Modal Analysis of Functionally Graded Material Plates by Experimentally and FEA.

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Abstract— The present work aims to carry out modal analysis of a functionally graded material (FGM) plate to determine its natural frequencies and mode shapes by using experimentally and FEA method. Functionally graded material can be differentiated by varying the composition and structure progressively over its volume, consequentially in corresponding changes in the material constituents. The mechanical properties of a FGM plate change continuously from one surface to another through its thickness direction. For modal analysis of FGM plate is studied with the help of ANSYS software. Some examples are solved, and the results are compared with those available in the literature. The mode shape and natural frequencies of rectangular FGM plate are found at different boundary conditions. It has been observed the effect of thickness ratio, which indicates the percentage of ceramic and metal composition in the FGM. In addition, the effects of crack on the FGM plate natural frequency and mode shapes with different boundary conditions are studied.

Index Terms— Boundary condition, functionally graded material, natural frequencies, mode shapes.

I. INTRODUCTION

The Modal analysis is a technique used to determine structure's vibration characteristics: Natural frequencies, and Mode shapes. These are most fundamental of all dynamic analysis types. Whereas technology development at an ever increasing rate, the need for advanced capability materials becomes a main concern in the modal analysis of engineering structures of more intricate and higher performance systems. These requirements can be seen in many fields in which engineers are exploring the applications of engineering materials. Functionally Graded Materials (FGMs) are a relatively new technology and are being studied for the use in components exposed to high temperature gradients. An FGM material property allows the designer to tailor material response to meet design criteria. An FGM composed of ceramic on the outside surface and metal on the inside surface eliminates the abrupt change between coefficients of thermal expansion, offers thermal protection, and provides load carrying capability. This is possible because the material constituents of an FGM changes gradually through-the-thickness; therefore, stress concentrations from abrupt changes in material properties are eliminated. The FGM plates has numerous applications in advanced engineering fields such as thin-walled structural components in space vehicles, nuclear reactors, and other high thermal application areas. The Functionally graded material (FGM) can be intended for specific function and applications. The material property such as young's modulus; density and poisons ratio values are varying continuously throughout the thickness direction.

II. LITERATURE REVIEW

This section includes the literature survey of earlier research work made by various researchers on functionally graded material. This section presents the summary of these research works.

M.K. Singha, et al. (2011) presented work on the nonlinear behaviors of functionally graded material (FGM) plates under transverse distributed load are investigated here using a high precision plate bending finite element. Material properties of the plate are assumed to be graded in the thickness direction according to a simple power-law distribution in terms of volume fractions of the constituents. The effective material properties are then evaluated based on the rule of mixture. The formulation is developed based on the first-order shear deformation theory considering the physical/exact neutral surface position. The shear correction factors are evaluated employing the energy equivalence principle. The transverse shear stresses and transverse normal stress components are obtained using the in-plane stresses evaluated from the constitutive equations and the three-dimensional equilibrium equations. Even for a thin FGM plate, the value of shear correction factor highly depends on distribution of material through the thick- ness of the plate. The transverse deflection depends on the in-plane fixity for the simply supported boundary condition whereas it is not so for the clamped boundary condition.

Ramu I, Mohanty S.C et al. (2014)the present work aims to carry out modal analysis of a functionally graded material (FGM) plate to determine its natural frequencies and mode shapes by using Finite Element Method (FEM).

Functionally graded material can be differentiated by varying the composition and structure progressively over its volume, consequentially in corresponding changes in the material constituents. The mechanical properties of a FGM plate change continuously from one surface to another through its thickness direction according to power law. Some examples are solved, and the results are compared with those available in the literature. The mode shape and natural frequencies of rectangular FGM plate are found at different boundary conditions. It has been observed the effect of volume fraction index, which indicates the percentage of ceramic and metal composition in the FGM. In addition, the effects of power law index on the FGM plate natural frequency and mode shapes with different boundary conditions are studied. Effects of power index ‘n’ and the four different boundary conditions on the natural frequencies are investigated. Natural frequencies for CCCC boundary conditions are the largest than the corresponding frequencies obtained for SCSC, SSSS and SFSF boundary conditions. Increasing the power law index value n reduces the first five natural frequencies for all the different boundary conditions. The effect of power law index is more prominent within the range up to 4, after which the effect is not that much significant.

