Multi-State Component Criticality Analysis For Reliability Improvement of Process Plant

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Abstract- This paper proposes a development of model based on markov process for a bread baking system to evaluate the system performance. The system comprises of continuous different five units. Analysis of reliability, long-run availability and mean time before failure of the bread baking system can help in increasing the production and quality of the bread. The results obtained in the present work are considered to be very useful for devising the best possible maintenance strategies. The differential equations have been developed on the basis of probabilistic approach using a transition diagram. These equations have further been solved using normalizing condition in order to develop the steady state availability. The effect of repair rate on most vulnerable items of the system is examined to realize the highest level of performance. Increase in availability of system confers many benefits such as more profit, improved delivery performance and reduced lead times.

Keywords-Availability, Markov Process, Performance Modeling, maintenance.

I. INTRODUCTION

Reliability engineering had gained its importance in recent years due to the results it had shown. Reliability, in general can be defined as the probability of a system/device performing its anticipated purpose adequately for the intended period of time under the given operating conditions. Effective maintenance would certainly result in higher availability of the component and higher profits. Many engineering techniques are used in reliability risk assessments, such as reliability hazard analysis, failure mode and effects analysis (FMEA), fault tree analysis (FTA), Reliability Centered Maintenance, material stress and wear calculations, fatigue and creep analysis, human error analysis, reliability testing, etc. It is essential to have high productivity and maximum profit from process plants for their survival. To achieve this end, availability and reliability of equipment in process must be maintained at the highest order. Further the failure is a random phenomenon, always associated with the operating state of any physical system and its causes are either deterioration in the components of the system and/or man handling errors. Therefore the main concern is to maintain system performance measures such as reliability and availability to achieve high profit goals and productivity in regard to system failures. These measures are considered as most significant factor associated with non-repairable and repairable system.

II. SYSTEM DESCRIPTION

A bread baking system is composed of various sub systems such as proofing unit, baking unit, cooling unit, slicing unit and packing unit is shown in Fig.1. The different sub systems and processes are discussed as follows:

Proofing Unit: - Proving or proofing refers to the dough resting period during fermentation after moulding has been accomplished and moulded dough pieces are placed in bread pans or tins. During this resting period the fermentation of dough continues. The dough finally proofed or fermented in baking pan for desired dough height. It is generally carried out at 30-35°C and at 85% relative humidity. Proofing takes about 55-65 minutes. During proofing the dough increases remarkably in volume. The dough expands by a factor of 3-4 during proofing.

Baking: - After proofing the dough is subjected to heat in a baking oven. Baking temperature generally varies depending up on oven and product type but it is generally kept in the range of 220-250°C. The baking time of bread may range from 25 to 30 minutes depending up on size of bread loaf.

Cooling: - After baking, bread is cooled prior to slicing and to prevent condensation of moisture in the wrapper.

Slicing: - In this section, cutting/slicing of bread before facilitate to packaging takes place. Slicing is done with high grade stainless steel cutters as per requirement of lot sizes.

Packing: - After slicing, predefined sizes of bread loafs are packed in different wrappers as per lot requirements for further palletizing.
III. NOTATIONS

Proofing unit (R) : One unit subjected to major failure only.
Baking unit (B) : One unit subjected to major failure only.
Cooling unit (C) : One unit subjected to major failure only.
Slicing unit (N) : One unit subjected to major failure only.
Packing unit (P) : One unit subjected to major failure only.

Superscript ‘o’ : Indicate component/subsystem is operative
Superscript ‘g’ : Indicate component/subsystem is good but not operative.
Superscript ‘r’ : Indicate component/subsystem is under repair.

\[ \lambda_i \] : Indicate failure rates of components R, B, C, N, P (k=1, 2, 3, 4, 5).

\[ \mu_i \] : Indicate repair rates of components R, B, C, N, P (k=1, 2, 3, 4, 5).

\[ P_i(t) \] : state probability that the system is in \( i^{th} \) state at time \( t \).
\[ s \] : Laplace transform variable

Dash (') : represent derivatives w.r.t. ‘t’

IV. ASSUMPTIONS

a) All the subsystems are initially operating.
b) All the sub-systems are initially in good state.
c) Each unit has two states viz., good and failed.
d) Each unit is as good as new after repair.
e) The failure rates and repair rates of all units are taken constant.
f) Failure and repair events are statistically independent

V. PERFORMANCE MODELING OF THE SYSTEM

Probability consideration gives the following differential equations associated with the state transition diagram as shown in figure 2.

\[ P_{i}(t) + \beta_i P_{i}(t) = \mu_i P_{i-1}(t) + \mu_i P_{i+1}(t) + \mu_i P_{i+1}(t) + \mu_i P_{i+1}(t) \]

