Comparison between Interval and Fuzzy Load Flow Methods Considering Uncertainty

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Abstract— This paper presents a power flow algorithm to carry out load flow by analysing uncertainties. The proposed algorithm is compared with Fuzzy arithmetic to deal the effect of uncertainty. The proposed method is Interval load flow i.e. Interval Improved Fast Decoupled Power Flow (IIFDPF) that considers input parameters as interval variables and considered method is trapezoidal method have been employed to solve the uncertainties in load flow analysis where LR (left-right) fuzzy arithmetic have been applied to model the uncertainties. Uncertain parameters are treated as fuzzy numbers with given membership functions. Both techniques are tested on a typical Load flow solution, considering loads and generator are as uncertain. In order to provide a basis for comparison between above two approaches the shapes of the membership function used in fuzzy L-R as inputs and same range of uncertainty consider in interval method.

Interval method provides better output results compared to Fuzzy L-R method. The IIFDPF algorithm is validate with INTLAB toolbox /MATLAB environment using the IEEE standard test data.

Keywords— Interval arithmetic, Interval Improved Fast Decoupled, Interval Newton’s method, Fuzzy load flow

I. INTRODUCTION

Most important task in front of power engineers is to obtain solution methods for uncertainty in the planning and operation of power systems. As a basic tool to power system analysis, power flow calculation encounters the difficulty in dealing with uncertainty.

At present, there are three kinds of algorithms of power flow calculation considering uncertainty in power systems:

1) Fuzzy power flow algorithm using fuzzy numbers to express uncertainty and calculate on the basis of fuzzy theory.

2) Probability power flow algorithm using probability theory to deal with the uncertainty.

3) Interval algorithm using interval number and interval arithmetic to process uncertainty.

A. Interval Arithmetic Analysis:

Interval method, the input variables are defined as interval variables. An interval variable is a closed set of real numbers [x1, x2], such that any x in the interval belongs to the set. All numbers in the interval are equally privileged.

The power flow problem consists of a system of nonlinear equations, which is solved by the use of iterative methods. The problem reduces to the solution of a system of linear equations for each iteration, and this can be achieved by the use of interval mathematics. Three popular iterative operators for the solution of the interval non-linear equations are the Newton operator, the Krawczyk operator and the Hansen-Sengupta operator [1-3]. The solution set has a very complex, non-convex structure and can’t be characterized as an interval. For this reason, it is generally preferred to work with the hull of the solution set, where the hull is the smallest interval vector containing the solution set. So, solving the interval linear equations means that we need to obtain the hull of the solution set. As it was in international literature [4], where this method was validated against Monte Carlo simulations and Stochastic Power Flow analysis, interval methods have proven computationally superior to Monte Carlo simulations and Stochastic Power Flows.

B. Fuzzy Arithmetic (Possibilistic) Analysis:

Qualitative uncertainty is initially expressed in vague, nonnumeric (usually verbal) terms such as “approximately equal to” or “a small percentage”. By using the concept of “degree of membership” of a value to a set, it is possible to establish the notion of fuzzy sets and fuzzy arithmetic. Qualitative uncertainty is quantified using fuzzy sets [5, 6]. The notion of fuzzy methods originates from an extension of the notion of a set, where membership in a set is permitted to be something other than a binary variable (member/non-member). A set membership function (smf) \( p(x) \) is defined so that a value \( 0 \leq p(x) \leq 1 \) denotes the belief in the possibility that \( x \) belongs to the set. This is the idea used in formal mathematics in order to describe vague concepts. The extreme values 1 and 0 are used to denote that \( x \) belongs to the set or not. Many applications of this methodology can be found in the literature on power systems planning [7] but also for the solution of the LF problem [8].

Probabilistic Analysis:
In this methodology, the input variables are converted to random variables (r.v.), with a known probability density function (pdf) [9]. The results are not fixed numbers, but pdfs showing the possible range of the result quantities and the corresponding probability of each value to occurrence.