R.C. Batraet et al.(2005) presented work on the first-order shear deformation theory (FSDT) coupled with the finite element method (FEM) to study free vibrations of a functionally graded (FG) anisotropic rectangular plate with the objective of maximizing one of its first five natural frequencies. The following edge conditions are considered: (i) all edges clamped, (ii) all edges simply supported, (iii) two opposite edges clamped and the other two free, and (iv) two opposite edges simply supported and the other two free. An advantage of a functionally graded plate over a laminated plate is that material properties vary continuously through the plate thickness. Thus no sudden discontinuities in stresses occur across an interface between any two adjoining laminae thereby eliminating the delamination mode of failure. Whereas there have been numerous works on studying the response of FG plates made of isotropic elastic constituents with the homogenized material also modeled as isotropic elastic the only other study on FG anisotropic plate has assumed that all elastic constants vary exponentially through the plate thickness at the same rate. It is highly unlikely that elastic moduli of a FG anisotropic plate will exhibit this property. Here we consider a FG anisotropic plate in which the fiber orientation varies smoothly through the plate thickness. That in-plane pure distortional mode of vibration are admissible in a simply supported laminated orthotropic plate only if the speeds of shear waves of the same amplitude and propagating in the same direction in every two adjoining laminae are equal.

A. Allahverdizadehetal et.al (2008) in this paper, a semi-analytical approach for nonlinear free and forced axisymmetric vibration of a thin circular functionally graded plate is developed. The plate thickness is constant. Functionally graded material (FGM) properties vary through the thickness of the plate. For harmonic vibrations, by using assumed-time-mode method and Kantorovichtime averaging technique, the governing equations are solved. Steady-state free and forced vibration analysis is investigated in detail and corresponding results at uniform ambient temperature are illustrated. Some of these results in special cases are verified by comparing with those in the literature. The results show that the free vibration frequencies are dependent on vibration amplitudes, and that the volume fraction index has a significant influence on the nonlinear response characteristics of the plate. The FGM properties vary through the constant thickness of the plate. The results of this study in special cases were compared with the response of free and forced vibration of a clamped circular metallic plate in linear and nonlinear cases. In the results, the influences of vibration amplitude, variation of Poisson’s ratio and volume fraction index have been examined. Several conclusions may be drawn from this study. It is considered that variation of volume fraction index is influential in FGM properties, dynamic treatment and the amount of stresses. The vibration frequencies are dependent on large vibration amplitudes, and for a clamped circular plate the effect of variation of Poisson’s ratio on the fundamental frequencies is negligible.

Xiaobai Li et al. (2016) presented work on a size-dependent Timoshenko beam model, which accounts for through-thickness power law variation of a two-constituent functionally graded (FG) material, is derived in the framework of the nonlocal strain gradient theory. The equations of motion and boundary conditions are deduced by employing the Hamilton principle. The model contains a material length scale parameter introduced to consider the significance of strain gradient stress field and a nonlocal parameter introduced to consider the significance of nonlocal elastic stress field. The influence of through-thickness power-law variation and size-dependent parameters on vibration is investigated. It is found that through-thickness grading of the FG material in the beam has a great effect on the natural frequencies and therefore can be used to control the natural frequencies. The vibration frequencies can generally increase with the increasing material length scale parameter or the decreasing nonlocal parameter. When the material characteristic parameter is smaller than the nonlocal parameter, the FG beam exerts a stiffness-softening effect. When the material characteristic parameter is larger than the nonlocal parameter, the FG beam exerts a stiffness-hardening effect. Through-thickness grading of the FG material in the beam has a significant effect on the natural frequencies and the natural frequencies can be controlled by choosing appropriate values of the power-law index. The effect of shear deformation on high-order frequencies is more significant than that on low-order frequencies. The vibration frequencies can generally increase with the increasing material length scale parameter or the decreasing nonlocal parameter.

Shyang-Ho Chi et al. (2006) studied on an elastic, rectangular, and simply supported, functionally graded material (FGM) plate of medium thickness subjected to transverse loading has been investigated. The Poisson's ratios of the FGM plates are assumed to be constant, but their Young's moduli vary continuously throughout the thickness direction according to the volume fraction of constituents defined by power-law, sigmoid, or exponential function. Based on the classical plate theory and Fourier series expansion, the series solutions of power-law FGM (simply called P-FGM), sigmoid FGM (S-FGM), and exponential FGM (E-FGM) plates are obtained. The analytical solutions of P-, S- and E-FGM plates are proved by the numerical results of finite element method reveal that the formulations of the solutions of FGM plates and homogeneous plates are similar, except the bending stiffness of plates. The bending stiffness of a homogeneous plate is Eh3/12(1 _ m2), while the expressions of the bending stiffness of FGM plates are more complicated combination of material properties.