Where \( \beta_i = (\lambda_i + \lambda_i + \lambda_i + \lambda_i) \)

Similarly

\[ P_{i}(t) + \mu_i P_{i}(t) = \lambda_i P_{i+1}(t) \]

With initial conditions at time \( t = 0 \):
\[ P_i(0) = 1 \quad \text{When } i = 0 ; \]
\[ P_i(0) = 0 \quad \text{When } i \neq 0 \]

I) Solution of Equations:

Solving above equations after taking Laplace transform of equations, the following Laplace transforms of state probabilities are obtained

\[ sP_{i}(s) + \beta_i P_{i}(s) = \mu_i P_{i-1}(s) + \mu_i P_{i+1}(s) + \mu_i P_{i+1}(s) + \mu_i P_{i+1}(s) \]

\[ sP_{i}(s) + \mu_i P_{i}(s) = \lambda_i P_{i+1}(s) \]

\[ sP_{i}(s) + \mu_i P_{i}(s) = \lambda_i P_{i+1}(s) \]
\[
\begin{align*}
    sP_i(t)+\mu_iP_i(t) = \lambda_iP_i(t) & \quad \ldots \text{5} \\
    sP_i(t)+\mu_iP_i(t) = \lambda_iP_i(t) & \quad \ldots \text{6}
\end{align*}
\]
Solving Equations (3), (4), (5) & (6)
\[
P_i(s) = \frac{\lambda_i}{s+\mu_i}P_i(s) \quad \text{where} \quad M_i = \frac{\lambda_i}{s+\mu_i}
\]
Similarly
\[
P_i(s) = \frac{\lambda_i}{s+\mu_i}P_i(s)
\]
\[
M_j = \frac{\lambda_j}{s+\mu_j}
\]
On Putting the values of \( P_i(s) \) to \( P_j(s) \) in equation (2) and by using the normalizing condition, we get
\[
(s+\beta_j)P_i(s) - 1 + (\mu_iK_i + \mu_iK_i + \mu_iK_i + \mu_iK_i + \mu_iK_i)P_i(s) = 0
\]
\[
P_i(s) = \left[(s+\beta_j) - (\mu_iK_i + \mu_iK_i + \mu_iK_i + \mu_iK_i + \mu_iK_i)\right]^{-1}
\]
\[
\ldots (7)
\]
Availability Function \( A(s) \) for the system is given as:
\[
A(s) = P_0(s)
\]
Where \( P_0(s) \) is given by equation (7)
Inversion of \( A(s) \) gives the availability function \( A(t) \)

**Steady State Behavior of the System**

The steady state behavior of the system can be analyzed by setting \( t \to \infty \) and \( d/dt \to 0 \), the limiting probabilities from equations (2) – (6) are:
\[
\beta_iP_i+\mu_iP_i = \lambda_iP_i \quad \text{or} \quad P_i = \frac{\lambda_i}{\mu_i}P_i \quad \ldots (8)
\]
Similarly
\[
P_j(s) = \frac{\lambda_j}{s+\mu_j}P_j(s)
\]
Using normalizing condition (i.e sum of all the probabilities is equal to one) i.e. \( \sum_{i=0}^{S} P_i = 1 \) gives:
\[
P_0 = \left[1 + \sum_{i=0}^{S} \frac{\lambda_i}{\mu_i}\right]^{-1} \quad \ldots (9)
\]

Steady state availability of the system is given as
\[
A(\infty) = P_0
\]
Where \( P_0 \) is given by equation (9)
So \( A(\infty) = 0.8375 \) (taking values as mention in the table-1)

**Table 1**

<table>
<thead>
<tr>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( \lambda_3 )</th>
<th>( \lambda_4 )</th>
<th>( \lambda_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>0.02</td>
<td>0.01</td>
<td>0.001</td>
<td>0.01</td>
</tr>
<tr>
<td>( \mu_1 )</td>
<td>( \mu_2 )</td>
<td>( \mu_3 )</td>
<td>( \mu_4 )</td>
<td>( \mu_5 )</td>
</tr>
<tr>
<td>0.33</td>
<td>0.50</td>
<td>0.50</td>
<td>0.33</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**VI. AVAILABILITY ANALYSIS**

The effect of change in various parameters on long run availability is studied in this section. The long run availability of the system is computed by alternating the failure and repair rates of the subsystem. This effect, for some important parameters, is shown in the following tables.