This paper proposes an Interval Improved Fast Decoupled Power Flow (IIFDPF) algorithm under data uncertainty, which introduces interval arithmetic into traditional fast decoupled power flow used in power systems. In this proposed algorithm, interval numbers are used to express the uncertainty. Load and generator data due to measurement error.

IIFDPF algorithm leads to solve the interval power flow method and the resulting non linear model is solved by using interval Newton's method and two sets of linear interval equations i.e. the decoupled P equations and Q equations. Based upon different strategies of updating the voltage, angle and the bus voltage in each iteration, the strategic solution is obtained by saving computing time and faster convergence and better accuracy and reliable in consideration of uncertainty. The proposed algorithm has been applied to IEEE-5, IEEE-9 bus system. The implementation was performed in MATLAB environment, using intlab toolbox [10]. Results are compared with fuzzy trapezoidal method employed to solve the uncertainties in load flow analysis using LR (left-right) fuzzy arithmetic.

II. INTERVAL ARITHMETIC AND MATLAB TOOLBOX INTLAB:-

The concept of interval analysis is to compute with intervals of real numbers in place of real numbers. While floating point arithmetic is affected by rounding errors, and can produce inaccurate results, interval arithmetic has the advantage of giving rigorous bounds for the exact solution. An application is when some parameters are not known exactly but are known to lie within a certain interval algorithms may be implemented using interval arithmetic with uncertain parameters as intervals to produce an interval that bounds all possible results.

A real interval $x$ is a nonempty set of real numbers

$$x = [\underline{x}, \bar{x}] = \{x \in \mathbb{R} : \underline{x} \leq x \leq \bar{x}\};$$

Where $\underline{x}$ is called the infimum and $\bar{x}$ is called the supremum. The set of all intervals over $\mathbb{R}$ is denoted by $\text{IR}$ where

$$\text{IR} = \{[\underline{x}, \bar{x}] : \underline{x}, \bar{x} \in \mathbb{R} : \underline{x} \leq \bar{x}\};$$

The midpoint of $x$, $\text{mid}(x) = \frac{1}{2}(\underline{x} + \bar{x})$

And the radius of $x$, $\text{rad}(x) = \frac{1}{2}(\bar{x} - \underline{x})$

Interval arithmetic operations are defined such that the interval results enclose all possible real results. Which guarantees reliability of interval method. Given $x=[\underline{x}, \bar{x}]$ and $y=[\underline{y}, \bar{y}]$ the four elementary operations are defined by

\[
x \text{ op } y = [x \text{ op } y : x \in x, y \in y]\text{ for op}{\{+ , - , \times, \div\}.}
\]

They are given by

\[
x + y = [\underline{x} + \underline{y}, \bar{x} + \bar{y}]
\]

\[
x - y = [\underline{x} - \underline{y}, \bar{x} - \bar{y}]
\]

\[
x \times y = \left[\min\{\underline{x} \underline{y}, \underline{x} \bar{y}, \bar{x} \underline{y}, \bar{x} \bar{y}\}, \max\{\underline{x} \underline{y}, \underline{x} \bar{y}, \bar{x} \underline{y}, \bar{x} \bar{y}\}\right]
\]

\[
1/x = \left[1/\underline{x}, 1/\bar{x}\right] \text{ if } \underline{x} > 0 \text{ or } \bar{x} < 0,
\]

\[
x + y = x \times 1/ y.
\]

An interval Newton method has been developed for solving systems of nonlinear equations. While inheriting the local quadratic convergence properties of the ordinary Newton method, the interval Newton method can be used in an algorithm that is mathematically guaranteed to find all roots within a given starting interval.

We consider the problem of solving nonlinear system of equations using the interval Newton's method. The problem is to find bounds on the solution of nonlinear continuous function

\[
f : \mathbb{R}^n \rightarrow \mathbb{R}^n \text{ in a given box } X^{(0)} \in \mathbb{I}\mathbb{R}^n.
\]

Using the mean value theorem

We have, for any $x'$, that

\[
f(x') \in f(\bar{x}) + J(X)(x' - \bar{x})
\]

Where $J(X)$ is the interval Jacobian matrix, and $\bar{x} \in X$.