Mohammad Rezaiee-Pajand et al. (2016) presented work on free vibration analysis of two different double-beam systems is presented in this article. In both systems, the ends of beams are elastically restrained against translation and rotation. Besides, the beams are interconnected via a mass–spring system. For the double-beam made up of an axially functionally graded beam and a homogeneous beam, a system of three differential equations, including a PDE with variable coefficients, a PDE with constant coefficients and an ODE with constant coefficients is solved. In the second case of two parallel axially functionally graded beams interconnected, a system of three differential equations, including two PDE with variable coefficients and an ODE with constant coefficients is solved. The behavior of the connecting mass–spring system is investigated in detail. Due to the complexity of the studied problem, two different methods are utilized to solve the problem. First, the problem is analytically solved, and closed-form solution is obtained. Second, a finite element solution is found. Comparing the results of both methods with the available ones in the literature shows the accuracy of the proposed methods. The effects of system parameters on the natural frequencies and mode shapes. It is shown that FE results are in excellent agreement with the exact ones due to using inhomogeneous elements in the FE analysis. The effects of system parameters such as springs' stiffness, suspended mass and gradient parameters are thoroughly examined.

Luan C. Trinh et al. (2016) presented work on an analytical method for vibration and buckling behaviours of Functionally Graded (FG) beams with various boundary conditions under mechanical and thermal loads is presented. Based on linear strain displacement relations, equations of motion and essential boundary conditions are derived from Hamilton's principle. In order to account for thermal effects, three cases of the temperature rise through the thickness, which are uniform, linear and nonlinear, are considered. The exact solutions are derived using the state space approach. Numerical results are presented to investigate the effects of boundary conditions, temperature distributions, material parameters and slenderness ratios on the critical temperatures, critical buckling loads, and natural frequencies as well as load-frequencies curves, temperature frequencies curves of FG beams under thermal/mechanical loads. The present model is found to be appropriate and efficient in analyzing the vibration and buckling of FG beams under mechanical/thermal loads.

Ramu Iet al. (2012) the present research work aims to determine the natural frequencies of an isotropic thin plate using Finite element method. The calculated frequencies have been compared with those obtained from exact Levy type solution. Based on this Kirchhoff plate theory, the stiffness and mass matrices are calculated using Finite Element Method (FEM). This methodology is useful for obtaining the natural frequencies of the considered rectangular plate. Numerical results obtained from FEM of the simply supported rectangular plates are giving close agreement with the exact solutions results. The approximate technique finite element method is used for analyzing a rectangular plate based on the Classical plate theory. In this paper free vibration analysis has been carried out for a rectangular plate. By varying the thickness of the plate, it has been concluded that the frequency parameter is constant.

Faruk Fırat Calim et al. (2016) the current study aims to analyze free and forced vibrations of axially functionally graded (AFG) Timoshenko beams on two-parameter viscoelastic foundation. In these kinds of beams, material properties alter through the axis. In the dynamic analysis, as research parameters, material inhomogeneity and foundation constants are considered. By using complementary functions method (CFM), in Laplace domain, differential equations are calculated. The calculations are performed in the Laplace domain. Calculations were transformed from Laplace domain to the time domain by applying Durbin's procedure. The free and forced vibrations of AFG Timoshenko beams on elastic/viscoelastic foundation are examined by solving different kinds of problems. It can be concluded that the material gradient index, spring constants, viscosity coefficients and various boundary conditions have remarkable influence on dynamic response of AFG beams on two parameter viscoelastic foundation.

Victor Birman et al. (2007) studied on functionally graded materials FGM are composite materials formed of two or more constituent phases with a continuously variable composition. FGMs possess a number of advantages that make them attractive in potential applications, including a potential reduction of in-plane and transverse through-the-thickness stresses, an improved residual stress distribution, enhanced thermal properties, higher fracture toughness, and reduced stress intensity factors. The studies considered in this section are concerned with stress, deformation, stability, and vibration problems of FGM beams, plates, and shells accounting for various effects, such as geometric and physical nonlinearity and transverse shear deformability. Material properties evaluated according to theoretical models often disagree with measured values of FGM constants. This indicates that a probabilistic approach to homogenization accounting for uncertainty in the actual material distribution throughout the volume may be justified. While there is a broad spectrum of successfully implemented FGM manufacturing techniques, there remains a need in the procedures and protocols that guarantee a reliable and predictable distribution of material constituent phases and properties throughout the structure.