**a) Effect of failure rate of Proofing unit on availability**

\( A_{L3} = 0.02, \lambda_3 = 0.01, \lambda_4 = 0.001, \lambda_5 = 0.01, \mu_1 = 0.33, \mu_2 = 0.50, \mu_3 = 0.50, \mu_4 = 0.33, \mu_5 = 0.25. \)

**Table-2 Steady state availability versus failure rate of proofing unit**

<table>
<thead>
<tr>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( \lambda_3 )</th>
<th>( \lambda_4 )</th>
<th>( \lambda_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>0.04</td>
<td>0.05</td>
<td>0.06</td>
<td>0.07</td>
</tr>
<tr>
<td>A</td>
<td>0.8375</td>
<td>0.8169</td>
<td>0.7971</td>
<td>0.7783</td>
</tr>
</tbody>
</table>

**b) Effect of failure rate of Baking unit on availability**

\( \lambda_1 = 0.03, \lambda_3 = 0.01, \lambda_4 = 0.001, \lambda_5 = 0.01, \mu_1 = 0.33, \mu_2 = 0.50, \mu_3 = 0.50, \mu_4 = 0.33, \mu_5 = 0.25. \)

**Table-3 Steady state availability versus failure rate of Baking unit**

<table>
<thead>
<tr>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( \lambda_3 )</th>
<th>( \lambda_4 )</th>
<th>( \lambda_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.04</td>
<td>0.06</td>
<td>0.08</td>
<td>0.1</td>
</tr>
<tr>
<td>A</td>
<td>0.8375</td>
<td>0.8104</td>
<td>0.7850</td>
<td>0.7611</td>
</tr>
</tbody>
</table>
c) Effect of failure rate of Cooling unit on availability
   \[ \lambda_1 = 0.03, \lambda_2 = 0.02, \lambda_3 = 0.001, \lambda_4 = 0.01, \mu_1 = 0.33, \mu_2 = 0.50, \mu_3 = 0.50, \mu_4 = 0.33, \mu_5 = 0.25. \]

   Table 4: Steady state availability versus failure rate of Cooling unit
   \[
   \begin{array}{cccccc}
   \lambda_1 & \mu_1 & 0.33 & 0.43 & 0.53 & 0.63 & 0.73 \\
   A & 0.8375 & 0.8238 & 0.8104 & 0.7975 & 0.7850 \\
   \end{array}
   \]

   d) Effect of failure rate of Slicing unit on availability
   \[ \lambda_1 = 0.03, \lambda_2 = 0.02, \lambda_3 = 0.01, \lambda_4 = 0.01, \mu_1 = 0.33, \mu_2 = 0.50, \mu_3 = 0.50, \mu_4 = 0.33, \mu_5 = 0.25. \]

   Table 5: Steady State availability versus failure rate of Slicing unit
   \[
   \begin{array}{cccccc}
   \lambda_1 & \mu_1 & 0.01 & 0.02 & 0.03 & 0.04 & 0.05 \\
   A & 0.8375 & 0.8238 & 0.8104 & 0.7975 & 0.7850 \\
   \end{array}
   \]

d) Effect of failure rate of Packing unit on availability
   \[ \lambda_1 = 0.03, \lambda_2 = 0.02, \lambda_3 = 0.01, \lambda_4 = 0.001, \lambda_5 = 0.01, \mu_1 = 0.33, \mu_2 = 0.50, \mu_3 = 0.50, \mu_4 = 0.33, \mu_5 = 0.25. \]

   Table 6: Steady State availability versus failure rate of Packing unit
   \[
   \begin{array}{cccccc}
   \lambda_1 & \mu_1 & 0.01 & 0.03 & 0.05 & 0.07 & 0.09 \\
   A & 0.8375 & 0.7850 & 0.7386 & 0.6974 & 0.6605 \\
   \end{array}
   \]

e) Effect of repair rate of Packing unit on availability
   \[ \lambda_1 = 0.03, \lambda_2 = 0.02, \lambda_3 = 0.01, \lambda_4 = 0.001, \lambda_5 = 0.01, \mu_1 = 0.33, \mu_2 = 0.50, \mu_3 = 0.50, \mu_4 = 0.33, \mu_5 = 0.25. \]

   Table 7: Steady State availability versus repair rate of Packing unit
   \[
   \begin{array}{cccccc}
   \mu_1 & 0.33 & 0.43 & 0.53 & 0.63 & 0.73 \\
   A & 0.8375 & 0.8526 & 0.8624 & 0.8691 & 0.8741 \\
   \end{array}
   \]

f) Effect of repair rate of Proofing unit on availability
   \[ \lambda_1 = 0.03, \lambda_2 = 0.02, \lambda_3 = 0.01, \lambda_4 = 0.01, \lambda_5 = 0.01, \mu_1 = 0.33, \mu_2 = 0.50, \mu_3 = 0.50, \mu_4 = 0.33, \mu_5 = 0.25. \]