If $x'$ is a zero of $f$ then $f(x') = 0$ and, therefore,

\[
-f(\bar{x}) \in J(X)(x' - \bar{x})
\]

This interval linear system can then be solved for $x'$ to obtain an outer bound on the solutions set, say $N(\bar{x}, x)$.

This gives
\[ 0 = f(\hat{x}) + J(\hat{x}) N(\bar{x}, x) - \ddot{x} \]

It gives into the following iteration

\[
solve \text{ for } N
\]

\[
f(\hat{x}^{(k)}) + J(\hat{x}^{(k)}) N(\bar{x}^{(k)}, \hat{x}^{(k)}) - \ddot{x}^{(k)} = 0
\]

\[
\hat{x}^{(k+1)} = \hat{x}^{(k)} \cap N(\bar{x}^{(k)}, \hat{x}^{(k)})
\]

\[
\ddot{x}^{(k)} \text{ is the mid value.}
\]

Linear equation can solved as to give newtons operator

\[
N(\bar{x}, x) = \ddot{x} - J(\bar{x})^{-1} f(\bar{x})
\]

We use the following notation \( \Delta x = -J(\bar{x})^{-1} f(\bar{x}) \) that results in the iteration

\[
\Delta x^{(k)} = - J(\hat{x}^{(k)})^{-1} f(\hat{x}^{(k)})
\]

\[
N(\bar{x}^{(k)}, \hat{x}^{(k)}) = \hat{x}^{(k)} + \Delta x^{(k)}
\]

\[
\hat{x}^{(k+1)} = \hat{x}^{(k)} \cap N(\bar{x}^{(k)}, \hat{x}^{(k)})
\]

Various interval methods are determined by how

\[
N(\bar{x}^{(k)}, \hat{x}^{(k)}) \text{is defined. The interval linear system can be solved using interval methods (e.g., Interval Gaussian elimination, Krawczyk operator and the Hansen-Sengupta operator, interval Gauss-Seidel iteration etc.) to give the operator}
\]

\[
N(\bar{x}, x) = \ddot{x} - J(\bar{x})^{-1} f(\bar{x})
\]

At each iteration, an interval linear system has to solved. Intlab provides the command "\" for solving interval linear systems, combining the iterative method above described. In this paper we decided to use Newton’s method to avoid preconditioning other problem of convergence usually faced as discussed in section I [13].

### III. INTERVAL IMPROVED FAST DECOUPLED POWER FLOW (IIFDPF) UNDER UNCERTAINTY:

All loads and generator bus data in the electric system are provided by measurement instruments which frequently are inaccurate. Moreover, the specified variables like real power at PV buses also can be uncertain since their values are obtained via measurement equipment. This uncertainty in the input data can be enlarged due to both rounding and truncating processes that occur in numerical computation. As a consequence the actual error presented in the final results cannot be easily evaluated. In order to rigorously control and automatically handle these numerical errors. This paper proposes to apply techniques of Interval Arithmetic for a more reliable load and generator modelling.

As the Fast Decoupled power flow method (FDPFM) is one of the improved method, which was based on a simplification of the Newton –Raphson’s method and reported by Stott and Alsac in 1974. This method answer due to its calculation simplifications, fast convergence and reliable results becomes widely used in power flow analysis. After simplifications and assumptions of NR method, equations (1) and (2) we get:

The basic idea of the Interval Improved Fast Decoupled Power Flow using interval arithmetic being proposed in this paper is as follows. Firstly, use interval numbers to express the uncertain variables in power system such as the uncertainty of all the load bus (PQ bus i.e. \( P_l \) and \( Q_l \)) and generator bus (PV bus i.e. \( P_g \) and \( Q_g \)). Then, the mathematical model of the Improved Fast Decoupled Power Flow using interval arithmetic is built.

\[
\frac{\Delta P}{|V|} = - B \ ' \Delta \delta \quad \ldots (1)
\]

\[
\frac{\Delta Q}{|V|} = - B " \Delta |V| \quad \ldots (2)
\]

The above two equations, the branch parameter matrices are interval matrices and state variable vectors are interval vectors, which mean that the elements of them are mostly interval numbers.