Dong Wei et al. (2009) in this paper, an analytical method is proposed for solving the free vibration of cracked functionally graded material (FGM) beams with axial loading, rotary inertia and shear deformation. The governing differential equations of motion for an FGM beam are established and the corresponding solutions are found first. The discontinuity of rotation caused by the cracks is simulated by means of the rotational spring model. Based on the transfer matrix method, then the recurrence formula is developed to get the eigenvalue equations of free vibration of FGM beams. The main advantage of the proposed method is that the eigenvalue equation for vibrating beams with an arbitrary number of cracks can be conveniently determined from a third-order determinant. Due to the decrease in the determinant order as compared with previous methods, the developed method is simpler and more convenient to analytically solve the free vibration problem of cracked FGM beams. Moreover, free vibration analyses of the Euler–Bernoulli and Timoshenko beams with any number of cracks can be conducted using the unified procedure based on the developed method. These advantages of the proposed procedure would be more remarkable as the increase of the number of cracks. A comprehensive analysis is conducted to investigate the influences of the location and total number of cracks, material properties, axial load, inertia and end supports on the natural frequencies and vibration mode shapes of FGM beams.

III. EXPERIMENTAL ANALYSIS

A. Experimental setup of cantilever condition

An experimental setup is as shown in Figure A. First a plate of test material is to be fixed at one end. Impact hammer is used to disturb the frequency or to oscillate the plate. After impact the plate will be oscillated, so accelerometer sense data and signal generated by DAQ device. To calculate the natural frequency of the cantilever condition experimentally, experiment is conducted with the specified cantilever condition specimen to record the data of time history (Acceleration-Time), and FFT plot. The natural frequencies of the system can be obtained directly by observing the FFT plot. The location of peak values corresponds to the natural frequencies of the system. Same procedure is also used for simply supported and both end fixed boundary conditions.

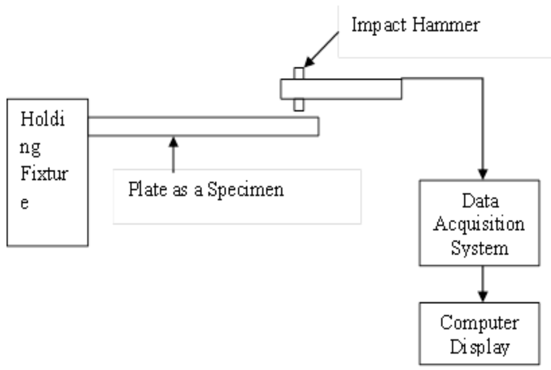

FIG .A. BLOCK DIAGRAM OF EXPERIMENTAL SETUP FOR CANTILEVER CONDITION

B. Dimension of Specimen

Functionally Graded Material Plates specimen is selected as composite specimen for test. The dimensions of all beams are considered as 250mm x 50mm x 3mm with different end conditions. The care taken that, all the specimens have same dimensions but with different thickness. The schematic representation of specimen is shown is a Fig.2.

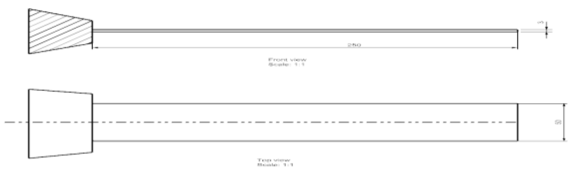

Fig.2 Dimension of Specimen (Plate)

Here, Length of the plate = 250 mm

Width of the plate = 50 mm

Thickness of the plate = 3 mm and 5 mm

C. Procedure for impact testing by using ADASH 4400VA PRO FFT

The basic procedure to conduct experimental impact test (BUMP test in ADASH 4400 VA PRO) is given below.

1. Fix the specimen with one end fixed, simply supported and both end fixed boundary conditions.
2. The connections of the impact hammer, vibration analyzer and user interference (computer) were properly made.
3. The FGM plate was struck with an impact hammer and is excited by means of it.
4. Perform the impact test by using Fast Fourier Transformation (FFT) on the cantilever beam to measure the frequency response.
5. Repeat the impact testing procedure and calculate natural frequency of the structure
6. Post process the modal data through the transfer function plots and take reading from response graphs which are in the user interface (computer).
7. Same procedure is also used for cracked specimen with same boundary condition.

