Abstract – A signal processing technique of efficiently acquiring and reconstructing a signal, from its sparse coefficients by finding solutions to under determined linear systems is termed as Compressed Sensing. This takes advantage of the signal’s sparseness or compressibility in some domain, allowing the entire signal to be determined from relatively few measurements/coefficients. Compressed sensing takes advantage of the redundancy in signals which are not pure noise. Many natural signals have got sparse representation when expressed in some transform domain or basis, that is, they contain many coefficients close to or equal to zero. Different basis functions like DCT or DFT or wavelets are used to obtain the sparse representation of the UWB signals. This paper contains the study of recovery of ultra wide band signals from its sparse representation using $L_2$ minimization algorithm.

Keywords – Ultra wide band(UWB) signal, Compressed Sensing (CS), Sparsity, In-coherence, Signal representation, Signal recovery, $L_2$ minimization Algorithm.

I. INTRODUCTION

The Nyquist-Shannon sampling theorem states that signals, images, videos, and other data can be exactly recovered from a set of uniformly spaced samples taken at the so called Nyquist rate of twice the highest frequency present in the signal of interest. Capitalizing on this discovery, much of signal processing has moved from the analog to the digital domain. Digitization has enabled the creation of sensing and processing systems that are more robust, flexible, cheaper and, consequently, more widely used than their analog counterparts. Unfortunately, in many important and emerging applications, the resulting Nyquist rate is so high that we end up with far too many samples. Here are considering the ultra wide band signal whose signal bandwidth is greater than 500 MHz or whose fractional band width is larger than 20 % and the radiated power cannot exceed - 43.3 dBm/MHz according to the Federal communication commission (FCC) [1]. UWB transmissions transmit information by generating radio energy at specific time instants with the shape of a pulse. These pulses are on the order of nanoseconds and are used as the elementary pulse shaping to carry the information. As these wide bandwidth signals requires greater computing resources, processing power if sampled at Nyquist rate[12]. Therefore, if the signal to be sampled has got sparse or compressible representation in certain transform domain or basis, then the signal can be represented using only a small amount of data or using fewer non zero coefficients which spans the entire band of the signal. A certain minimum number of samples is required in order to perfectly capture an arbitrary band limited signal, when the signal is sparse in a known basis thereby vastly reducing the number of measurements that is needed for signal acquisition. This is the fundamental idea behind CS rather than first sampling at a high rate and then compressing the sampled data. The data is directly sensed in a compressed form i.e., at a lower sampling rate. A finite dimensional signal having a sparse or compressible representation can be recovered from a small set of linear, non-adaptive measurements [6][7]. Here the ultra wide band signals is partially recovered from its sparse representation using $L_2$ minimization algorithm.

II. COMPRESSED SENSING (CS)

Compressed sensing (CS) is a signal processing technique for efficiently acquiring and reconstructing a signal, by finding solutions to underdetermined linear systems[11]. The entire signal can be determined from relatively few measurements by taking advantage of the signal’s sparseness or compressibility in certain transform domain or basis function like Discrete Cosine Transform (DCT), Discrete Fourier Transform (DFT), Wavelet etc; and the recovering these sparse signal using $L_2$ minimization algorithm by solving the under
determined linear systems (i.e.; the number of unknown coefficients are more than the number of equations (or measurements)).

Compressed sensing typically starts with taking a weighted linear combination of samples also called compressive measurements in a basis (sensing basis) different from the basis (known as representation basis) in which the signal is known to be sparse. The results showed that the number of these compressive measurements can be small and still contain nearly all the useful information [2][3]. Therefore, the task of converting the signal back into the intended domain involves solving an underdetermined matrix equation since the number of compressive measurements taken is smaller than the length of the signal. However, adding the constraint that the initial signal is sparse enables one to solve this underdetermined system.

2.1 Sparsity

Sparsity Signals can often be well-approximated as a linear combination of just a few elements from a known basis or dictionary. When this representation is exact we say that the signal is sparse[3][4]. Sparse signal models provide a mathematical framework for capturing the fact that in many cases these high-dimensional signals contain relatively little information compared to their ambient dimension. Signal processing has focused on signals produced by physical systems. Many natural and man-made systems can be modelled as linear. Thus, it is natural to consider signal models that complement this kind of linear structure. A signal \( f(t) \) is obtained by linear functional recording the values \( \phi_k(t) \) that complement this kind of linear structure. A signal \( f(t) \) is obtained by linear functional recording the values \( \phi_k(t) \) by simply correlating the object which has to be acquired with the waveforms \( \phi_k(t) \).

\[
x = \Psi C. \quad \text{where} \quad \|C\|_0 \leq K \tag{1}
\]

A vector \( f \in R_n \) (such as the N-dimensional signal in Fig.1) which is expanded in an orthonormal basis (such as a DFT/DCT basis). \( \Psi = [\Psi_1, \Psi_2, \Psi_3, ..., \Psi_N] \).

\[
f(t) = \sum_{i=1}^{N} x_i \Psi_i(t) \tag{2}
\]

where \( x \) is the coefficient sequence of \( f \), \( x_i=\langle f, \Psi_i \rangle \). It will be convenient to express \( f \) as \( x \) (where \( \Psi_i(t) \) is the \( N \times N \) matrix with \( \Psi_1, \Psi_2, \Psi_3, ..., \Psi_N \) as columns). A DCT expresses a sequence of finitely many data points in terms of a sum of cosine functions oscillating at different frequencies where small high-frequency components can be discarded to spectral methods for the numerical solution of partial differential equations. For compression, it turns out that cosine functions are much more efficient as fewer functions are needed to approximate a typical signal. DCT is an standard orthogonal transformation basis, it also belongs to a family of 16 trigonometric transformations.