   Table 8: Steady state availability versus repair rate of Proofing unit
   \[
   \begin{array}{cccccc}
   \mu_1 & 0.50 & 0.60 & 0.70 & 0.80 & 0.90 \\
   A & 0.8375 & 0.8423 & 0.8457 & 0.8482 & 0.8504 \\
   \end{array}
   \]

g) Effect of failure rate of Packing unit on availability
   \[ \lambda_1 = 0.03, \lambda_2 = 0.02, \lambda_3 = 0.01, \lambda_4 = 0.001, \lambda_5 = 0.01, \mu_1 = 0.33, \mu_2 = 0.50, \mu_3 = 0.33, \mu_4 = 0.25. \]

   Table 9: Steady state availability versus repair rate of Cooling unit
   \[
   \begin{array}{cccccc}
   \mu_1 & 0.50 & 0.60 & 0.70 & 0.80 & 0.90 \\
   A & 0.8375 & 0.8399 & 0.8416 & 0.8429 & 0.8439 \\
   \end{array}
   \]

h) Effect of repair rate of Packing unit on availability
   \[ \lambda_1 = 0.03, \lambda_2 = 0.02, \lambda_3 = 0.01, \lambda_4 = 0.001, \lambda_5 = 0.01, \mu_1 = 0.33, \mu_2 = 0.50, \mu_3 = 0.33, \mu_4 = 0.25. \]

   Table 10: Steady State availability versus repair rate of Slicing unit
   \[
   \begin{array}{cccccc}
   \mu_1 & 0.33 & 0.43 & 0.53 & 0.63 & 0.73 \\
   A & 0.8375 & 0.8381 & 0.8383 & 0.8386 & 0.8388 \\
   \end{array}
   \]

j) Effect of repair rate of Packing unit on availability
   \[ \lambda_1 = 0.03, \lambda_2 = 0.02, \lambda_3 = 0.01, \lambda_4 = 0.001, \lambda_5 = 0.01, \mu_1 = 0.33, \mu_2 = 0.50, \mu_3 = 0.50, \mu_4 = 0.33. \]

   Table 11: Steady State availability versus repair rate of packing unit
   \[
   \begin{array}{cccccc}
   \mu_1 & 0.25 & 0.50 & 0.75 & 1.0 & 1.25 \\
   A & 0.8375 & 0.8519 & 0.8568 & 0.8592 & 0.8607 \\
   \end{array}
   \]

VII. RESULTS AND DISCUSSION

Expressions to determine the reliability characteristics of the complex bread baking System are derived. Availability analysis revealed that increase in the failure rate of different sub systems such as proofing unit, baking unit, cooling unit, slicing unit and packing unit reduces the availability of the system. Tables 2 to 11 shows the effect of failure rate of different sub systems on the long run availability of the bread baking system. On the other hand the repair rates of the constituent component increase the availability of the system. These effects are shown in tables 7 to 11. As shown the failure rate of packing unit affects the availability of the system up to larger extent. On the other hand the improved repair rate of proofing unit and the packing unit increase the availability of the system to significant level. The improvements in the availability of the system are 3.67% and 2.32 % on increasing the repair rate of proofing unit and packing unit from 0.33 to 0.73 and 0.25 to 1.25 repairs per hour respectively. Thus it can be concluded that the system availability can be improved by increasing the repair rates which in turn can be achieved by incorporating efficient corrective maintenance and preventive maintenance of different sub systems. The appropriate target value of the system availability can be focused and accordingly the repair rate can be set by plant management.

Critical Ranking of Sub-Components of the System

<table>
<thead>
<tr>
<th>Sub-Systems</th>
<th>Components</th>
<th>Increase in availability</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bread Baking</td>
<td>Proofing Unit</td>
<td>3.66%</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Baking unit</td>
<td>1.29%</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Cooling unit</td>
<td>0.64%</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Slicing unit</td>
<td>0.13%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Packing unit</td>
<td>2.32%</td>
<td>2</td>
</tr>
</tbody>
</table>

The constituent components are ranked in order of 1 to 5 corresponding to the increase in system availability on improving the respective repair rates in descending order. This information helps in prioritizing the repair activities as well as suggesting that redundancy can be introduced at first ranked unit for improving the availability of the system. Based on the above analysis,
maintenance/repair schedule can be prepared which might help the maintenance managers to improve the system effectiveness by adopting suitable preventive maintenance actions.

REFERENCES


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