7.

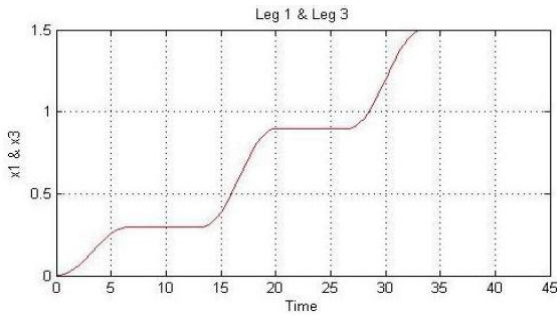


Fig 4: Movement of Leg 1 and Leg 3 in x-direction

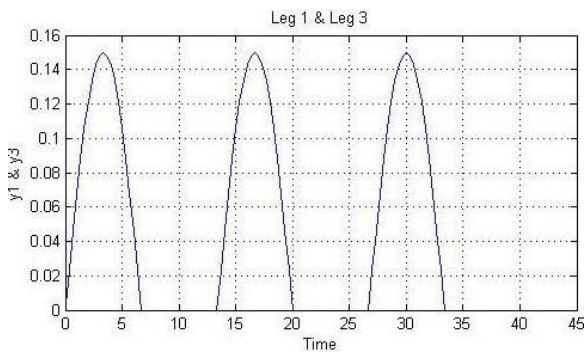


Fig 5: Movement of Leg 2 and Leg 4 in x-direction We can also check out how the robot moves along y-axis.

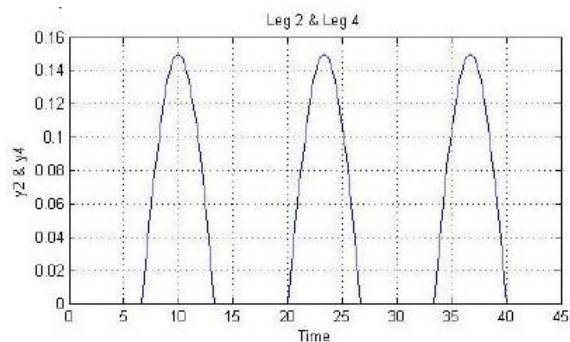


Fig 6: Movement of Leg 1 and Leg 3 in y-direction

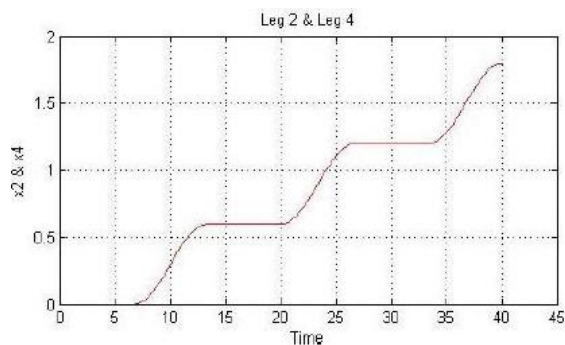


Fig 7: Movement of Leg 2 & leg 4 in y-direction.

There is the verification of the forward kinematic model and inverse kinematic model with the help of robotic tool box. The robotic toolbox provides many functions that are useful in robotics such as kinematics, dynamics and trajectory planning. The toolbox is useful for simulation as well as in analyzing results from experiments with real robots. Robotics mathematics can be programmed in MATLAB2011a and simultaneously results can be checked with MATLAB-Robotics toolbox.

The result that is obtained by direct and inverse kinematics is validated using Robotic Toolbox. There are two links L1 and L2 of length 0.5 m each.

L1=link (0 0.5 60 0)

L2=link (0 0.5 60 0)

R=robot ({L1 L2})

T=fkine(r,q)

q=ikine(r,T)

By using robotic toolbox, the transformation matrix obtained is given as follows :

$$T = \begin{bmatrix} 0.6559 & -0.7548 & 0 & -0.0779 \\ 0.7548 & -0.6559 & 0 & 0.8104 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This result is verified with the results which are obtained from direct and inverse kinematics.

q= [1.0471 1.2391];

Above are the angles of hip and knee in radians for a specified position.

## V. APPROACH FOR DIFFERENTIAL MOTION AND STATICS

The direct and inverse kinematic model establishes the relationship between the manipulator's joint displacement and position and orientation of its end effector. These relationships permit the static control of the manipulator to place the end effector at specified location and make it traverse a specified path in space. However, for the manipulator not only the final location of the end-effector is of concern, but also the velocities at which the end-effector would move to reach the final location are an equally important concern.