In the operation of actual power system, the influence of parameter uncertainty of electric lines and transformers factor is often small enough to be neglected. So the equations (3) & (4) can be simplified as below:

\[
\Delta \delta = - |B'| \ \left\{ \frac{\Delta P}{|V|} \right\} \quad \ldots (3)
\]

\[
\Delta V = - |B'| \ \left\{ \frac{\Delta Q}{|V|} \right\} \quad \ldots (4)
\]

IIFDPF algorithm leads to solve the interval power flow method and the resulting non linear model is solved by using interval Newton’s method and two sets of linear interval equations i.e. the Decoupled P equations and Q equations (3) and (4). Based upon different strategies of updating the voltage angle and the bus voltage in each iteration was based on the weak coupling between \( \Delta P \) and \( \Delta V \) and between \( \Delta Q \) and \( \Delta \delta \). In Fast decoupled instead of updating the voltage magnitude and the voltage angle once and simultaneously in each iteration, IIFDPF algorithm updated either the voltage angle on the voltage magnitude at each bus, to recalculate the real and reactive power and then updated the second variable based on what was updating first. Moreover for speed improvements and convergence reliability, the update of one of the two variables repeated several times, holding the other variable at its last calculated value, which reduces the number of floating point operations of the algorithm and thus lead to the faster convergence.
IV. NUMERICAL RESULTS:

To demonstrate the performance of proposed method, an IEEE-5 and IEEE-9 bus test systems are consider. To evaluate algorithm the uncertainty considered load values at PQ buses and generation data considered at all PV buses. Uncertainty representation will be represented and modelled by implementing the fuzzy numbers as Trapezoidal Fuzzy Membership Function. The input variables which are active and reactive loads and generations are varied in the same percentages for the simplicity as follow [11]:

\[
[0.90P, 0.975P, P, 1.025P, 1.1P] \\
[0.90Q, 0.975Q, Q, 1.025Q, 1.1Q]
\]

Where P and Q are the injected powers.

![Fig 1 IIFDPF method - interval Voltage IEEE-5](image)

The solution obtained by the algorithm is shown in Fig 1 and 4. The results of interval method encloses the results of traditional crisp method, when the parameters being set the midpoint of given interval. This demonstrates the validity of the proposed method. It satisfies the rule based on interval arithmetic. The Interval Fast Decoupled method and Interval Improved Fast Decoupled Power Flow method (IIFDPF) is validated. Above results are compared with fuzzy load flow, in proposed method and considered method same range of uncertainty taken modelling.

![Fig 2 Bus voltages in Interval and fuzzy IEEE-5](image)

![Fig 3 Interval vs Fuzzy width of interval- IEEE-5](image)
Fig 4 Interval Voltage IEEE - 9 BUS

From the fig.2 and fig. 3 it is observed that width of interval is small in comparison with fuzzy method. In IIFDPF method avoid drawbacks preconditioning and complex and time consuming in interval modelling [13].Two Iteration are necessary in order to obtain the convergence of iteration process. Interval Newton’s method is simple in comparison with other Interval methods is discussed. The proposed method is successfully tested with IEEE-14bus. The proposed method is proved to be a fast convergent algorithm.

V. CONCLUSION

This paper has presented a study based on IIFDPF method applied to the power flow tool in electrical power system. The main goal of the work was to evaluate the influence of measurement error on the voltage profile of energy network and compare with fuzzy method. The proposed algorithm is fast converging and takes the consideration of retaining the mid-point of the load flow studies and the conclusion for the output results and simulation which have been conducted, it shows that interval results are much more accurate, precise and width of IIFDPF method is small than the fuzzy load flow method. IIFDPF method has been validated against fuzzy method, results are enclosed the crisp values.

The proposed method can deal with uncertain input data in power flow problems. IIFDPF method has computationally superior to fuzzy arithmetic.

REFERENCES