\[
y(k) = w(k) \sum_{n=1}^{N} x(n) \cos \left( \frac{\pi(2n-1)(k-1)}{2N} \right) \tag{3}
\]

where \( k = 1, 2, ..., N \) and

\[
w(k) = \begin{cases} \frac{1}{\sqrt{N}} & k = 1 \\ \frac{\sqrt{2}}{N} & 2 \leq k \leq N \end{cases}
\]

\( N \) is the length of \( x \) and \( x \) and \( y \) are the same size. The series is indexed from \( n = 0 \) and \( k = 0 \). The DCT basis can be computed using the transformation kernel, which is the same for both forward DCT and inverse discrete cosine transformations (IDCT) and is given below by Eq 4.

\[
x(n) = \sum_{k=1}^{N} w(k)y(k) \cos \left( \frac{\pi(2n-1)(k-1)}{2N} \right) \tag{4}
\]

where \( n = 1, 2, ..., N \) and

\[
w(k) = \begin{cases} \frac{1}{\sqrt{N}} & k = 1 \\ \frac{\sqrt{2}}{N} & 2 \leq k \leq N \end{cases}
\]

and \( N = \text{length}(x) \), which is the same as \( \text{length}(y) \). The series is indexed from \( n = 1 \) and \( k = 1 \). When a signal has a sparse expansion, one can discard the small coefficients without much perceptual loss. Consider \( f(t) \) obtained by keeping only the terms corresponding to the \( S \) largest values of \( x \) in the expansion. This vector is sparse in a strict sense since all but a few of its entries are zero; we will call \( S \)-sparse such objects with at most \( S \) non-zero entries. In plain terms, one can throw away a large fraction of the coefficients without much perceptual loss. When measuring the approximation error using an ‘p’ norm [10], this procedure yields the better approximation of the original signal using only basis elements.

2.2 Incoherent Sampling

Suppose there is given a pair of ortho bases of \( R_n \), the first basis is used for sensing the object \( f \) as in Eq 1 and the second is used to represent \( f \). The coherence between the sensing basis and the representation basis
measures the largest correlation between any two elements and if contain correlated elements, the coherence is large. Otherwise, it is small. Compressive sampling is mainly concerned with low coherence pairs, in this example $\psi$ is the DCT basis.

Finally, random matrices are largely incoherent with any fixed basis $\psi$. Select an ortho basis function uniformly at random, which can be done by ortho normalizing $N$ vectors sampled independently and uniformly on the unit sphere. Then with high probability, the coherence between representation basis and sensing basis is about $\sqrt{2\log(n)}$. Random waveforms with independent identically distributed (i.i.d) entries, eg., Gaussian or ±1 binary entries, will also exhibit a very low coherence with any fixed representation $\psi$.

III. SIGNAL RECOVERY USING $L_2$ MINIMIZATION ALGORITHM

The reconstruction of sparse vectors, require a way to quantify the sparsity of a vector[5]. A d-dimensional signal $x$ is s-sparse if it has $S$ or fewer non-zero coordinates. Sampling the signal involves a linear operator, $b = \Phi f$. In the compressed signal is a vector $b$ of $m$ random samples of the original signal. A measurement matrix $A$ is constructed by extracting $m$ rows from the $N \times N$ DCT matrix, where $k$ is the vector of indices used for the sample $b$. The resulting linear system, $Ax = b$, is $m \times n$, which is $500 \times 5000$. There are 10 times as many unknowns as equations. If the expansion of the original signal as a linear combination of the selected basis functions has $m$ any zero coefficients, then its often possible to reconstruct the signal exactly which involve counting non-zero with $L_0$ Norm[10]. This is a combinatorial problem whose computational complexity makes it impractical[Its NP-hard][8] and $L_0$ can be replaced by $L_1$ norm [9], from that with overwhelming probability of the two problems have the same solution. Here it is using $L_2$ norm computation algorithm instead of the $L1$ norm computation because it can be posed as a linear programming problem and solved with the traditional simple algorithm. $L_1$ Norm minimization involves linear programming problem which involves modern interior point methods gives accurate result.

IV. RESULTS AND DISCUSSION

A raw signal of UWB frequency is represented in figure 1 based on following equation 5 $f = \sin(2\pi \times 2 \times 10^9 t) + \sin(2\pi \times 3 \times 10^9 t)$

The UWB signal can be represented as a vector $f$ with millions of components of $f$ using a linear combination of certain basis functions:

$$f = \Psi c$$

where $f = \sin(2\pi \times 2 \times 10^9 t) + \sin(2\pi \times 3 \times 10^9 t)$ and is a DCT basis function.

The basis functions must be suited to a particular application. In this example, $\Psi$ is the discrete cosine transform. It is assumed that, most of the coefficients $c$ are effectively zero so that $c$ is sparse. The Figure 2 shows the coefficients $c$, obtained by taking the inverse discrete cosine transform of $f$. Because the frequencies are incommensurate, this signal does not fall exactly within the space spanned by the DCT basis functions, and so there are a few significant non-zero coefficients.
Fig. 3: Sparse representation of recovered signal

Here this paper is using $L_2$ norm computation algorithm instead of the $L_1$ norm computation because it involves modern interior point methods. The recovered sparse signal is shown is figure 3 along with the recovered original signal using $L_2$ minimization algorithm as shown is figure 4.

Fig. 4: Recovered UWB signal

V. CONCLUSION

Compressed sensing is a new and fast growing eld of signal processing that addresses the short coming of conventional signal compression. Given a signal with few non-zero co-ordinates relative to its dimension, compressed sensing seeks to reconstruct the signal from few measurements. In this paper the signal of frequency is represented in its sparse form using DCT and then recovered partially using $L_2$ Minimization algorithm.

VI. REFERENCES