This requires the co-ordination of the instantaneous end-effector velocity and joint velocities. One way to achieve this is to take the time derivative of kinematic equations of the manipulator. The transformation from the joint velocities to the end-effector velocity is described by a matrix, called the Jacobian. The Jacobian

matrix which is dependent on manipulator configuration is a linear mapping from velocities in joint space to velocities in Cartesian space. The Jacobian is one of the most important tools for characterization of differential motions of the manipulator. At certain locations in joint space, the Jacobian matrix may lose rank and it may not be possible to find its inverse. These locations are referred to as ‘singular’ Configurations [10].

It is observed that the Cartesian velocities (linear as well as angular) of the end effector are linearly related to the joint velocities. The relationship between infinitesimal (differential) joint motions with infinitesimal (differential)

changes in the end-effectors position (and orientation) has been investigated. As these changes take place in infinitesimal (differential) time, there is a need to map instantaneous end-effector velocities,  $V_e$  (in Cartesian space) to instantaneous joint velocities (in joint space). This mapping between differential changes is linear and can be expressed as

$$V_e(t) = J(q) \dots \dots \dots (1)$$

Where,

$V_e(t) = 6 \times 1$  Cartesian velocity vector (End-effector velocity),

$J(q) = 6 \times n$  Manipulator Jacobian or Jacobian matrix,

$q = n \times 1$  vector of  $n$  joint velocities.

$$V_e(t) = [J_1(q) J_2(q) \dots \dots \dots J_n(q)] \dots \dots \dots (2)$$

In the above equation,  $J_i(q)$  is the  $i$ th column of the Jacobian matrix.

We can write the end-effector velocity as,

$$V_e(t) = \begin{bmatrix} v \\ w \end{bmatrix} \dots \dots \dots (3)$$

Equation (3) represents the forward differential motion model or differential kinematics model presented schematically in following figure which is similar to the forward kinematic model. Note that the  $J(q)$  is the function of the joint variables [11]. The first three rows of Jacobian  $J(q)$  are associated with the linear velocity of the end effector  $v$ , while last three rows correspond to the angular velocity  $w$  of the end effector. Each joint of the manipulator generate some linear and/or some angular velocity of the manipulator. Column (i) of the Jacobian matrix (q) is made up of three linear velocity components  $j_{vi}$  and three angular velocity components and  $j_{wi}$  can be expressed as,

$$J_i(q) = \begin{bmatrix} j_{vxi} \\ j_{vyi} \\ j_{vzi} \\ j_{wx_i} \\ j_{wyi} \\ j_{wzi} \end{bmatrix} \dots \dots \dots (4)$$

Where  $j_{vi}$  and  $j_{wi}$  represents the component  $k$  of linear velocity and angular velocity respectively, contributed by joint  $i$  with  $k=x, y, z$  or  $z$  and  $i=1, 2 \dots n$ . The contribution of joint  $i$  to column is computed depending on whether joint  $I$  is prismatic or rotary.

Each column of Jacobian matrix is computed separately and all the columns are combined to form the total Jacobian matrix. Jacobian matrix for rotary joint is given by following equation,

$$J_i(q) = \begin{bmatrix} j_{vi} \\ j_{wi} \end{bmatrix} = \begin{bmatrix} P_{i-1} * {}^{i-1}P \\ P_{i-1} \end{bmatrix} \dots \dots \dots (5)$$

The Jacobian matrix column  $J_1$  for joint 1, which is a rotary joint, is determined as follows,

The joint axis vector  $P_0$  ( $P_{i-1}$  for  $i=1$ ) is

$$P_0 = R_0 u \dots \dots \dots (6)$$

The transformation matrix and the rotation matrix are the identity matrix. Thus,

$$P_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \dots \dots \dots (7)$$

The end-effector position vector (for  $i=1$  and  $n=2$ ) is determined from following equation,

$${}^{i-1}P = {}^0T_n O_n - {}^{i-1}T_n O_n \dots \dots \dots (8)$$

$${}^0P = {}^0T_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - {}^0T_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$${}^0P = \begin{bmatrix} a_1 C_1 + a_2 C_2 - a_2 S_1 S_2 \\ a_1 S_1 + a_1 C_1 S_2 + a_2 C_2 S_1 \\ 0 \\ 0 \end{bmatrix} \dots \dots \dots (9)$$