D. Crack specimen

It is assumed that an FGM beam of length L and thickness h is shown in Fig. 3. An open crack with depth a, being perpendicular to the beam surface, is located at a distance L1 from the left end of the beam. The shear modulus G, Young's modulus E and mass density r of the beam only vary in the thickness direction and follow the exponential distributions.

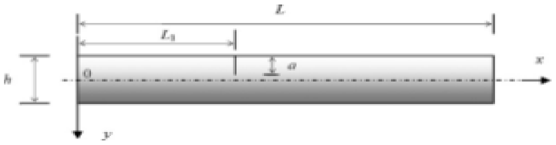

Fig. D. FGM plate with an open edge crack.

IV. NUMERICAL ANALYSIS

Simulations and computer aided design comes in to sort these things up. ANSYS is a general purpose of finite element modeling package for numerically solving a wide variety of mechanical problems.

A. Modal Analysis by Using ANSYS

The modal analysis is performed to find out the natural frequencies of a model. The following steps are involved to perform a modal analysis:

a. Set the analysis preference.

b. Create or import the geometry into ANSYS Workbench.

c. Define element attributes (element types, real constants and material properties).

d. Define meshing attributes.

e. Generate a mesh for the model.

f. Specifying the analysis type, analysis options, and applies loads.

g. Obtain the solution.

h. Review the results.

V. RESULT TABLE AND DISCUSSION

In this chapter natural frequencies of FGM plate without crack and with crack are studied experimentally as well as numerically. This chapter consists of the effect of different end conditions with varying thickness ratio along length and location of crack on the vibration behaviour of FGM plate is presented. For free vibration analysis, numerical results are compared with experimental results.

A. Natural frequency of FGM Plate

Here, the natural frequency along the length of FGM plate is analyzed. The variations in natural frequency with different end conditions along varying thickness and without crack are shown in the Table V.1.

TABLE.V.1 : NATURAL FREQUENCY OF FGM PLATE WITHOUT CRACK WITH DIFFERENT THICKNESS ALONG LENGTH

Sr. No.	**Boundary Conditions**	**Experimental Natural Frequency**			**FEA (ANSYS) Natural Frequency**		
		ω_1 (Hz)	**ω_2 (Hz)**	**ω_3 (Hz)**	**ω_1 (Hz)**	**ω_2 (Hz)**	**ω_3 (Hz)**
1.	One end fix 3mm	29.50	113	197	31.227	115.49	194.88
	5mm	49	187.5	320.5	51.98	189.44	322.01
2	Simply supported 3mm	135	239.5	349	136.26	238.73	348.12
	5mm	220.5	385	-	222.97	387.93	572.3
3.	Both end fix 3mm	129.5	214.5	346	127.01	216.61	348.32
	5mm	209	349.5	-	210.03	354.03	570.7

Here, the natural frequency along the length of FGM plate is analyzed. The variations in natural frequency with different end conditions along varying thickness and with crack are shown in the Table V.2.

TABLE.V.2 : NATURAL FREQUENCY OF FGM PLATE WITH CRACK AT ONE END WITH DIFFERENT THICKNESS ALONG LENGTH

Sr. No.	**Boundary Conditions**	**Experimental Natural Frequency**			**FEA (ANSYS) Natural Frequency**		
		ω_1 (Hz)	**ω_2 (Hz)**	**ω_3 (Hz)**	**ω_1 (Hz)**	**ω_2 (Hz)**	**ω_3 (Hz)**
1	One end fix 3mm	27.5	107.5	182	31.319	113.99	187.59
	5mm	53.50	192.5	306.5	52.125	187.52	309.47
2	Simply supported 3mm	118	213	311.5	125.11	219.88	314.52
	5mm	204.5	359.5	-	205.35	358.94	516.69
3	Both end fix 3mm	118.5	201.5	328.5	124.91	201.61	332.18
	5mm	206.5	328.5	-	206.16	330.3	543.33

Here, the natural frequency along the length of FGM plate is analyzed. The variations in natural frequency with different end conditions along varying thickness and with crack are shown in the Table V.3.