The first column of Jacobian is calculated by substituting equation (7) and (9) in equation (5). Thus,

$$J_1 = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} * \begin{bmatrix} a_1 C_1 + a_2 C_2 - a_2 S_1 S_2 \\ a_1 S_1 + a_1 C_1 S_2 + a_2 C_2 S_1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} -a_1 S_1 + a_1 C_1 \bar{S}_2 + a_2 C_2 S_1 \\ a_1 C_1 + a_2 C_2 - a_2 S_1 S_2 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

By following the similar steps for joint 2, we can get the Jacobian  $J_2$  as

$$J_2 = \begin{bmatrix} -a_1 C_1 S_2 + a_2 C_2 S_1 \\ a_2 C_2 - a_2 S_1 S_2 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J = [J_1 \quad J_2]$$

$$J = \begin{bmatrix} -a_1 S_1 + a_1 C_1 S_2 + a_2 C_2 S_1 & -a_1 C_1 S_2 + a_2 C_2 S_1 \\ a_1 C_1 + a_2 C_2 - a_2 S_1 S_2 & a_2 C_2 - a_2 S_1 S_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \quad (10)$$

Equation (10) provides the linear and angular velocities of the end effector i.e foot of the robot.

$$\text{That is, } \begin{bmatrix} v_x \\ v_y \end{bmatrix} = J' \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Considering xy plane,

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} w_x \\ w_y \end{bmatrix} = \begin{bmatrix} -a_1 S_1 + a_1 C_1 S_2 + a_2 C_2 S_1 & -a_1 C_1 S_2 + a_2 C_2 S_1 \\ a_1 C_1 + a_2 C_2 - a_2 S_1 S_2 & a_2 C_2 - a_2 S_1 S_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

From the above equation, we get the linear velocity of the foot position. By using Jacobian, we can find out the linear and angular velocity of the final foot position. With the help of Jacobian, we get the three linear velocity and three angular velocity components of the foot.

## VI. CONCLUSION

The kinematic analysis of quadruped robot is the basic of space planning, motion control and optimal design. Here the direct and inverse kinematic models for the robot are calculated. Direct kinematics gives exact position and orientation of robot feet and the inverse kinematics gives the angles of the joints of the legs. The results are verified with robotic toolbox of Matlab2011a. It provides the real time analysis of the joint variables, which lays foundation for following up motion control.

## VII. REFERENCES

- [1] J.A.Tenreiro Machado and Manuel F.Silva , *An Overview Of Legged Robots*, Department of Electrical Engineering, Institute of Engineering of Porto, Porto, Portugal.
- [2] Sven Böttcher, *Principles of robot locomotion*, Seminar 'Human robot interaction'.
- [3] Freyr Hardarson, *Locomotion for difficult terrain*, Mechatronics Division, Department of Machine Design Royal Institute of Technology, Stockholm, Sweden.A Survey Study 1997-04-01.
- [4] Thanhtam Ho, Sunghac Choi, Sangyoon Lee, *Development of a Biomimetic Quadruped Robot*, Department of Mechanical Design and Production Engineering, Konkuk University, Seoul 143-701, Korea.
- [5] "MIT Leg Lab Robots." 13 Oct 2008 <<http://www.ai.mit.edu/projects/leglab/robots/robotmain.html>>.
- [6] "Boston Dynamics: The Leader in Lifelike Human Simulation." 13 Oct 2008 <<http://bostondynamics.com/content/sec.php?section=BigDog>>
- [7] Shibendu Shekhar Roy, Ajay Kumar Singh1, and Dilip Kumar Pratihar, Department of Mechanical Engineering, National Institute of Technology, Durgapur, India *Analysis of Six-legged Walking Robots*, Department of Mechanical Engineering, Indian Institute of Technology, Kharagpur, India.
- [8] Raghu Mandayam Anand, *Kinematic Analysis of a Quadruped Robot capable of functioning as a machine tool*, University of Texas at Arlington.
- [9] Filipe Miguel Teixeira Pereira da Silva, Department of Mechanical Engineering, University of Aveiro and José Antônio Tenreiro Machado, Department of Electrical and Computer Engineering, *Kinematic Analysis and Modeling of Biped Locomotion Systems*, J. Braz. Soc. Mech. Sci. vol.21 no.3 Rio de Janeiro Sept. 1999
- [10] Modeling and Control of Robot Manipulators, By Lorenzo Sciavicco & Bruno Siciliano.
- [11] Robotics and Control, By R.K.Mittal and I.J.Nagrath.