TABLE.V.3 : NATURAL FREQUENCY OF FGM PLATE WITH CRACK AT BOTH ENDS WITH DIFFERENT THICKNESS ALONG LENGTH

Sr. No.	**Boundary Conditions**	**Experimental Natural Frequency**			**FEA (ANSYS) Natural Frequency**		
		ω_1 (Hz)	**ω_2 (Hz)**	**ω_3 (Hz)**	**ω_1 (Hz)**	**ω_2 (Hz)**	**ω_3 (Hz)**
1	One end fix 3mm	24.5	105.5	180.5	27.721	105.11	183.46
	5mm	45	175	284	46.114	172.77	283.56
2	Simply supported 3mm	113	201.5	286.5	116.07	197.93	288.68
	5mm	190	320.5	-	191.57	323.94	475.82
3	Both end fix 3mm	120.5	182.5	319.5	123.25	185.61	321.72
	5mm	201.5	301.5	-	203.37	304.36	526.6

B. Mode Shapes

Following are results of ANSYS are only for one end fixed boundary condition.

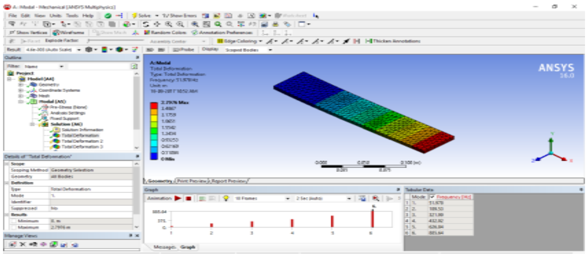

Fig.B.1. First mode shape

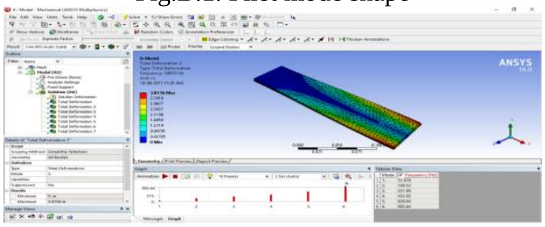

Fig.B.2. Second mode shape

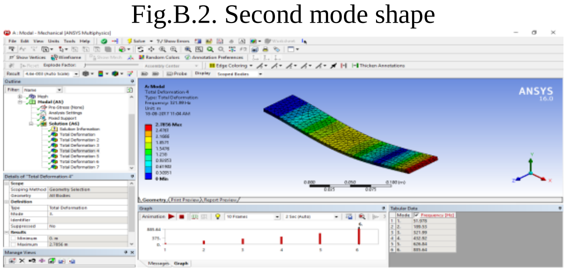

Fig.B.3. Third mode shape

VI. CONCLUSION

Based on the experimental and numerical results following conclusions can be drawn:

1. The natural frequencies of FGM plate for different boundary conditions have been reported. Results obtained from experimental analysis shows very good agreement with numerical analysis.

2. The FGM plate has highest magnitude of frequency for fixed Boundary condition, lowest magnitude of frequency for cantilever boundary condition, and in between simply supported plate lies.

3. The % error in frequency is higher for fixed Boundary condition compared to simply supported Boundary condition followed by cantilever boundary conditions.

4. Mode shapes were plotted for differently boundary conditions of FGM plate with the help of FFT and ANSYS, to get exact idea of mode shape. Vibration analysis of FGM plate was also done on FFT to get natural frequency.

5. As the number of crack increases, therefore reduction in natural frequency also increases.

6. Along the boundary conditions of the FGM plate thickness is increased therefore results in increasing natural frequencies.

7. As the location of the crack changes there is a change in the natural frequency of FGM plate.

8. As the varying thickness ratio along the length of FGM plate changes the natural frequency of FGM plate increases.

9. The use of finite element analysis tool ANSYS is a successful tool to investigate the effect of crack on the natural frequency of FGM plate.

10. Results obtained from experimental analysis shows very good agreement with numerical analysis.

11. Natural frequencies for both end clamped boundary condition are the largest than the corresponding frequencies obtained for simply supported and cantilever boundary conditions.

12. As the size of crack increases, therefore reduction in natural frequency also increases.

13. Along the boundary conditions in the FGM plate thickness is increased therefore results in increasing natural frequencies.

14. As the location of the crack changes there is a slight change in the natural frequency of FGM plate.

15. As the varying thickness ratio along the length of FGM plate changes the natural frequency increases.

16. The use of finite element analysis tool ANSYS is a successful tool to investigate the effect of crack on the natural frequency of FGM plate.

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